Functions, Sequence and Series

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Contents:

1. Learning Outcome

After reading this lesson you will be able to know the various types of functions i.e. linear, quadratic, polynomial, rational functions and their graphs. Besides sequence and series will also are covered in this lesson. Various types of sequences i.e. Bounded sequence, Finite sequence and Infinite sequence, Limit of a sequence, Convergent sequence, Divergent Sequence, Oscillatory Sequence are discussed in detail. Similarly different types of series i.e. Arithmetic Series, Geometric Series are also explained below.

2. Graphs and Functions

Cartesian coordinate system is composed of a horizontal line and a vertical line perpendicular to each other. These lines are called coordinate axis. The point where they intersect each other in called the origin (0). Horizontal axis or x–axis (abscissa) gives the distance of a point from vertical axis or y–axis (ordinate) give the distance of a point from horizontal axis. To the right of y–axis x coordinates are negative. Above the x–axis y coordinates are positive and below x–axis they are negative.

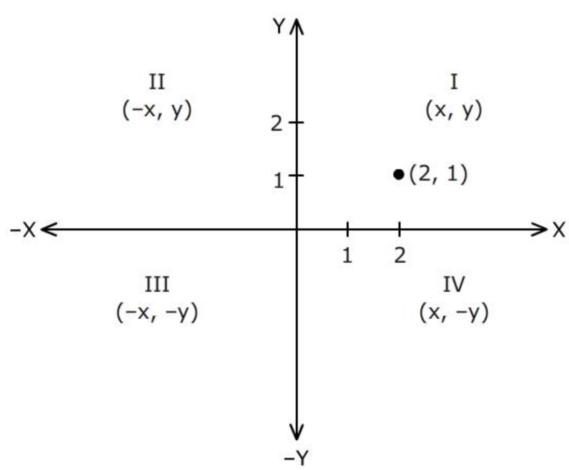

Figure 1

The sign of the coordinate in each quadrant are shown in the figure. Quadrants are numbered anticlockwise.

Each point in the coordinate system is associated with ordered pair of numbers known as coordinates, showing the location of point in relation to origin. The point (2, 1) is 2 units right of y-axis and 1 unit above x-axis.

There are several functions which are utilized in economics and some of them are:

Linear functions $f(x) = mx + c$

Quadratic function $f(x) = ax^2 + bx + c$ $(a \neq 0)$

Polynomial function :

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_0$ n is a nonnegative integers and $a_n \neq 0$

Rational functions :

$$f(x) = g(x) / h(x)$$

where g(x) and h(x) are polynomials and $h(x) \neq 0$.

Power function :

$f(x) = ax^n$ (where x is any real number)

The domain of linear, quadratic and polynomial functions is the set of all real numbers; the domain of rational and power functions any value of x for which the function is not defined. We shall now discuss these functions use by one.

2.1 Linear functions:

Given any equation in x and y, we can depict the set of points in the coordinate system; which satisfy this equation. This set of points is called the graph of the equation. The graph of a linear equation is a straight line.

The general equation of a straight line is

$$Ax + By + C = 0$$

This can be written as

$$y = -\frac{A}{B}x - \frac{C}{B}$$

Where the slope of the line is $-A/B$ and the y–intercept is $-C/B$.

The equation has been written in the slope intercept form. We can write it as

$$y = mx + c$$

Where slope = m and intercept = c

In the figure we have drawn two straight lines. In this case use line the slope is m_1 and the intercept is b. In the case of other line the slope is m_2 and the intercept is c.

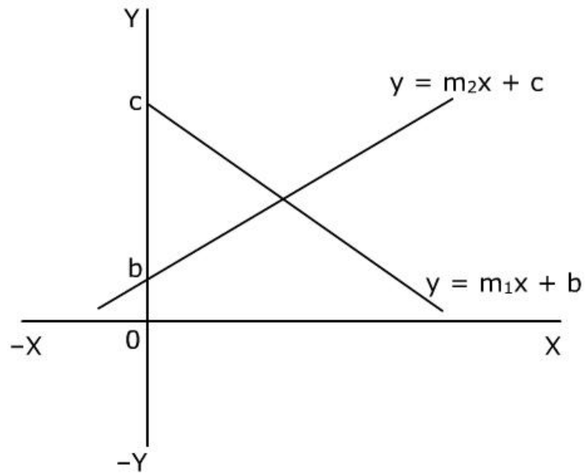

Figure 2

The slope of a straight line conveys the steepness and direction of the line.

See the figures given below.

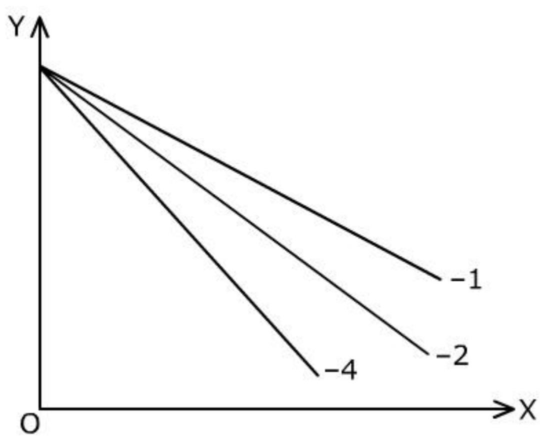

Figure 3(a)

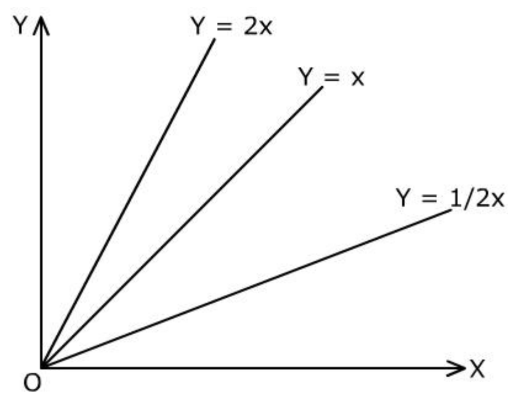

Figure 3(b)

In the figure (a) the negative sign convey that the straight line is negatively sloping. The magnitude of the slope (m) conveys the steepness of line.

The graph of a linear function can be drawn easily by different values of x in the equation and finding out the values of y. Now we have ordered pairs of x and y (x, y) which can be plotted in the figure.

Another way of plotting the graph is if we have the y– intercept and one point (x, y). Plotting this two in the figure, we can find the straight line by joining this two points by a straight line passing through then points.

2.2 Point–point formula:

If two points on a straight line are given. Then we can find the slope and equation of straight line given two points (x,y) and (x_2, y_2)

$$slope = m = \frac{y_1 - y_2}{x_1 - x_2}$$

Given two points satisfying the equation. We can write

$$y_1 = mx_1 + c \qquad \text{........ (1)}$$

$$y_2 = mx_2 + c \qquad \text{........ (2)}$$

Subtracting equation (2) from (1) we get

$$y_1 - y_2 = m(x_1 - x_2)$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Point slope formula for straight line is

$$y - y_1 = m(x - x_1)$$

For any point (x, y) to be on a straight line passing through the point (x_1 , y_1) and has a slope m, it must be true that

$$m = \frac{y - y_1}{x - x_1}$$

Rearranging this

$$y - y_1 = m(x - x_1)$$

This is the equation of straight line and can be written as

$$y = m(x - x_1) + y_1$$

$$= mx + y_1 - mx_1$$

or $$= mx + c$$

when $c = y_1 - mx_1 = \text{constant}$

Line parallel to y = mx + c

Find the equation of a line parallel to

$y = mx + c$, which is y = 3x + 2 passing through point (2, 5)

Solution: Parallel lines have equal slope so the slope of the line m equation is given by;

$$y - y_1 = m(x - x_1)$$

Suppose m = 3 then

$$y - 5 = 3(x - 2)$$

$$y = 3x - 6 + 5 = 3x\ s - 1$$

Linear passing through point (8, 3) and perpendicular t_0 another line y = 2x + 10. Perpendicular line have slope that are negative reciprocal of each other.

So m = -1/2

Equation is $y - 3 = -\frac{1}{2}(x - 8) \Rightarrow y = -\frac{1}{2}x + 7$

2.3 Quadratic Function:

An equation of the form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$ is called a quadratic equation .

This is an example of non–linear equation. The graph of the function will not look like a straight line. It will be non–linear graph.

We take an example

$$y = x^2$$

In this function a = 1, b = 0 and c = 0.

T0 graph this function, simple pick some representative values of x; solve for f(x) which is usually referred t0 as y in graphing. Plot the resulting ordered paris [x, f(x)] and connect them with a smooth line. The procedure is shown below for

$y = x^2$

x	F(x) = y	[x, f(x)]
–3	9	(–3, 9)
–2	4	(–2, 4)
–1	1	(–1, 1)
2	4	(2, 4)
3	9	(3, 9)

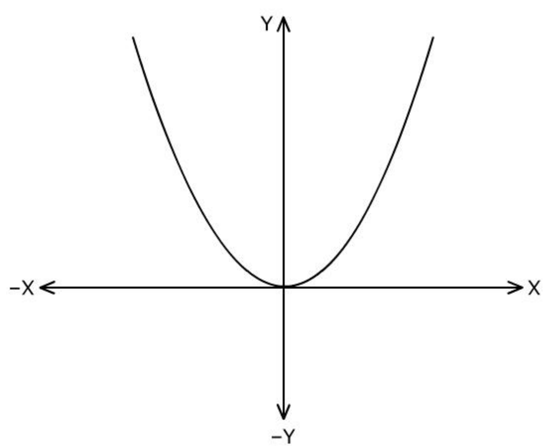

Figure 4

The graph of a quadratic function $ax^2 + bx + c = 0$ where $a \neq 0$, is a parabola. In the figure where a = 1, b = 0 and c = 0 is graphed. The vertex of parabola is (0, 0). In the case of general equation of parabola then vertex is $\left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

The quadratic function can be solved by factoring, completing square, or using the quadratic formula.

The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

An equation of the from

$x^2 + bx = 0$ where a = 1 and c= 0

can be converted into a perfect square by taking one half of the coefficient of x $\left(b/2\right)$, squaring it $\left(b/2\right)^2$ and adding to the original expression to obtain

$$x^2 + bx + \frac{b^2}{4} = \left(x + b/2\right)^2$$

Example:

$x^2 + 12x + 35 = 0$

i). Move the constant to right hand side

$x^2 + 12x = -35$

ii). Take the half of the coefficient of x (12) which is $12/2 = 6$

square it $6^2 = 36$

iii). Add 6^2 to both sides

$x^2 + 12x + 6^2 = -35 + 6^2$

$(x + 6)^2 = 1$

Take the square root of both sides and then solve for x

$x + 6 = \sqrt{1} = \pm 1$

$x = 7 \quad and \ 5$

For an expression in the form

$ax^2 + bx + c$

Write

$$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = 0$$

Now take half of b/a which is $= \frac{b}{2a}$. Take the square of $\frac{b}{2a} = \left(\frac{b}{2a}\right)^2$. Add and subtract it in the bracket expression

$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right]$$

$$= a\left[\left(x + b/2a\right)^2\right] + \frac{4ac - b^2}{4a} = 0$$

$$or\ a\left[x + b/2a^{\ 2}\right] - \frac{b^2 - 4ac}{4a} = 0$$

The vertex of this parabola is

$$\left(\frac{-b}{2a}, \frac{e-b^2}{4a}\right)$$

From the above exercise; a quadratic function can be expressed in this form:

$y = a(x – h)^2 + k$

where the axis is (x–h) = 0, x = h and the vertex is (h, k). The expression h shifts the function by h units from the origin. The function will shift to the right or left will depend on the sign of h for example if

For Example if :

$y = (x–3)^2 + 16$

Then the function will be shifting to the right of y axis.

The term K shifts the function up or lower it depending upon the sign of k. In our example.

$y = (x–3)^2 + 16$

The graph has been shifted 3 units to the left of origin and 16 units above the x–axis.

If a > 0, the parabola opens up and the vertex is the lowest point of the function.

If a < 0, the parabola opens down and the vertex is the highest point.

If |a| > 1 the parabola is narrower than if |a| = 1 If 0 < |a| < 1 it is wider then if |a| = 1

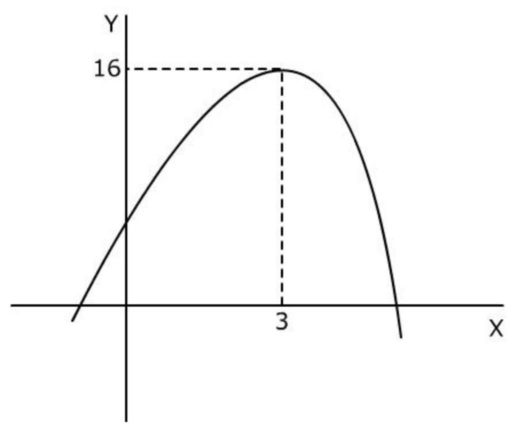

Figure 5

2.4 Polynomial Functions:

The function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + -------- + a_1 x + a_0 = 0$$

(where; a's are constant ; $a_n \neq 0$)

is call the polynomial of degree n. Linear quadratic and cubic function are also examples of polynomial.

The polynomial equation has at the most n real solution or roots, but it need not have any.

Cubic function or polynomials of degree greater than or equal to 3 are complicated because the shape of the graph changes substantially as the coefficient a_1, a_2, a_3 and a_0 change.

Zeros of polynomial Equation

If r is a root of equation

$f(x)=0$ i.e. if $f(r)=0$

then (x – r) is a factor of f(x) conversely if (x–r) is a factor of f(x) the r is a root of f(x)= 0

or f(r) = 0

If b/c , a rational function in it lowest terms, is a root of the equation .

$a_n x^n + a_{n-1} x^{n-1} + ------- + a_1 x + a_0 = 0$ with intergral coefficient, then b is a function a_0 and c is a function of a_n.

Example :

$3x^3 + 5x^2 - 3x - 2 = 0$

value of b are limited to factors of 2. Which are ±1, ±2 and vector of c are limited to factions of 3 which are ±1, ±3. Hence the only possible real roots are ±1, ±2 and $\pm 1/3$, $\pm 2/3$.

Integral root theorem

It follows that if an equation f(x)= 0 has integral coefficient and the lead coefficient is 1 (i.e. $a_n = 1$)

$x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ------- + a_1 x + a_0 = 0$

then any rational root in an integer and a function a_0.

Example :

$3x^3 + 5x^2 - 3x - 2 = 0$

possible roots are ±1 and ±2.

Whether there possible roots are actually the root of the equation or not can be found out by putting these values in the equation in place of x and then finding out whether f(r) = 0 or not. If the equation is satisfied then this a root of the equation.

Another method of finding out whether r is a root of the equation or not is synthetic division. Synthetic division is a simplified method of dividing the polynomial f(x) by (x–r). Where r any assigned number.

Example :

$x^3 + 2x^2 - 23x - 60 = 0$

find functions of 60 ±1, ±2, ±3, ±4, ±5 and so on let us try 5

So we want to divide $x^3 + 2x^2 - 23x - 60 = 0$ by (x – 5) using the synthetic division steps

1.) Write the terms of dividend in descending power of the variable and fill in missing terms using zero for the coefficient (In our example there is no missing term)

$$x^3 + 2x^2 - 23x - 60 \div (x - 5)$$

Write the constant terms a from the divisor on the left of a ⌋ and write the coefficient from the divided to the write of the symbol.

$$5 \rfloor\ 1 + 2 - 23 - 60$$

Bring down the first term in the divisor to the third row for now

$$\begin{array}{l|l} 5 & 1 + 2 - 23 - 60 \\ \hline & 1 \end{array}$$

Multiply the term in the quotial row (third row) by the divisor and write the product between the second row below the second term in the first row, add the numbers in the column formed and write the sum as the second term in the quotient row

$$\begin{array}{l|l} 5 & 1 + 2 - 23 - 60 \\ & \quad + 5 \\ \hline & 1 + 7 \end{array}$$

Multiply the last term in the quotient row by the divisor under the term in the top row, add the sum and write the sum in the quotient row. Continue this process until all of the terms in the top row have a number under them.

$$\begin{array}{l} 5|1 + 2 - 23 - 60 \\ \quad + 5 + 35 + 60 \\ \hline \quad 1 + 7 + 12 + 0 \end{array}$$

The third row is the quotient row with the last terms being the remainder. The degree of the quotient polynomial is one less than the degree of the dividend because we have divided by a linear factor. The term in the quotient row are the coefficients of the quotient polynomials. The degree of the polynomial is 2

$$x^3 + 2x^2 - 23x - 60 \div (x - 5)$$

$$= x^2 + 7x + 12 + \frac{0}{x-5}$$

or $$x^2 + 7x + 12$$

The existence of zero remainder proves that 5 is the root of the equation.

2.5 Rational Functions:

$$f(x) = g(x)/h(x) \qquad h(x) \neq 0$$

where g(x) and h(x) are polynomials

The graph of the rational functions can be drawn by taking representative values of x and finding f(x) or y. Then the ordered paris can be plotted.

Drawing the graph of rational function is made easier by funding the asymptote.

Vertical asymptote : The horizontal asymptote is the line x = k where k is found after all cancellation of common factor in the numerator and denominator are completed, and then solving the denominator by setting it equal to zero.

Example :

$$f(x) = \frac{2x - 3}{x^2 - 4}$$

$x^2 - 4 = 0$ $x = -2$ and 2

are the vertical asymptotes

<u>Horizontal asymptotes</u> : The vertical asymptote is line y = h where h is found by comparing the degree g(x) and h(x).

i) If the degree of f(x) is less than the degree of h(x) then the rational function has a horizontal asymptote of y = 0

ii) If the degree of g(x) is equal to the degree of h(x) than f(x) has a horizontal asymptote of $y = a_n / b_n$ where a_n is the coefficient of the highest degree term of g(x) (Numerator) and b_n is the coefficient of the highest degree term of the h(x) (the denominator)

iii) If the degree of g(x) is greater than the degree of h(x), then f(x) does not have a horizontal asymptote.

The graph of f(x) may cross the horizontal asymptote in the interior of its domain. This is due to the fact that we are concerned with how f(x) behave as $x \to \infty$ or $x \to -\infty$ in determining the asymptote.

2.6 Graphing Rational functions:

i) $f(x) = g(x) / h(x)$ we first determine the holes: Values of x for which both g(x) and h(x) are zero. After any holes are located, we reduce f(x) to lowest terms.

ii) Once f(x) is in lowest terms we find the asymptote, symmetry, zeros and y intercept if they exist.

iii) Graph the asymptotes as dashed lines.

iv) Plot the zeros and y intercept and plot other points to determine how the graph approaches the asymptotes.

v) Sketch the graph through plotted points and approaching the asymptotes.

Example :

$$y = \frac{x^3 - 2x^2 - 3x}{x}$$

i) The graph has a hole at x = 0

$$y = \frac{0}{0}.$$

ii) Reduce it lowest term

$$\frac{x^3 - 2x^2 - 3x}{x} = x^2 - 2x - 3.$$

when x ≠ 0

There is a hole at (0, 3)

iii) There is no asymptote for the graph.

There is no y intercept but there are zero at (3, 0) and (–1, 0). We plot the zero and place an open circle around the point (0, –3) to indicate the hole in the graph. Now select the corresponding points and plot them

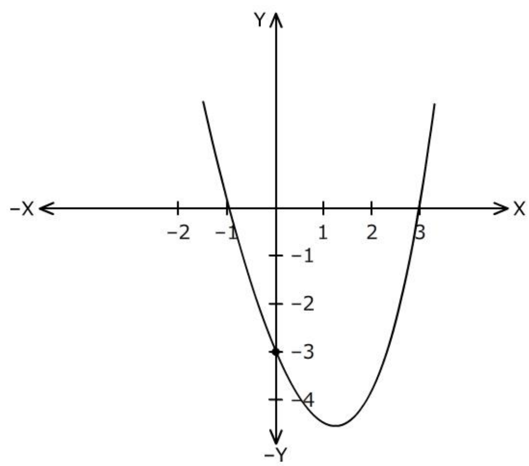

Figure 6

Example :

$$\frac{5}{x - 2}$$

i) Set the denominators equal to zero

$$x - 2 = 0$$

$x = 2$ is the vertical asymptote

ii) Since the degree of g(x) less then degree of h(x) the horizontal asymptote is

$$y = 0.$$

iii) when $x = 0$ $\quad y = {-5}/{2}$

There are no holes nor there are zero (the graph does not cross the x–axis)

Plotting different point satisfying the equation the graph would look like as shown in the figure 7.

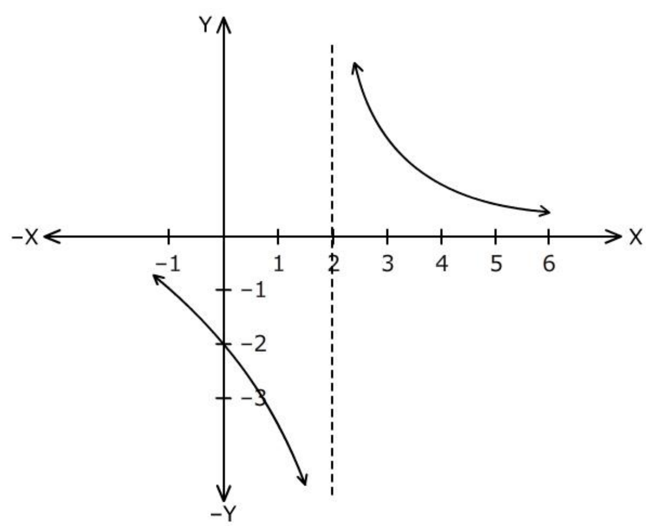

Figure 7

3. Sequence:

A sequence is a function whose domain is the subset (or set of) Natural numbers N.

For example

$$f(x) = \frac{1}{n}$$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\ldots\ldots\ldots \frac{1}{k} --------$$

The dots indicate the sequence.

The numbers in the list are called the terms of the sequence.

Usually we write terms of the sequence as a_1, a_2, a_3, a_4 and so on

$$a_1 = 1 \qquad a_2 = \frac{1}{2} \qquad a_3 = \frac{1}{3}$$

The nth term of the sequence

$$a_n = \frac{1}{n}$$

We write the sequence by placing braces around the formula for nth term.

$$f(x) = \frac{1}{n} \qquad n \in N$$

$$a_n = \left\{ \frac{1}{n} \right\}$$

<u>Example</u> :

(1) $a_n = \frac{1}{\alpha^{n-1}}$ $\qquad 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots\ldots\ldots\ldots$

(2) $a_n = (-1)^{n+1}$ $\qquad 1, -1, 1, -1, \ldots\ldots\ldots\ldots\ldots$

(3) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$ $\qquad a_n = \frac{1}{2n-1}$

Sequence is $\frac{2}{4}, \frac{4}{5}, \frac{6}{6}, \frac{8}{7}, ...$

$$a_n = \frac{2n}{n+3}$$

Given the nth term of sequence one can find out different terms of sequence.

Sequence for which

$a_1 > a_2$

$a_2 > a_3$

$a_n > a_{n+1}$

For any $n \in N$ is said to be a decreasing sequence.

A sequence for which

$a_1 < a_2$

$a_2 < a_3$

$a_n < a_{n+1}$

For any $n \in N$ is said to be increasing sequence.

3.1 Bounded sequence:

A sequence $\langle a_n \rangle$ is bounded above, if and only if there exists a number M such that.

$$a_n \leq M \quad \forall \quad n \in N$$

For all $n \in N$

The number M is called the upper bound of the sequence.

The sequence $\langle a_k \rangle$ is bounded below, if and only if there exists a number m such that

$$a_n \geq m \qquad \forall\, n \in N \qquad\qquad \textit{for all} \;\; n \in N$$

A sequence is said to be bounded if it is bounded above and below.

3.2 Finite sequence and Infinite sequence:

A finite sequence has a terminal value.

$a_1, a_2, a_3, \ldots\ldots, a_n$

The values a_1, a_2, and so on are the terms of the sequence. The terminal value is a_n. So it is a finite sequence.

Take the sequence

$$\langle a_k \rangle = \left\{ \frac{1}{k} \right\} \qquad \text{where } K \in N$$

The term are

$$1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}, \ldots\ldots\ldots$$

There is no terminal value of the sequence. The sequence is an infinite sequence.

3.3 Limit of a sequence:

Let $\langle a_n \rangle$ be sequence of real numbers. L is the limit of this sequence if for any arbitrarily chosen small number $\in$ there exists a positive number N such that for all $n \geq N$ we have

$$|a_n - L| < \in$$

This can be written as

$$\lim_{x \to \infty} \langle a_n \rangle = L$$

In simple words it means nth terms gets closer and closer to L if n tends to infinity (Note L is a finite number.)

3.4 Convergent sequence:

If $\underset{n\to\infty}{Lim}\langle a_n\rangle = L$ (a finite number the sequence is convergent sequence.)

3.5 Divergent Sequence:

If $\underset{x\to\infty}{Lim}\cdots\langle a_n\rangle = \pm\infty$

then sequence is a divergent sequence.

3.6 Oscillatory Sequence:

When $\langle a_n\rangle$ jump back and forth along a number line, it is an oscillatory sequence. Determine whether the given sequence convergent, divergent or oscillates. If sequence converges them heel out the limit of the sequence

(i) $\frac{1}{2^{n-1}}$

(ii) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

(iii) $a_n = \frac{1}{2}\left[1+(-1)^{n+1}\right]$

(iv) $a_n = \frac{n}{n+1}$

(v) $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \ldots\ldots$

Determine whether the following sequence are increasing, decreasing or neither

(i) $a_n = {(5n-2)}/{(4n+1)}$

(ii) $a_n = {3^n}/{(1+3^n)}$

(iii) $a_n = {n!}/{2^n}$

Show that the sequence diverge or not if diverges then to $\infty \, or - \infty$

(i) $< n^2 >$

(ii) $< 2^n >$

(iii) $< -2n >$

(iv) $< (-1)^n$

Oscillates or not

(i) $< (-1)^n$

(ii) $< (-1)^n >$

(iii) $< 1, 2, \frac{1}{2}, 3, \frac{1}{3} \ldots\ldots >$

4. Series:

A series is a special type of sequence If a_i , i = 1, 2, 3, 4………. is a sequence them

$$S_n = a_1 + a_2 + a_3 + \ldots\ldots a_n$$

The sum of terms of a sequence is called a series.

$$S_n = a_1 + a_2 + a_3 + \ldots\ldots a_n$$

The series S_n is finite series since it is a sum of finite sequence. We can use symbol $\sum$ (sigma) for the summation

$$s_n = \sum_{i=1}^{\infty} a_n$$

Series is a special type of sequence. Any result derived for sequence also applies to series.

4.1 Convergence and Divergence of Series:

If a series is monotonic and bounded it then it has a limit. The series is convergent. We can use ratio test to determine whether series associated with sequence $\langle ai \rangle$ is convergent or not.

If $S_n = \sum_{i=1}^{n} ai$ is series associated with a sequence $\langle ai \rangle$ and

$$\lim_{x \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

If (i) L<1 the series S_n converges

(ii) L>1 the series diverges

(iii) L = 1 then series may converges or diverges.

4.2 Arithmetic Series:

A series in which each term is obtained by adding a constant quantity to its preceding term is known as arithmetic series.

The constant quantity is the common difference.

Look at the sequences

a, a + d, a + 2d, a + 3d,……. , a + (n-1)d

Where n denotes number of terms a + (n-1) d is the nth term or last term of the arithmetic series.

The sum of arithmetic series in

$$S_n = n/2 \{2a + (n-1)d\}$$

4.3 Geometric Series:

A series in which each successive term is obtained by multiplying the previous term by a constant quantity is called a geometric series. The constant quantity is called common a ratio. The general form of geometric series with first term equal to a and common ratio equal to r is

$a, ar, ar^2, ar^3, \ldots. ar^{n-1}$

The sum of n terms of a geometric series is

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

Sum of an infinite geometric series where $|r| < 1$ is $S_n = \dfrac{a}{1-r}$

When $|r| < 1$ the series is convergent and when $|r| \geq 1$ the series diverges.

<u>Example</u> :

Test the convergence of $\sum\limits_{n=1}^{\infty} \dfrac{n^n}{n!}$

Using the ratio test $\lim\limits_{n \to \infty} \dfrac{a_{n+1}}{a_n}$

$$a_1 = \frac{n}{1} \qquad a_2 = \frac{2^2}{2!} \qquad a_3 = \frac{3^3}{3!}$$

$$a_n = \frac{n^n}{n!} \qquad a_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!}$$

$$\frac{a_n + 1}{a_n} = \frac{(n+1)^{n+1} / (n+1)!}{n^n / n!}$$

$$= \frac{(n+1)^{n+1}}{n^n} \times \frac{n!}{(n+1)!}$$

$$= \frac{(n+1)^{n+1}}{n^n . n} = \frac{(n+1)^{n+1}}{n^{n+1}}$$

$$= \left(1 + \frac{1}{n}\right)^n$$

$\lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right) = e > 1 \quad \{e = 2.71828\}$ series is diversant

5. Exercises:

(1) How many terms of arithmetic sequence 24, 22, 20 ..... are receded to give a sum of 150?

(2) How long it take to pay off n debt of Rs. 880 if Rs. 25 is paid in the first month, Rs. 27 in the second and Rs. 29 in the third month.

(3) The second term of a geometric sequence is 3 and the fifth term in 81/8. Find the eight th term.

(4) The first term of a geometric series is 375 and the forth term in 192. Find the common ratio and the sum of first for terms.

(5) A man agrees to work at the rate of Rs. 1 for the first day, Rs. 2 for the second day, Rs. 4 the third day, Rs. 8 for the forth th day etc. How much would he receive at the end of 15 days.

(6) The population of a certain tour will increase 3% each year for four years. What is the percentage increase in population after for years?

6. References

K. Sydsaeter and P. Hammond, Mathematics for Economic Analysis, Pearson Educational Asia, Delhi, 2002.