DC-1

Semester-II

Paper-IV: Mathematical Methods in Economics-II

Lesson: Constrained Optimization

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Learning outcomes:

After you have read this chapter, you should be able to:-

1. Explain the importance and relevance of constraint.
2. Differentiate between free and constrained optimum.
3. Solve problems of constrained optimization in economics.
4. Define parameters.
5. Interpret lagrangian multiplier.

Introduction

In the last chapter, we covered optimization if objective function of two or more choice variable. But that optimization was unconstrained. It was unconstrained in the sense that for example, in case of discriminating monopolist; there was no restriction or limit to what level of output to be produced. But there could have been constraints, that given the level of technique, & machinery, the total maximum output that could be produced is suppose; 1000 units. In such a case, optimized maximum may differ if value of extrema is greater than provided this constraint.

In this chapter, we will cover optimization with equality constraints. The new optimum referred here is constrained optimum, which is likely to differ from free optimum.

This chapter is divided into sections. In the first section, we will analyze geometric properties of constraint.

1. Geometrically analyzing constraint

The primary purpose of imposing a constraint is to give due cognizance to certain limiting factors present in the optimization problem under discussion. In the last chapter, we saw hills and valleys in 2D and bowls and domes in 3D and found relative extrema in all such cases. But there were no constraints.

Let us take example of a consumer who consumes only good & has utility function:

$U(x) = -(x-2)^2+4$

Its free optimum (maximum) is at x=2. But if government imposes some restriction that no one can consume more than 1 unit of x then constrained optimum is at x=1 .This is shown in fig 1 below.

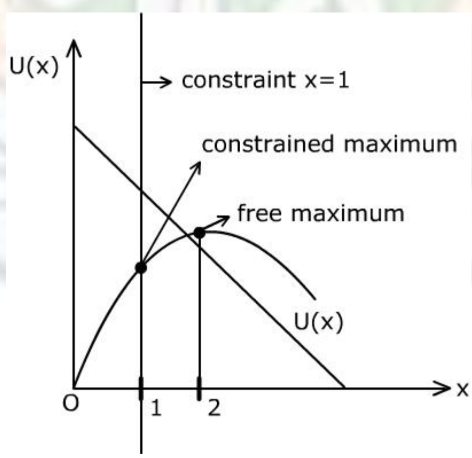

Fig.1

Let us consider another example that utility of a consumer depends on two goods, x_1 & x_2, with utility function as follows:-

$U = x_1x_2+2x_1$

If one finds, partial derivative of U then one would find that marginal utilities are positive and increasing function of x_1 & x_2. Hence, an unconstrained optimum would give result for purchasing infinite amount of goods. But a consumer has constraint known as budget constraint. If price of x_1 is Rs. 4 and price of x_2 is Rs.2; income of consumer is Rs.100 then the constraint becomes:

$4x_1+2x_2=100$

This constraint now narrows down the choice of x_1 & x_2 & one can find optimum x_1& x_2.

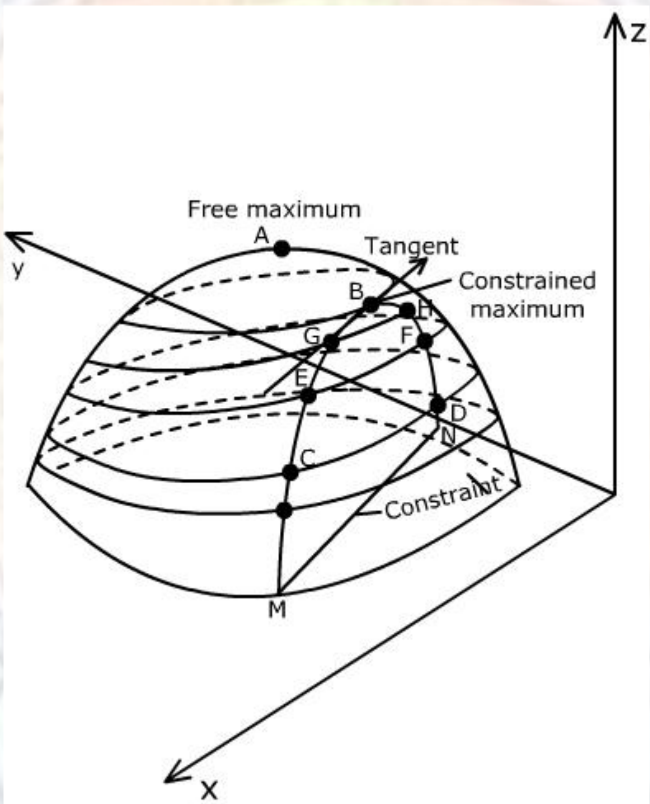

Fig.2

If one considers a general function z = f(x,y) and assumes it appears like a dome in 3D then free extremum is peak of the dome but constrained extremum is at the peak of u-shaped curve situated on top of the constraint. In fig.2, MN is constraint line indicating that sum of x & y cannot go beyond this line. Then constrained maxmium is at point B.

2. Algebrically setting constant into optimization equations.

All the points viz. C,D,E,F,G & H are feasible (Infact, the entire section of the bowl in the right hand side of constraint is feasible section). In Fig.3, various level curves of the function Z=f(x,y) are drawn. It is 2D projection of a 3D dome. M N is the constraint on sum of X & Y say of the form g(x,y) =c.

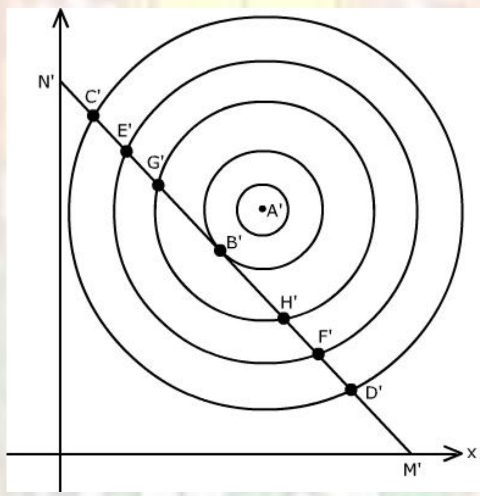

Fig.3

A' in fig.3 corresponds to A in fig.2 & is unconstrained maximum. B' (corresponding to B in fig.2) is constrained maximum. In Fig.3, at B', slope of the level curve f(x,y) = z_1, is equal to the slope of constraint g(x,y) =c.

Free & constrained maximum

A constrained maximum can be expected to have a lower value than the free maximum. It could also be that free optimum is also constrained maximum in which a case, constraint is not binding. If constraint is binding, which generally it is; free maximum is higher than constrained maximum. But constrained maximum can never exceed free optimum.

Algebraically setting constraint

The condition for optimum of f(x,y) = z requires that steep of level curve f (x,y) = z_1 is equal to the slope of constraint g(x,y) = c, which can be expressed as follows:-

$$\frac{-g_x'(x,y)}{g_y'(x,y)} = \frac{-f_x'(x,y)}{f_y'(x,y)}$$

Example1:

Suppose one wishes to maximize f(x,y)= xy subject to 2x+y = m.The constraint can also be written as y=m-2x. Then objective function becomes f(x,y(x)) = x (m-2x). Now, the objective function becomes function of one variable. So, for its optimization

$$\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial f}{\partial x} = m - 4x = 0$$

$$\Rightarrow \quad 4x = m$$

$$\Rightarrow \quad x = \frac{m}{4}$$

and $y = m - \frac{2}{m} \times \frac{m}{4} - \frac{m}{2}$

Similar result can also be desired by using calculus techniques as follows:-

$$\frac{f_x'(x,y)}{f_y'(x,y)} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{2}{1}$$

$$\Rightarrow \quad y = 2x$$

Putting y = 2x into 2x+y = m yields

x= m/4 and y = m/2.

<u>Lagrange Multiplier method</u>

Suppose f (x_0,y_0) is optimum value of f(x,y) = z with the constrains g (x,y)= c. Then we know that:

$$\frac{f'_x(x,y)}{f'_y(x,y)} = \frac{g'_x(x,y)}{g'_y(x,y)}$$

$$\Rightarrow \qquad \frac{f'_x(x,y)}{g'_x(x,y)} = \frac{f'_y(x,y)}{g'_y(x,y)}$$

At (x_0,y_0); the above ratios would have some common value. The common value of these ratios is known as Lagrange multiplier and then the above equation becomes:

$$f'_x(x,y) - \lambda g'_x(x,y) = 0$$

$$f'_y(x,y) - \lambda g'_y(x,y) = 0$$

Now let us define lagrangean function ,Lby:

$$L(x,y) = f(x,y) - \lambda(g(x,y) - c)$$

The partial derivatives of L(x, y) with respect to x and y are $L'_x(x,y) = f'_x(x,y) - \lambda g'_x(x,y)$ and $L'_y(x,y) = f'_y(x,y) - \lambda g'_y(x,y)$, respectively.

Equate these partial derivatives equal to zero and solve these equations along with constraint

g (x,y) = c. Solve these equations for optimum valves of x,y and λ .

Lagrangean Function: a better technique

The advantage of lagrangean function over slope equality method is that this method can involve more than two variable and more than one constraint (which we will solve is coming examples).

Example 1 contd.:-

The lagrangean is $L(x, y) = xy - \lambda(2x + y - m)$

$$L'_x(x, y) = y - 2\lambda = 0 \quad (1)$$

$$L'_y(x, y) = x - \lambda = 0 \quad (2)$$

$$L'_\lambda(x, y) = 2x + 2\lambda - m = 0 \quad (3)$$

Solving 1 & 2 we get,

y = 2x

Putting y = 2x in third equation , we get x=m/4 and y = m/2. The result obtained here again is same as done by previous techniques. Hence, any of these techniques is equally applicable. Also y = x from equation 2, so x= m/4. Notice that x,y & λ: all are function of m. m, here is referred as parameter because optimal value of f(x,y) is also function of m as optimum value equals (m/4) (m/2) i.e.(m/8).

Economic Interpretation of lagrange multiplier.

Suppose consider the objective is to maximize f(x,y) subject to g (x,y) = c . Suppose that, x* & y* are the values of x and y that solve for this problem. In general, x & y depend on (parameter of this model) we assume x=x*(c) and y = y*(c) are differentiable functions of c. The associated value f* of f(x,y) is then also function of c,

$$f^*(c) = f(x^*(c), y^*(c))$$

Here f*(c) is also called optimal value function. Also, λ is a function of parameter: c. taking differential of above equation, we get:

$$df^*(c) = df(x^*, y^*)$$

$$= f'_x(x^* + y^*)dx^* + f'_y(x^*, y^*)dy^*$$

We also know that $f'_x(x^*, y^*) = \lambda g'_x(x^*, y^*)$ and $f'_y(x^*, y^*) = \lambda g'_y(x^*, y^*)$ so

$$df^*(c) = \lambda g'_x(x^*, y^*)dx^* + \lambda g'_y(x^*, y^*)dy^*$$

Taking the total differential of constraint

$$g(x^*(c), y^*(c)) = c \text{ yields}$$

$$g'_x(x^*, y^*(c)dx^* + g'_y(x^*, y^*)dy^* = dc$$

So it implies $df^*(c) = \lambda dc$

In particular if dc is small charge is c, then

$$f^*(c+dc) - f^*(c) \approx \lambda(c)dc$$

Also $\dfrac{df^*(c)}{dc} = \lambda(c)$

Thus, the Lagrange multiplier is the rate at which the optimal value of the objective function changes with respect to changes in the parameter c.

In economic applications, c after denotes the available stock of resources which acts as constraint on utility or profit function: f (x,y). λ becomes then the shadow price of the resource as it indicates how utility or profit changes as dc more units of recourses are provided.

Lagrangian multiplier as benefit-cost ratio

Consider again that objective is to maximize f(x,y) subject to g (x,y) = c. Then, we know that

$$\frac{f_*}{g_x} - \frac{f_y}{g_y} = \lambda$$

In other words, at maximum point ratio of f_i to g_i is same for every choice variable, i (x & y here). The numerators (f_i) are the marginal contributions of each choice variable to function f. They show the marginal benefit that one more unit of x or y will have for the function to be maximized (f(x,y).Denominators i.e. g_i:s are marginal cost of each choice variable. That is, they reflect the added burden on the constraint of using slightly more of x (or y).

Example 1 contd:-

Objective was to maximize f (x,y)= xy subject to 2x+y = m. As solved earlier, x(m) = m/4, y(m)= m/2, and λ(m) = m/4. So the value function is f* (m) = (m/4) (m/2) = $m^2/8$.

$$\frac{df^*(m)}{dm} = \frac{m}{4} = \lambda(m)$$

Suppose m is 100 so f *(100) = 100/8. If m increases to 101 then, new optimized value would be f *(101) = 101/8. f*(101) – f*(100) = 25.125.

Also from above results, we know that:

$$\frac{df^*(c)}{dc} = \lambda(c) = \lambda(100) = 25$$

which is a good approximation to the actual change is the optional value function

Example 2 :

Sufficient Conditions

The conditions that we studied till now were necessary conditions but not sufficient. To make this clear, let us consider following example:

$$\max f(x, y) = 2x + 3y \text{ subject to}$$

$$\boxed{\sqrt{x} + \sqrt{y} = 5}$$

The Lagrangean is $L(x, y) = 2x + 3y - \lambda(\sqrt{x} + \sqrt{y} - 5)$. So the three first order conditions becomes:

$$L'_x(x, y) = 2 - \lambda \frac{1}{2\sqrt{x}} = 0$$

$$L'_y(x, y) = 3 - \lambda \frac{1}{2\sqrt{y}} = 0$$

$$\sqrt{x} + \sqrt{y} = 5$$

Solving first two equations yields $y = 4x/9$. Putting this value is third equation yields x = 9 and y = 4. This is indicated by point P in Fig. 4. But as it is evident (9, 4) does not solve the problem (of maximizing $f(x, y)$). Rather solution to this problem is Q = (0, 25) where constraint is satisfied and $2x + 3y$ optimized value of 75 (instead of 30 at point P).

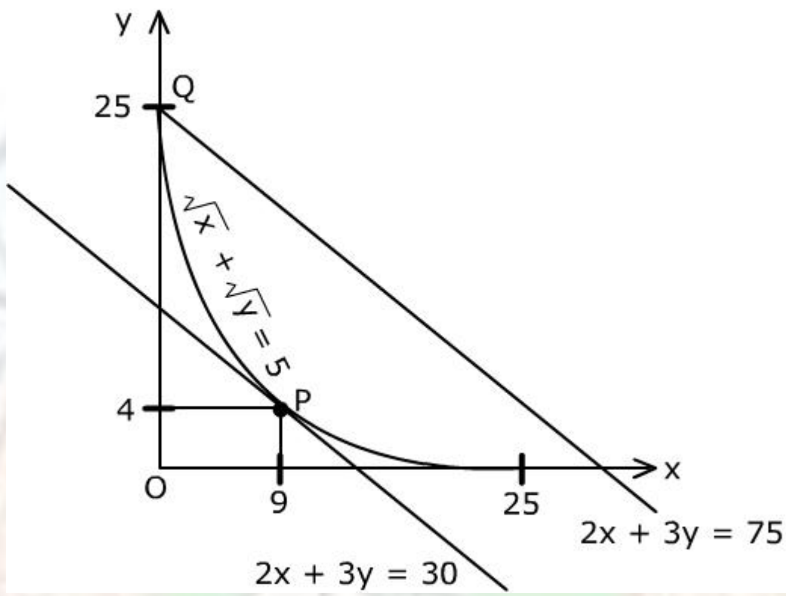

Figure 4

So, this lays the ground that these first o5rder conditions are though necessary but not sufficient.

Consider the same problem of maximising $z = f(x, y)$ subject to $g(x, y) = c$.

Let there be some stationary point (x_0, y_0). By implicit function theorem, the equation $g(x, y) = c$ defines g as differentiable function of x in some neighbourhood of (x_0, y_0). Let this be denoted by $y = h(x)$, then

$$\boxed{y' = h'(x) = -g'_x(x, y)/g'_y(x, y)}$$

The problem of maximisation of $f(x, y)$ is reduced to maximisation of $z = f(x, h(x))$ i.e. with respect to single variable, x.

Then

$$\frac{dz}{dx} = f_x'(x,y) + f_y'(x,y)y'$$

$$\Rightarrow \qquad \frac{dz}{dx} = f_x'(x,y) - f_y'(x,y)\frac{g_x'(x,y)}{g_y'(x,y)}$$

The necessary condition becomes $\frac{dz}{dx} = 0$.

The sufficient condition for maximum of z becomes that second order derivative of z with respect to x becomes less than zero (for max.).

$$\frac{d^2z}{dx^2} = f_{xx}'' + f_{xy}''y' - (f_{yx}'' + f_{yy}''y')\frac{g_x'}{g_y'} - f_{xy}'\frac{(g_{xx}'' + g_{xy}''g')g_y' - (g_{yx}'' + g_{yy}''y')g_x'}{(g_y')^2}$$

$$\frac{d^2z}{dx^2} = \frac{1}{(g_y')^2}[f_{xx}'' - \lambda f_{xx}''(g_y')^2 - 2(f_{xy}'' - \lambda f_{xy}'')g_x'g_y'(f_{yy}'' - \lambda g_{yy}'')(g_x')^2]$$

The expression is bracket becomes determinant

$$D(x,y) = \begin{vmatrix} 0 & g_x' & g_y' \\ g_x' & f_{xx}'' - \lambda g_{xx}'' & f_{xy}'' - \lambda g_{xy}'' \\ g_y' & f_{xy}'' - \lambda g_{xy}'' & f_{yy}'' - g_{yy}'' \end{vmatrix}$$

$$\frac{d^2z}{dx^2} = -\frac{1}{(g_y')^2}D(x,y)$$

A sufficient condition for (x_0, y_0) to solve constraint problem is that (x_0, y_0) satisfies the first order conditions and moreover, that the bordered Hessian D(x_0, y_0) given above is > 0 in the maximization case and, is < 0 in the minimization case.

Envelope Results

Optimization problems in economies usually involve functions that depend on a number of parameters, like prices, income levels, taxes etc. These are parameters as these act as constant during optimization but they vary according to economic situation. For example, in case of utility maximization, income is fixed and we find optimal values of x_i's that maximize utility. But then in next period if income changes, then optimal solution would also change. To see, how optimal solution changes when parameters change, we would encounter here Envelope Theorem.

Consider the problem

$$\max_x f(\mathbf{x},\mathbf{r}) \text{ subject to } g_j(x,r)=0,\ \text{j = 1, ..., m}$$

where $\mathbf{r} = (r_1, ..., r_k)$ is vector of parameters and $\mathbf{x} = (x_1, ..., x_n)$ is vector of choice variables.

The optimization would give solution as $x_1^*(\mathbf{r}), ..., x_n^*(\mathbf{r})$ and optimal value of $f(\mathbf{x},\mathbf{r}) = f^*(\mathbf{r})$, where $f^*(\mathbf{r})$ is optimal value function for this problem.

$$f^*(\mathbf{r}) = f(\mathbf{x}^*(\mathbf{r}),\mathbf{r}) = f(x^*,(\mathbf{r}),...,x_n^*(\mathbf{r}),\mathbf{r}$$

Suppose now we wish to study how our optimal value function changers when nth parameter r_h changes. One way is to assume this new r_h, set lagrangean function, obtain value of $f(\mathbf{x},\ \mathbf{r})$. To avoid such tedious process is study how optimal value function changes as r_h changes.

$$\boxed{\frac{\partial f^*(r)}{\partial r_h} = \sum_{i=1}^{n} \frac{\partial f(x^*(\mathbf{r}),r)}{\partial x_i}.\frac{\partial x_i^*(\mathbf{r})}{\partial r_h} + \frac{\partial f(\mathbf{x}^*(r),r)}{\partial r_h}}$$

The above equation implies that $f^*(x)$ changes on two accounts : first, change in r_j changes vector $\mathbf{r}$ and it changes $f(\mathbf{x},\ \mathbf{r})$ directly and second, r_h changes all the functions $x_1^*(\mathbf{r})$ and hence indirectly changes $f(\mathbf{x}^*,\ \mathbf{r}),\mathbf{r})$. Let the $L(\mathbf{x}^*,\ \mathbf{r}) = f(\mathbf{x},\mathbf{r}) - \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x},\ \mathbf{r})$. The first order condition for the given problem is given as follows:

$$\boxed{\frac{\partial f(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial x_i} = \sum_{j=1}^{m} \lambda_j \frac{\partial g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial x_i} \quad \text{for all } i = 1,...,n}$$

So
$$\frac{\partial f^*(\mathbf{r})}{\partial r_h} = \sum_{i=1}^{n} \left[\sum_{j=1}^{m} \frac{\lambda_j \partial g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r}}{\partial x_i}\right] \frac{\partial x_i^*(\mathbf{r})}{\partial r_h} + \frac{\partial f(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial r_h}$$

$$\Rightarrow \qquad \frac{\partial f^*(\mathbf{r})}{\partial r_h} = \sum_{j=1}^{m} \lambda_j \left[\sum_{i=1}^{n} \frac{\partial g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial x_i} \frac{\partial x_i^*(\mathbf{r})}{\partial r_h} \right] + \frac{\partial f(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial r_h}$$

Differentiating identity $g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r})$ = 0 w.r.t. r_h yields

$$\sum_{i=1}^{n} \frac{\partial g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial x_i} . \frac{\partial x_i^*(\mathbf{r})}{\partial r_h} + \frac{\partial g_j(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial r_h} = 0$$

which holds for all *j* = 1, ...*m*.

So $$\frac{\partial f^*(\mathbf{r})}{\partial r_h} = -\sum_{j=1}^{m} \lambda_j \frac{\partial g_i(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial x_i} + \frac{\partial f(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial r_h}$$

$$= \frac{\partial L(\mathbf{x}^*(\mathbf{r}),\mathbf{r})}{\partial r_h}$$

Example 2 : Utility maximization. Consider the problem $U(x, y) = 100 - e^{-x} - e^{-y}$ subject to the constraint $px + qy = m$.

The lagrangean function L(x,y)=$100 - e^{-x} - e^{-y} - \lambda(px + qy - m)$

The first order conditions are as follows:

$$L'_x = e^{-x} - \lambda p = 0 \qquad (1)$$

$$L'_y = e^{-y} - \lambda q = 0 \qquad (2)$$

$$L'_\lambda = px + qy - m = 0 \qquad (3)$$

Solving first two equations yields $\frac{e^{-x}}{e^{-y}} = \frac{p}{q}$

$$\Rightarrow \qquad e^{-x} = e^{-y} \frac{p}{q}$$

Taking on both sides

$$\Rightarrow \qquad -x = -y + \ln\left(\frac{p}{q}\right)$$

$$\Rightarrow \qquad x = y - \ln\left(\frac{p}{q}\right)$$

Putting value of *x* in (3)

$$P\left(y^* - \ln\left(\frac{p}{q}\right)\right) + q_y^* = m$$

$$y^* = \frac{\ln\left(\frac{p}{q}\right) + m}{p+q}$$

$$x^* = \frac{\ln\left(\frac{p}{q}\right) + m}{p+q} - \ln\left(\frac{p}{q}\right)$$

x and *y* here becomes function of parameters price of *x* : *p*, price of *y* : *q* and income : *m*

Example 3 : Cost minimization

Cost function of the firm that uses capital K and labour, L to produce single output q. and production function is $Q = F(K,L) = K^{1/2}L^{1/4}$. The prices of labour and capital are w and *r* respectively.

So problem is $\min C = rk + wL$ subject to

$$K^{1/2}L^{1/4} = Q$$

$$L(K,L) = k + wL - \lambda(K^{1/2}L^{1/4} - Q)$$

The necessary conditions are as follows:

$$L'_k = r - \lambda \frac{1}{2} K^{-1/2} L^{1/4} = 0 \qquad (1)$$

$$L'_L = w - \lambda \frac{1}{4} K^{1/2} L^{-3/4} = 0 \qquad (2)$$

$$L'_\lambda = -K^{1/2} L^{1/4} + Q = 0 \qquad (3)$$

Solving (1) and (2), we get

$$\lambda = 2rK^{1/2}L^{-1/4} = 4wK^{-1/2}L^{3/4}$$

$$\Rightarrow \qquad L = \left(\frac{r}{2w}\right)K$$

Putting this into (3) we get

$$K^{3/4} = 2^{1/4} r^{-1/4} w^{1/4} Q$$

$$K^* = 2^{1/3} r^{-1/3} w^{1/3} Q$$

$$K^* = 2^{1/3} r^{-1/3} w^{1/3} Q^{4/3}$$

$$L^* = \left(\frac{r}{2w}\right)K^* = 2^{-2/3} r^{2/3} w^{-2/3} Q^{4/3}$$

Corresponding minimal cost is

$$C^* = rK^* + wL^* = 3.2^{-2/3} r^{2/3} w^{1/3} Q^{4/3}$$

Example 4:

Suppose a firm produces TV sets at two different locations and x_1 units are produced at first location and x_2 at second location. The joint cost function is given by

$$C = 0.1x_1^2 + 0.2x_2^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$$

If the firm has to supply an order of 100then this being constraint x_1^* and x_2^* would be produced so as to reduce cost.

$$L(x_1, x_2) = 0.1x_1^2 + 0.2x_2^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000 - \lambda(x_1 + x_2 - 100)$$

$$L'_x = 0.2x_1 + 0.2x_2 + 180 - \lambda = 0 \qquad (1)$$

$$L'_y = 0.4x_2 + 0.2x_1 + 60 - \lambda = 0 \qquad (2)$$

$$L_\lambda = -(x_1 + x_2 - 1000) = 0 \qquad (3)$$

Solving above three equations yields :

$$x_1^* = 400$$

$$x_2^* = 600$$

So firm should 400 units of TV sets at first location and 600 units at second location.

Let us check for the sufficiency condition also

$$D(x_1, x_2) = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0.2 & 0.2 \\ 1 & 0.2 & 0.4 \end{vmatrix} = -0.2 < 0$$

Hence (400, 600) stationary point is a minima and min. cost is 2,69,000.

Example 3 : (Contd..) Envelope results

Suppose now that we wish to know how C^* changes when r changes.

$$\frac{\partial C^*}{\partial r} = \frac{\partial}{\partial r}(3.2^{-2/3} r^{2/3} w^{1/3} Q^{4/3})$$

$$= \frac{2}{3} \times 3 \times 2^{-2/3} r^{-1/3} w^{1/3} Q^{4/3}$$

$$= 2^{1/3} r^{1/3} w^{1/3} Q^{4/3}$$

$$= K$$

Now, if instead envelope theorem is used, which states that

$$\frac{\partial C^*(r, w, Q)}{\partial r} = \frac{\partial L(K, L, r, w, Q)}{\partial r} = K$$

Similarly, $\frac{\partial C^*}{\partial w} = L$.

Exercises

Q1. If the utility of a consumer depends on two goods: x and y and utility function is given by U= (x+2)(y+1). If prices of x and y are Rs. 2 and Rs.5, respectively and income is Rs.51. find the optimal levels of x and y purchased by the consumer and indirect utility function.

[Hint: Indirect utility function is utility as function of parameters]

Q2. The production function of a firm is given by X=$K^{1/2}L^{1/2}$ and prices of capital and labor are fixed at Rs. r and Rs. w, respectively.

i) Find the cost minimizing combination of capital and labor.
ii) Derive cost function of firm.

Q3. The incomes of an individual in current and next year are Rs.500 and Rs. 792 respectively.his utility function of two consumption expenditures x and y is U=$x^{1/2}y^{1/2}$. If the market interest rate is 10% p.a. ,determine optimum consumption expenditures and amount consumer should borrow or lend in current year.

[Hint: constraint is income in two periods but consumption could differ from income of that period as consumer can borrow or lend in the market.]

Q4. A monopolist has the following demand functions for each of his products X and Y; x=72-0.5p_x and y=120-p_y. the combined cost is C=x^2+xy+y^2+35 and the maximum product is 40 units. Find

i) Profit maximizing level of output
ii) Price of each product
iii) The total profit

Q5. Find the optimal mix and its cost in the case when a producer chooses an output corresponding to isoquant k2l=16 and respective prices of c apital and labor are Re.1 and Rs.2, respectively. Also find the expansion path.

[Hint: expansion path shows change in optimal values when output changes parameter changes]

References

K. Sydsaeter and P. Hammond, Mathematics for Economic Analysis, Pearson Educational Asia, Delhi, 2002