LESSON: Budget Constraint

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Learning Outcomes:

After studying this chapter, a student should be able to:-

1. Sketch budget constraint.
2. Interpret slope of budget constraint.
3. Shift/ pivot budget constraint.
4. Compare budget sets when consumer's context (income and prices) differs.
5. Understand impact of government's actions on budget set of a consumer.

1. Introduction

A consumer has to go market, he faces different prices, choose among different alternatives, he chooses the best bundle he likes, etc. But, wait! What makes him a consumer? Or from where does he start? Yes, he has some money income at hand to spend which then follows the pursuit.

Two different persons have difference in incomes; this could alter their tax liability and hence disposable incomes differ too. These two persons, if residing at different locations, could face different prices. A consumer with lower level of income could be more happier if his income commands greater amount of goods & this is possible when prices are lower (relatively) compared to that faced by other person .

Different permutations & combinations of prices & income will be tried & tested in this chapter. This chapter basically provides you with one of the apparatuses used in consumer theory in microeconomics and is called 'budget constraint'. This chapter is divided into four sections and further into subsections. First section covers budget constraint algebraically & in second, it is analyzed graphically. In third section, slope of budget line is calculated & interpreted. In the last section, changes in budget set are analyzed via prices changes, income change, taxes, and subsidies & rationing.

2. Budget Constraint

"Budget"! This word is often heard when you ask your parents for some expensive toy, latest version of smart phone or when you ask for some trip. The reply that you get is 'It is not in our budget, this time!". So, one has basic understanding that, one can't consume infinitely any amount of the goods. There is a binding constraint and as above example makes it clear it is 'your parents' Income'; beyond which things become unaffordable.

In this section, we will understand feasible set of goods that can be consumed, given somebody's income. Some common notations and concepts would be used, which are explained below:-

1. There is money income , M, given to you
2. There are two goods: x & y. Even if we take two goods it would not change our analysis. Instead, of y you could also assume all other goods on the y – axis.
3. The prices of x & y are P_x & P_y respectively.
4. (x_1,y_1) represents a bundle of two goods x & y when a consumer consumes: x_1 of good x and y_1 of goods y .

2.1 Budget constraint equation

If there are two goods x & y and their respective prices are P_x & P_y ; then total expenditure made on two goods is   :

$$P_x . x + p_y . y$$

This expenditure need be less than or equal to one's money income M, so budget set becomes:

$$P_x . x + p_y . y \leq M \qquad \qquad …………..(1)$$

If one wishes to spend entire income on the two goods, then above equation has to satisfied with equality & budget constraint / line is

$$P_x . x + p_y . y = M \qquad \qquad …………..(2)$$

If there are three goods then, budget constraint becomes:

$$P_x . x + p_y . y + pz . z = M$$

There could be n goods & one can extend it to n – goods case.[1]

2.2 Drawing Budget Constraint

The budget constraint $P_x . x + p_y . y = M$ should be plotted by asking two questions:-

i)   How much x a consumer can buy if he spends entire income on x? Answer. Since y = 0, so x = M/p_x

ii)   How Much y a consumer can buy if he spend entire income on y? Answer, Since x = 0, y = M/p_y .
So, vertical intercept is M/p_y & horizontal intercept is M/Px, & budget line is a line joining these two points: (0 , M/p_y) & (M/p_x,0)

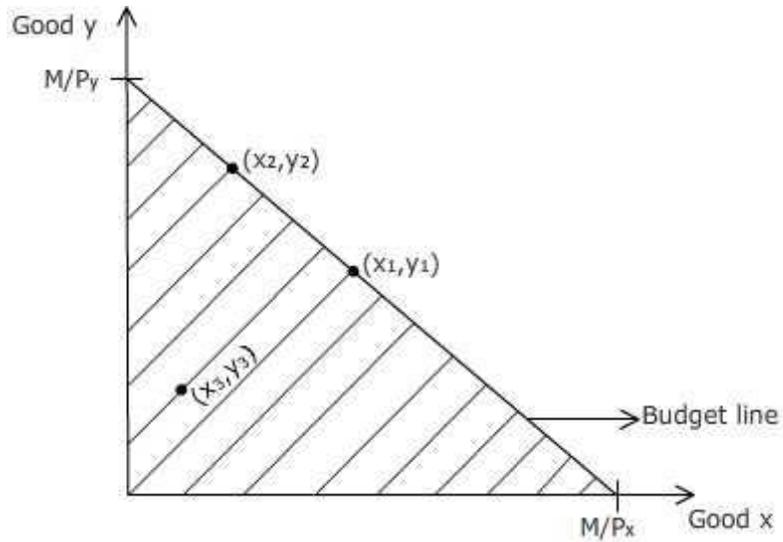[1] See the appendix to this chapter to understand n goods case.

Figure 1: Budget Set

The shaded region in fig.1 is budget set including the line since all consumption bundles like (x_1,y_1), (x_2,y_2) and (x_3,y_3) are affordable at prices (p_x,p_y) and with money income, M. In three goods case, budget constraint $P_x x + p_y y + p_z z = M$; which is a budget plane that can be drawn in 3 D.
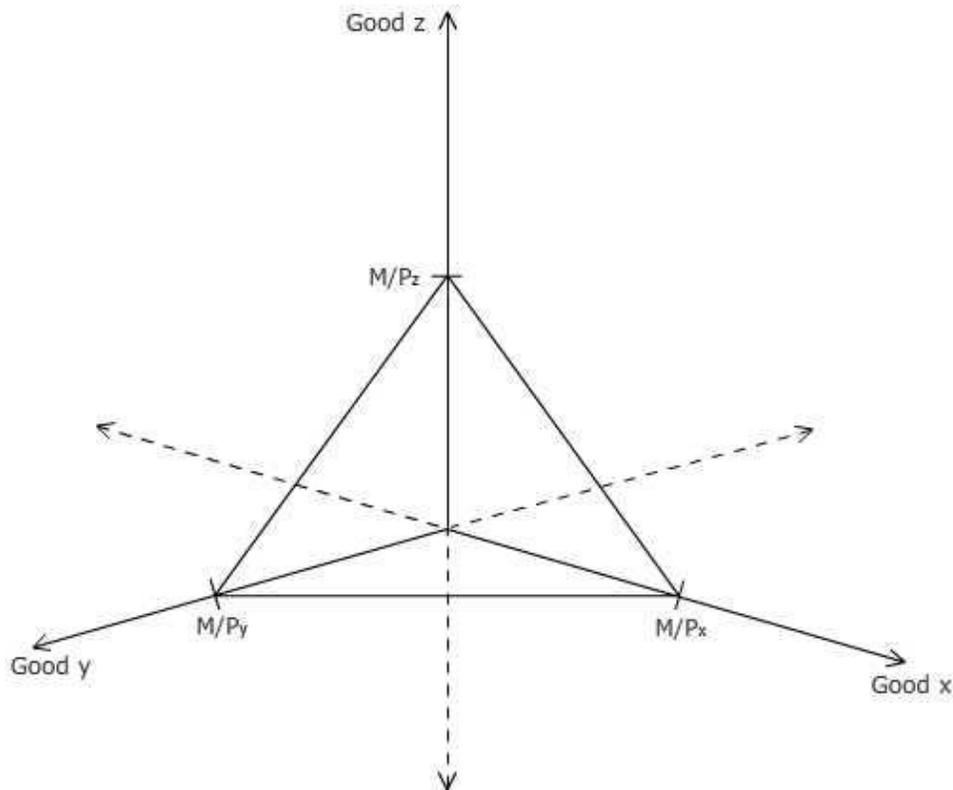
Figure 2: Budget Plane

2.3 Working with slope of Budget line algebraically

Rewriting budget line in the form y =mx+c,where 'm' used to be slope of the line & c, intercept.

$$y=\frac{M}{P_y}-\frac{P_x}{P_y}x_1 \qquad ....(3)$$

So the slope of budget line is- $\frac{P_x}{P_y}$ [2] .The negative sign shows downward slope of budget line.Let (x_1, y_1) be a affordable bundle on budget line then :-

$$P_x x_1+p_y \ y_1 =M \qquad ....(4)$$

Suppose, consumer changes bundle but satisfies budget constraint then:

$$P_x (x_1 +\Delta x)+ p_y(y_1+\Delta y)=M \qquad ....(5)$$

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[2] See appendix to this chapter for calculus treatment of slope of budget line.

On subtracting equation (4) from (5), we get:

$P_x \ \Delta x + p_y \ \Delta y = 0$

Or, $\Delta y / \Delta x = -p_x / p_y$ …………..(6)

The negative sign signifies the fact that to be on the same budget line, one has to make changes in such a manner that consumption of x & y moves in opposite directions i.e, if a consumer increases consumption of good x then consumption of y must fall (since income is constant; one can't afford increase in both commodities, simultaneously).

Slope of Budget line is price ratio of the goods or the relative price of one good is terms of other good. $\dfrac{\Delta y}{\Delta x}$ is interpreted as by how much consumption of y changes if consumption of x increases by 1 i.e. $\Delta x = 1$. It is equivalent to $-p_x/p_y$. So, a consumer is willing to substitute good x for good y at the rate of $-p_x/p_y$. Rewriting (6), we get:

$$\Delta y / \Delta x = -(1 / p_y / p_x)$$

A consumer has to give up less of y to add one unit of x if either P_y is higher or P_x is lower or both i.e. the relative price of x in terms of y is lower. This means if you give up some expensive commodity like a laptop (represents "y" here), you can add more of cheaper commodity like bread (represents "x" here). Put it other way, you have to sacrifice a smaller fraction of laptop (not literally) to add a unit of bread loaf.

2.4 What alters budget line?

For any line, a change can be made if either intercepts changes or slope change or both. Budget line either shifts parallely or non-parallely or it pivots.

2.4. 1   Budget line pivot

A budget line is called pivoted when slope of budget line changes & purchasing power stays constant.
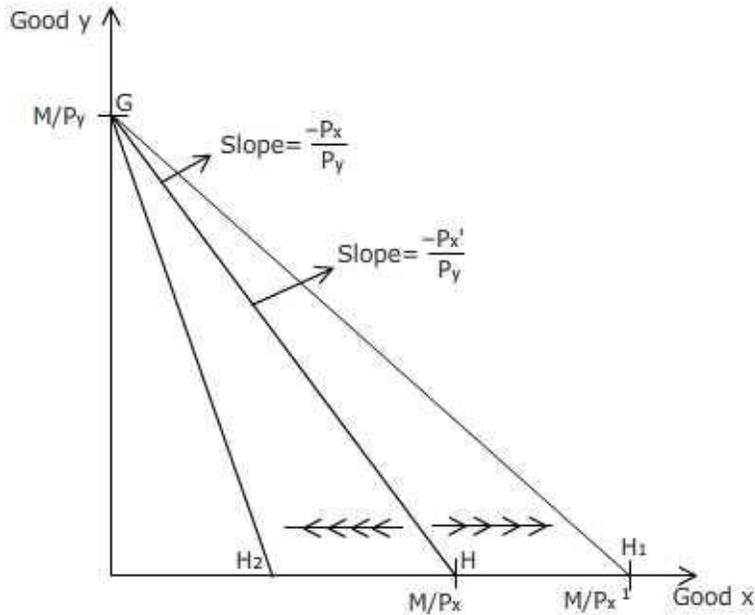
Figure 3: Budget Line pivot around y-axis

Budget line pivots to the right from GH to $GH_1(GH_2)$ if price of good x falls(increases) from p_x to P_x^1 (P_x^2). This fall in price of x allows for increased (decreased) consumption of x while keeping maximum possible consumption of y unchanged. The budget line GH_1 (GH_2) is flatter(steeper) than GH.[3]

Budget line pivot (like in fig.3) occurs in following cases:-

a. Market price of x increases (falls) from P_x to P_x^1 (P_x^2)
b. Good x is taxed & hence its price increases as P_x^1
   A good can be taxed either by levying a tax on per unit of good consumed or by levying tax on the value of good. If a tax is levied on quantity of x consumed then it is called quantity tax. Thus P_x^1 = P_x + t, where 't' is tax per unit x. If a tax is levied on value (price) of good x then it is called ad valorem tax. Thus P_x^1 = $p_x(1+t)$, where P_x is given to the supplier & $P_x t$ is tax collected by government.
c. A Subsidy on good x if announced then consumer faces a lower price. Like taxes, subsidy could be either quantity subsidy or ad valorem subsidy. In quantity subsidy case, $P_x^2 = P_x - s$, where "s" is per unit subsidy. In ad valorem subsidy case, $P_x^2 = P_x(1-s)$.

Point to remember is that a consumer has greater budget set when price of good falls & vice-a – versa.

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[3] To remember slope assume you are on x-axis at points H, H_1 or H_2 you have to drive up to point G. Ask yourself, when it most difficult ? Your answer must be when you are at point H_2, so it is a steep road and flattest is x-axis itself.

2.4.2   Parallel shift is Budget line

When the slope (which is given by given by ratio of prices) of Budget line is assumed constant than budget line shifts parallely.[4]
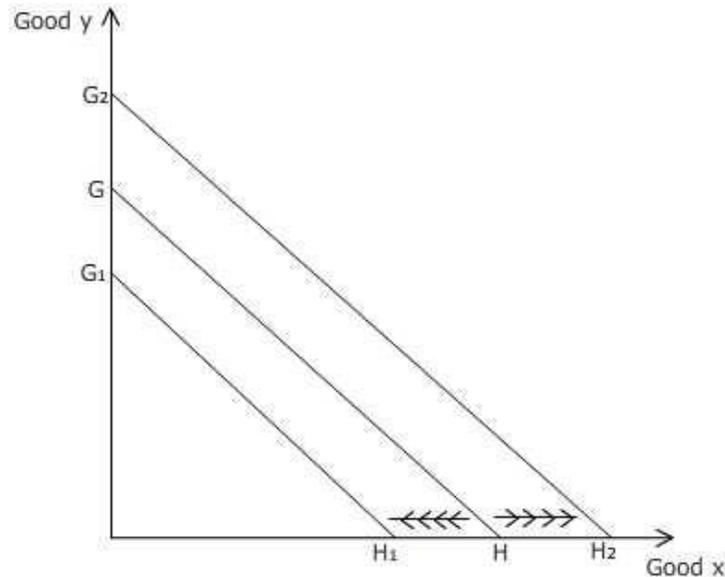

Figure 4: Parallel shifts in budget line

Parallel shift in budget line is brought about in following cases:-

a) When Money income rises(falls) from M toM_1(M_2)   then budget line shifts from GH to GH_1(G_2H_2)

b) When a lump sum tax[5] is changed from consumer, of the amount T (=M-M_1) then budget line shifts from GH to G_1H_1.
   When a lump sum subsidy of amount S (=M_2-M)is given to a consumer then budget line shifts from GH to G_2H_2.

c) When prices change proportionately i.e. the ratio P_x/p_y is held constant then also budget line shifts parallel. If P_x and P_y increase (decrease), then consumer can afford less (more) of both goods, then budget line shifts from GH to G_1H_1(G_2H_2).

2.4.3 Non-Parallel shifts in Budget line

To understand non-parallel shifts one should understand that there is both change of slope and change of both intercepts. A non – parallel rightward shift of budget line from GH to G_1H_1 implies a flatter budget line compared to GH.

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[4] Slope of parallel lines is same.
[5]In lump sum case, a fixed amount is taken away from income irrespective of consumption bundle or prices.
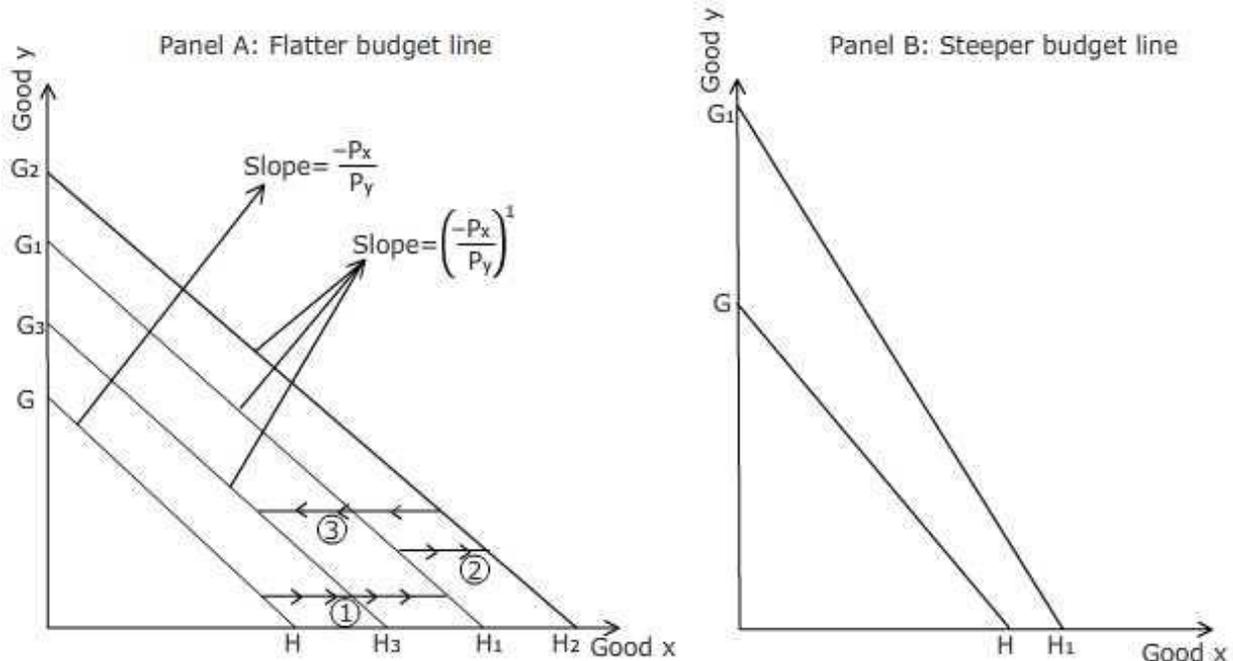
Figure 5 : Non-parallel shifts in budget line

G_1H_1 is flatter implies a lower slope. Let GH's slope be $-P_x/p_y$. A lowering of slope means either P_x falls or P_y increases or both.[6] Now, lets analyze cases in which budget constraint shifts non-parallely rightward:-

    a. P_x & P_y changes disproportionately.

When P_x & P_y both fall but disproportionately, then more of both goods could be consumed. If P_x/p_y falls, then change in budget line to G_1H_1 is depicted in panel A; else if p_x/p_y increases it is depicted is panel B of Fig.5.

    b. P_x & M Changes

For a rightward shift, M must increase and for slope to lower down p_x must fall which is shown by shift from GH to G_1H_1 in panel A of Fig 5. Otherwise, if P_y falls budget line shift is as shown in Panel B of Fig.5.

    c. P_y & M Changes

If M increases along with fall in P_y, then budget constraint shift outward indicating greater quantities of both goods & additional increase in y as its price has fallen would mean a steeper budget line shifted outward as depicted in panel B of fig.5.

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[6] While changing slope take absolute of $-p_x/p_y$ and then analyze steepness or flatness.

SOFT AND HARD BUDGET CONSTRAINT

When we say income of the consumer is given, he can purchase only out of his income; we are referring to 'hard' budget constraint. Softening of the budget constraint means that consumer can spend more than his income due to paternalistic role of the government. This sort of softening is relevant not for consumer households but also for private firms, NGOs and other economic organizations.

Also, apart from this support from government is at all times, is expected by consumer and consumer now behave taking this support into account.

d. P_x & P_y changes disproportionately & M changes.

Assume case 'a' & you reach G_1H_1 from GH in a panel of Fig.5 due to disproportionate change in prices.(1) Assume then, you have extra money income with you which shifts G_1H_1 to G_2H_2 ,(2). If your some money income would have been taken back from your to for the fall in prices your budget line would shift parallel leftward to G_3H_3, which is non-parallel rightward shift of budget line of GH.

For working out, leftward shift reverse all the cases. Start from G_1H_1 & arrive at GH.

2.4.4. Schemes conditioned on/after certain quantity of commodity

a) Rationing[7] Constraint

A constraint could be binding or not. A constraint is said to be binding if could alter your feasible set. Suppose in any economy good x can't be consumed more than quality x_1 by any individual. Then, a constraint is binding if $M/p_x > x_1$ & Consumer's budget line is lopped off, as shown is panel A of fig.5. Else wise, if $M/p_x < x_1$, then budget set is unaltered as shown in panel B of figure 6.

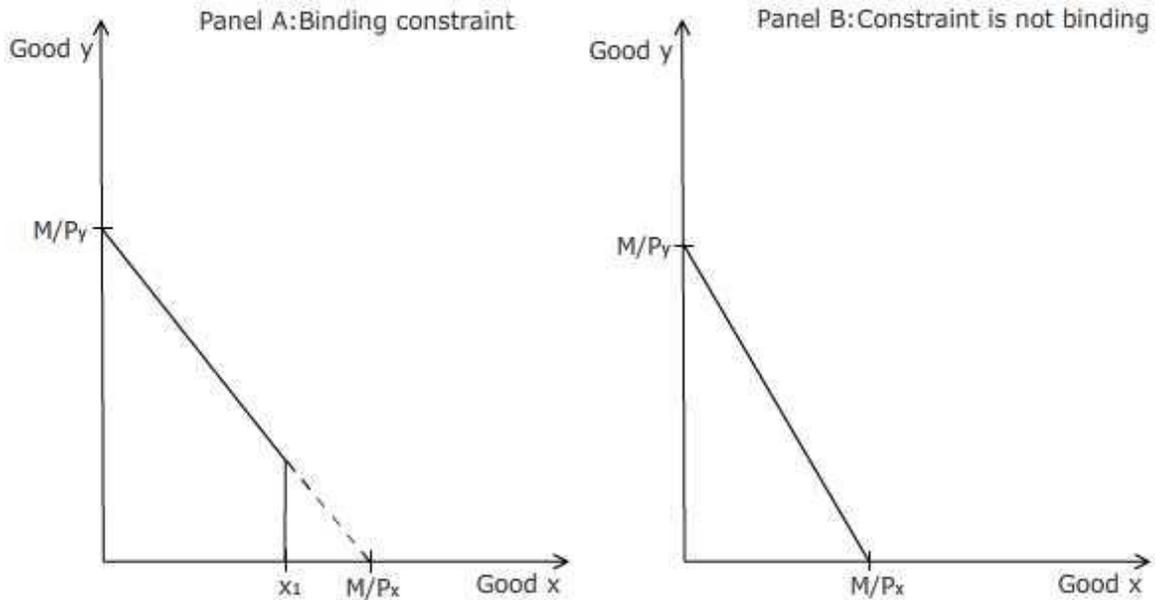[7]Rationing is any method of allowing a scare product or service other than by price mechanism.

Figure 6: rationing Constraints

b) Rationing, taxes & subsidies

If suppose a good is taxed after x_1 units of x are consumed then budget line will have slope $\frac{-P_x}{P_y}$ up to x_1 and then since x is taxed P_x rises & slope changes and budget line becomes steeper thereafter. So there is a link at x_1. This scenario is depicted is following figure.
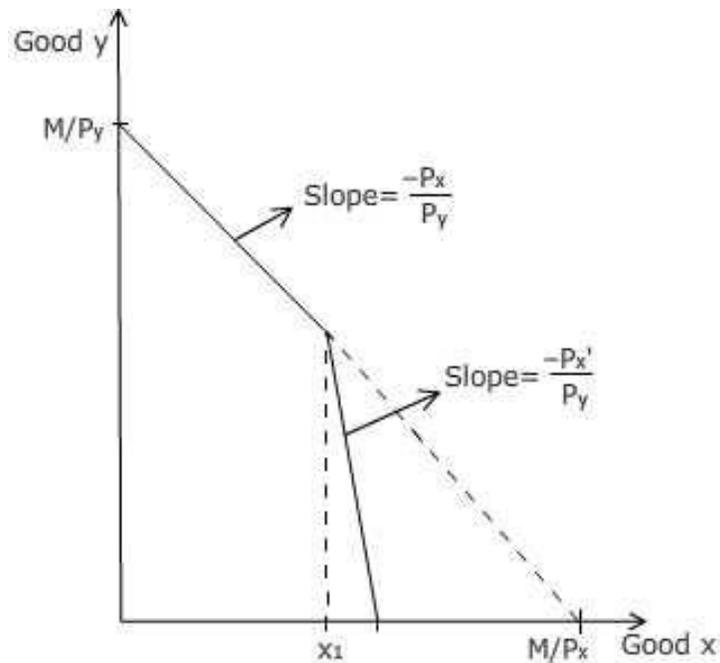

Figure 7: Effect of tax or subsidy, upto x_1

After x_1, since x is costlier you have to give up more of y to get one unit of x. In case of subsidy, subsidy is given up to x_1 change in budget line would be same just – with this assumption, that p_x^1 is original price and p_x is subsidized price.

c) Few free units of Good x to all

Suppose that government wants everyone to consume at least 4 kgs. of wheat (good x here). So, to ensure nutritional security, government announces zero price of wheat up to 4 units, after which market price, P_x is charged. So, now budget line change from GH to GFH.
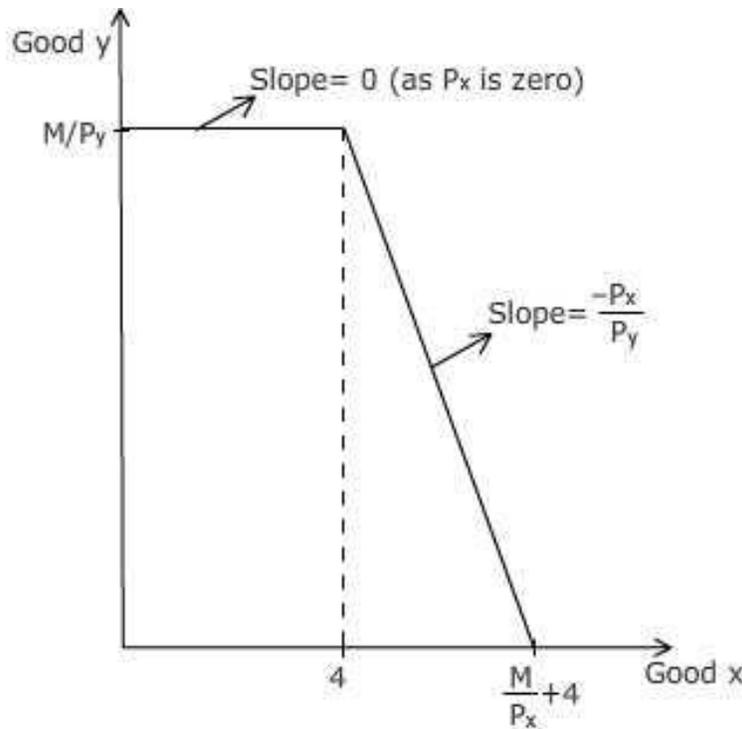

Figure 8: '4' units of x free / costless

d) Buy three get one free

There are schemes where in 4 lux soaps are bundled together & it is labeled that '3+1 free'. In such cases, budget line again changes, which is shown in fig 8. There is a discontinuous portion since as you consume three units of x, you have one extra of good & nothing in between is allowed to consume.
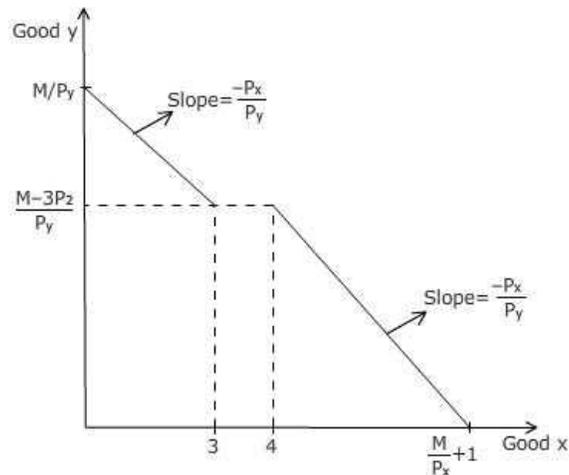
Figure 9: Buy '3' get '1' free

e) Cash Back Offer

Again there are schemes, in which you show wrapper(s) of good x & you some cash in return. It is much like kids collecting points in funflips. Suppose you show 5 wrappers of good x and you get back p_x [8] i.e. price of one unit of x. It is like increase in income after 5 units of x. It is like increase in income after 5 units of x.
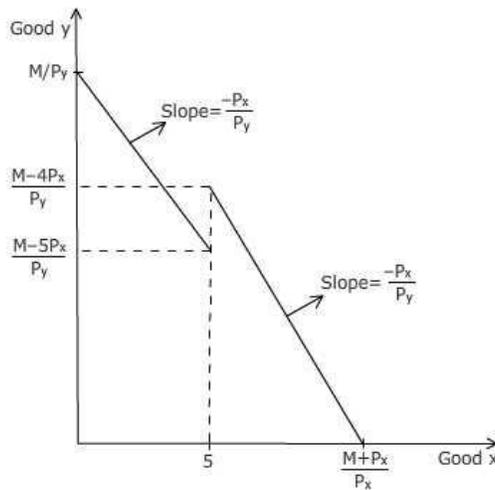

Figure 10: Cash- back offer

Summary

- Budget set is the set of all consumption bundles that are affordable at the ongoing prices in the market; given a consumer's income.

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[8] It could be of any amount and need not only be equivalent to P_x.

- Slope of the budget constraint is negative showing that a consumer has to substitute one good for another. The rate at which consumer is willing to substitute good y for good x is relative price of good x to good y and is denoted by $-p_x/p_y$.
- If price of either or both good falls then consumer can consume more as purchasing power increases and hence, budget set enlarges due to fall in price(s). Price changes can change the slope of budget constraint unless both goods' prices changes proportionately.
- Similarly, if income rises consumer has enlarged budget set. But change in income alone does not change the slope of budget constraint.
- A tax on commodity is viewed by consumer as price rise and subsidy as price fall. So taxes and subsidy treats budget constraint in the same manner as price change. Lump sum tax or subsidy is like decrease or increase in income respectively and hence alters budget constraint like income change.
- Various marketing schemes and government's rationing schemes could also change consumer's budget constraint.

Exercises

Q1. There was a consumer Abel who resided in a country where people only consumed Pepsi and Burger. The price of Pepsi was $1.5 per bottle and price of Burger was $ 2 per unit. Abel's income was $ 30.

a. Draw Abel's budget constraint. Properly label the axis.
b. Suppose he consumes 8 Pepsis and 9 Burgers. Mark such a consumption bundle and check whether is it feasible?
c. Suppose now Pepsi is taxed at the rate of $0.5 per bottle. Show graphically and interpret the impact.
d. If had it been the case that he consumed 8 bottles of Pepsi despite of tax then, his tax would have sumed to $4. Assume instead of tax per bottle, $4 has been taken away from Abel's income and prices remain ($1.5,$2). Compare this tax with earlier sort of tax and its impact on Abel's budget set.

Q2. In a food stamp program in a country, food coupons upto 5 kgs. of grain amounting to Rs.250 are given for Rs. 100. After a consumer consumes this limit, there are no coupons for him and he pays market price for the grain. Assume all other goods on the other axis and its price Rs. 1. Draw a person's budget constraint before and after the food stamp program, if his income is Rs. 1000.

Q3. What happens to the budget constraint in following cases :

a. When both prices and income doubles.
b. Price of good x doubles and that of good y triples.
c. Income of consumer doubles and price of good x is halved.

Q4. Rewrite budget constraint in following cases:

a. Government announces lump sum tax' T', quantity tax on good x of 't' and quantity subsidy on good y of 's'.

b. Price of good x doubles, the price of good y becomes four times larger and income become eight times larger.

Glossary

- **Budget set:** Budget set is the set of all consumption bundles that are affordable at the ongoing prices in the market; given a consumer's income.

- **Budget constraint:** Budget constraint is a line showing (locus of) all affordable bundles at which entire income is spent.

- **Budget line pivot:** A budget line is called pivoted when slope of budget line changes & purchasing power stays constant.

- **Lump sum tax(subsidy):** In lump sum case, a fixed amount is taken away from income irrespective of consumption bundle or prices.

- **Quantity tax(subsidy):** . If a tax is levied on quantity of x consumed then it is called quantity tax. Thus $P_x^1 = P_x + t$, where 't' is tax per unit x.

- **Ad valerom tax(subsidy):** If a tax is levied on value (price) of good x then it is called ad valorem tax.

- **Rationing :** Rationing is any method of allowing a scare product or service other than by price mechanism.

Appendix

A.1 Budget constraint for n-goods case

Let,   **P** be price vector of order n.

X be goods quantity sector of order n.

Dot product of P & X gives total expenditure which is equated to money income M on budget constraint:

PX=M

A.2   slope of Budget Line

Budget line is given by: $P_x .x + p_y . y = M$

Take total differential of above equation, and we get

$P_x \Delta x + p_y \Delta y = 0$

On rearranging we get,

$\Delta y/\Delta x = -p_x/p_y$

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C. Snyder and W. Nicholson, Fundamentals of Microeconomics, Cengage Learning (India), 2010.