

LESSON: Preferences and Indifference Curves

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## **Learning Outcomes:**

After studying this chapter, a student should be able to:-

1. Explain rational behavior of a consumer.
2. State axioms of preferences.
3. Define utility function.
4. List properties of Indifference curve.
5. Sketch indifference curve map for every preference.
6. Calculate and interpret slope of indifference curves.
7. Define satiation point.

## 1. Introduction

Consumers are rational decision makers in the sense they know their preference over goods and buy goods that get them maximum satisfaction. In this chapter we will assume consumers to be rational and study preferences and utility derived from various bundles of commodities. Only difference between actual consumer in the market and our study is that here we would have calculus applied to the situations consumer faces in the market.

This chapter is divided into three sections. First section covers axioms of preferences. In the second section, utility function is derived. In the last section, indifference curves are detailed at length.

## 2. Preferences

If I ask you, what would you like to have in dinner: fried rice or chowmien? I would receive three of these answers: (a) fried rice, (b) chowmein, (c) either of these. Irrespective of your answer choice, I would conclude something about your preferences either you like fried rice over chowmein or chowmein over fried rice or like both equally. As in this example commodities are ordered/ranked so could be bundles of commodities.

### 2.1 Axiom of preferences

While ranking of commodity bundles, consumer should, however follow some logical reasoning, and we can say a consumer is rational and consumer's preferences are consistent. Following are listed few of the assumption about consumer preferences which are called axioms of consumer theory.

Let us understand the notation used for it. '>' symbol is used when one bundle is strictly preferred over other. '~' symbol is used when two bundles give equal level of satisfaction to the consumer. When consumer prefer or is indifferent between two bundles over other and  $\geq$  symbol is used to compare two same bundles.

#### 2.1.1 .Complete

Axiom of completeness states that two bundles can be compared. Assume two bundles 1 and 2; both comprising of goods x and goods y.

- (a) Either  $(x_1, y_1)$  would be preferred over  $(x_2, y_2)$  :  $(x_1, y_1) \geq (x_2, y_2)$
- (b) Or  $(x_2, y_2)$  would be preferred over  $(x_1, y_1)$  :  $(x_2, y_2) \geq (x_1, y_1)$
- (c) Or both, which means that you consumer is indifferent between two bundles.

## 2.1.2 Reflexive

If a bundle is as good as itself then it follows reflexive. For example: a cold drink bottle on left is as good as on the right. Both the bundles contain one cold drink bottle. For kid, this sort of assumption could be seen invalid but not for adult, at least.

## 2.1.3 Transitivity

If bundle 1 is preferred over bundle 2 is preferred over bundle 3. Suppose you prefer studying in India over US and US over Australia, then it is seen that given all three choices, you must choose India

This assumption helps in forming the best bundle choice.

### Example 1

Consider the following binary relation defined over where  $x$  is set of human beings. Check if each of these relations satisfies reflexivity, completeness and transitivity.

i). At least as tall as

Assume three individuals; A, B, C

A could be taller than, shorter than or of same height as of, B.

A is at least as tall as B could be used in first & third instance or B is at least as tall as A can be used in second & third instance. Since, this relation helps in comparing A & B. This relation is complete.

A is at least as tall as A also makes sense hence this relation is reflexive.

If A is at least as tall as B, & B is at least as tall as C then A is at least as tall as C. Even if three would be of same height this statement would still hold. Hence, this relation is transitive.

ii) 'Taller Than'

A could be taller than, shorter than or of same height as of, B.

This relation is not complete since in third case of same height this relation cannot be used. It is not reflexive since A can't be taller than her/herself.

If A is taller than B & B is taller than C then it follows that A is taller than C. So this relation is transitive.

### 3. Utility

In the last section, consumer preferences were discussed, where in at a time two consumption bundles were compared & ordered (ranked). Indifferent, strictly preferred and weakly preferred, all are binary relationships defined over bundles of commodities. It would be easy if we could assign numeric values to the bundles in such a manner that preserve the ordering of bundles.<sup>1</sup> Preserving the order of consumption bundles, refer to assigning higher value to preferred bundle than less-preferred bundles.

#### Did You Know??

Utility is a concept that was introduced by Daniel Bernoulli. He believed that for the usual person, utility increased with wealth but at a decreasing rate.

#### 3.1 Assigning Utility

It is a function mapping of a consumption of a bundle to a unique, real number representing utility desired from that bundle.

$U: (x,y) \rightarrow$

So, utility function is a real valued function

If a bundle  $(x_1, y_1)$  is strictly preferred over  $(x_2, y_2)$  then  $u(x_1, y_1) > u(x_2, y_2)$

There are two set of theories/approaches to measure utility.

##### a) Ordinal Utility

As the name suggests, ordinal is, only order matters. Utility can be assigned to different consumption bundles irrespective of the magnitude, as long as the ordering of preference is maintained for a particular consumer. For ex: A consumer prefers Bundle 1 to bundle 2 then, Utility assigned to bundle 1 can be 1, 10 or 1000 provided that every time bundle 2 has "7" and bundle 3 has 90, then it could be inferred that bundle 3 is most preferred bundle, next to it is bundle 1 and least preferred one is bundle 2. But it cannot be inferred from the above information that bundle 3 is 9 times better bundle than bundle 1.

##### b) Cardinal Utility

There are economists who hold that magnitude of utility is of significance. This is the known as cardinal utility assignment to consumption bundles.

If in above example, it could be said with precision that utility of bundle 1 is 10 and of bundle 3 is 90: then it means consumer like bundle 3 times as much as bundle 1 if consumer is ready to pay nine times the price of bundle 1, for bundle 3.

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<sup>1</sup>Here axiom of transitivity should be followed for consistency of values assigned.

### 3.2 Positive Monotonic Transformation

If  $g$  is some increasing function (like  $g(x) = x^2$  or  $x^2$  in  $x$  or  $e^x$ , Then  $g$  of utility function would not alter the order of preferences though the magnitude would so, while dealing with ordinal utility function, a positive monotonic transformation would not change the analysis.

If a bundle  $(x_1, y_1)$  is preferred over  $(x_2, y_2)$  then  $u(x_1, y_1) > u(x_2, y_2)$ , if  $g$  is positively increasing function then  $g(u(x_1, y_1)) > g(u(x_2, y_2))$  where  $g(u(\cdot))$  is positive monotonic transformation of  $u$ . This monotonic transformation of a utility function represents same preferences as the original utility function.

Example:

Bundle\Utility	First way $U_1$	Second way $U_2$	Positive monotonic transformation $U_1 * 2$	Negative monotonic transformation $U_1 * (-2)$
1	9	3000	18	-18
2	10	4000	20	-20
3	11	5000	22	-22

In column 2, bundles are given utility in two digit column 3 bundle 1-3 are assigned utilities in thousand but order of preference is bundle 3, 2 and 1.  $U_1$  and  $U_2$  are ordinal utilities where preference of order is important. In column 4, positive monotonic transformation of  $U_1$  function is computed by doubling  $U_1$ . This does not alter the order of preferences of bundle 1-3. But a negative monotonic transformation represented in column 5 by  $(-2 U_1)$ , reverse the order of preference. Least preferred bundle 1 becomes most preferred; as -18 is greater than -20 and -22.

### 3.3 Total and marginal utilities-one good case

When more of good is consumed, total utilities goes on increasing. Ask yourself, would you prefer two pair of shoes or one? Obviously, your answer would be two. But, when you buy first shoe its utility is highest because that one is first one in your wardrobe. Another one

Adam Smith presented classical paradox called ' Water-Diamond'. In statement of this paradox he proposes, that, water which is more valuable but is priced less than diamond. He could not resolve this paradox.



Later, the marginal-utility theory of value resolved the paradox. Water in total is much more valuable than diamonds in total because the first few units of water are necessary for life itself. But, because water is plentiful and diamonds are scarce, the marginal value of a pound of diamonds exceeds the marginal value of a pound of water. It can be assumed that for water we are at a point on MU curve at higher quantity and for diamonds we are still at a point with less of Diamond (i.e. commodity on x-axis) is consumed.

have some utilities but obviously less than first one and so on .These two argument are consistent since former is in relation to total utility and latter to marginal utility. Total utility always increase with increase in number of units consumed. While addition made to this total utility falls.

$$\text{Marginal utility} = \frac{\Delta U}{\Delta x} = \frac{U(x_2) - U(x_1)}{x_2 - x_1} = MU$$

If  $x_2 > x_1$ , then  $U(x_2) > U(x_1)$ , hence marginal utility is positive. Though it falls if  $x_2$  increases to  $x_3$ <sup>2</sup> that is  $MU'$  is smaller than  $MU$  measure, where  $MU'$  is given by

$$MU' = \frac{U(x_3) - U(x_2)}{x_3 - x_2}$$

## 4. Indifference Curve Analysis

There could be various consumption bundles amongst which a consumer is indifferent .For example two serving of rice with three chapattis could give same satisfaction as three serving of rice with two chapattis .All such a combination give same fixed level of utility to

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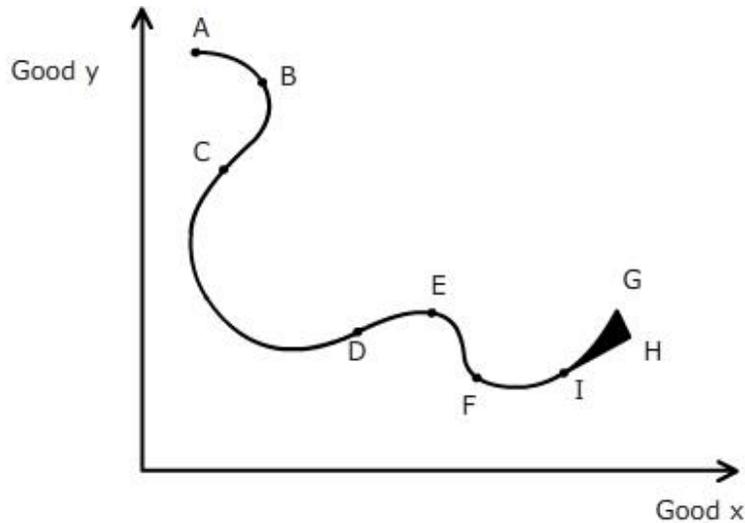
<sup>2</sup> Please see appendix to this chapter for the calculus treatment of it.

consumer. Indifference (referring to bundles for which a consumer is indifferent) Curve is locus of all such consumption bundles which yield same level of satisfaction/utility. Indifferent curves are also known as Iso-utility curve.

### 4.1 Well Behaved Indifference Curve:

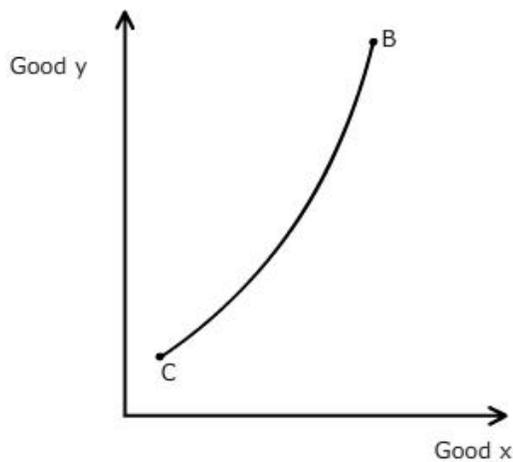
Let us consider following arbitrary indifferent curve.

Figure1 Assumed Indifference curve



Let us assume that the above curve is locus of all consumption bundles at which consumer has some constant utility throughout. There are upward sloping and downward sloping sections<sup>3</sup> of our assumed indifference curve. A positively sloped indifference curve would mean that more of both commodities like at point B and less of both commodities like at point C

Figure2 positively sloped indifference curve



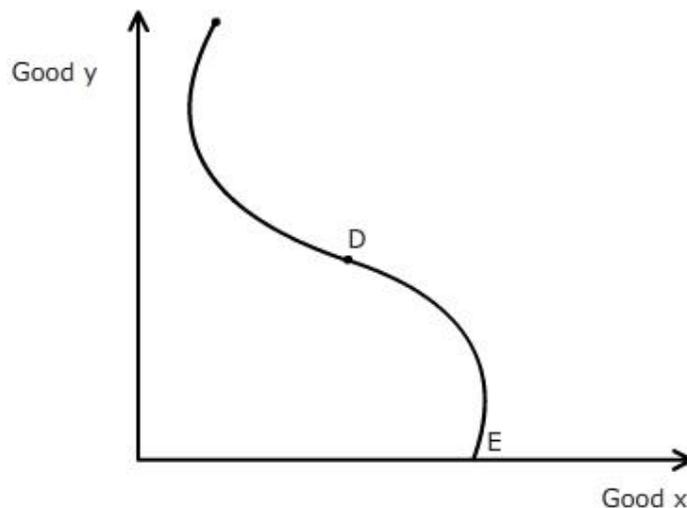
<sup>3</sup> BC ,DE are upward sloping section and AB,CD, and downward sloping section of indifference curve here.

both commodities; give same level of utility to consumer. But is it consistent? No, strictly more of both commodities should add utility for the consumer.<sup>4</sup>

So, now our indifference curve would not be upward sloping. We can now have only indifference curve which is downward sloping. A downward sloping indifference curve means that as we move along the indifference curve from point C to point D (in figure 3) would mean more of goods x with less of goods y. So good y is sacrificed so that addition made to utility from extra consumption of x, is equivalent to loss of utility due to reduction of y. CDE is negatively sloped<sup>5</sup> but with D as point of inflexion<sup>6</sup>.

CD section of assumed indifference curve is convex to the origin while DE section is concave to the origin.

Figure 3 negatively sloped indifference curve with point of inflexion



Now, next have to examine whether indifference curve need be convex or concave or could be both. Let us examine concave section DE, first. In figure 4, on x-axis change in units of x is assumed to be 1 unit and change in y is due to change in x.

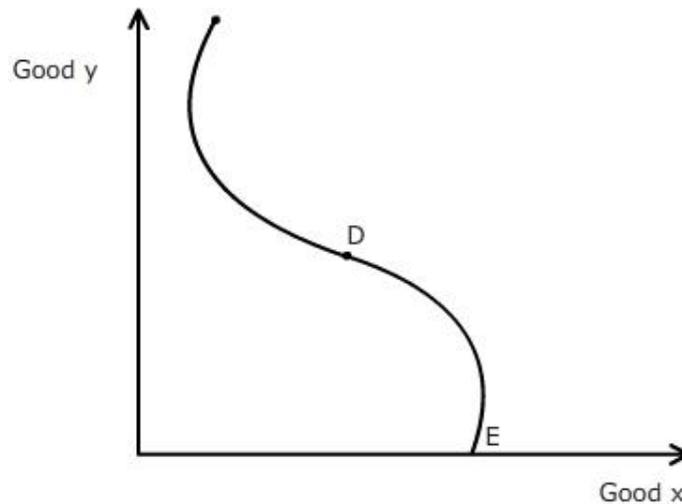
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<sup>4</sup> Though addition made would be declining (due to diminishing marginal utility), but both add to positive utilities. Unless either good becomes 'bad', this is discussed in sections to come.

<sup>5</sup> You can check its slope as negative or downward by drawing tangent at various points.

<sup>6</sup> Point of inflexion is a point at a curve from where the curvature of a curve changes.

Figure4 concave indifference curve



To increase consumption of x by 1 unit, change in y (along DE) goes on increasing. But from our knowledge of marginal utility, marginal utility of a good is high at lower levels of consumption and low at higher levels of consumption. So as we move from point 1 to 2 and further towards E ; loss of marginal utility from reduced y is greater than additional of marginal utility from increased x. The reason for the same being y is lost at greater pace than addition made to x .So, indifference curve cannot be concave<sup>7</sup>.

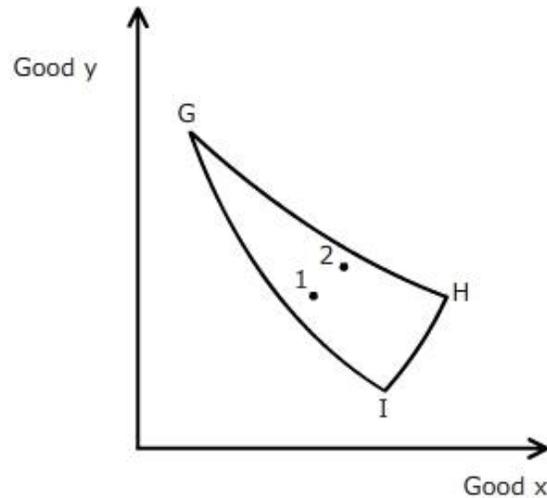
Indifference curve, so assumed in figure 1 had a thicker section GHI reconstructed here. It can have bundles 1 and 2 and many such bundles. Here bundle 2 contains more of both goods compared to bundle 1 and hence have higher utility.<sup>8</sup> So, assumption that these points are on same indifference curve is violated.

Figure5 Thicker section of indifference curve

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<sup>7</sup> Concave indifference curve can be observed and will be explained in coming sections.

<sup>8</sup> This property is known as assumption of monotonicity.



So, indifference curve sections excluded from our assumed shape of indifference curve are

- (1) upward sloping
- (2) concave to the origin
- (3) Thicker line.

## 4.2 Properties of indifference curves

### 4.2.1 Negatively sloped:

Indifference curve is negatively sloped as utility addition due to increase in one good should be compensated by decrease in other good.

### 4.2.2 Thinner lines :

Indifference curve should be thin curves as thicker lines would have areas of strict preference.

### 4.2.3 Convexity:

Indifference curve must be convex. Convexity would imply that fall in y- accompanied with increase in x- should get smaller as more of x is added. Look at the following two diagrams and the explanation that follows.

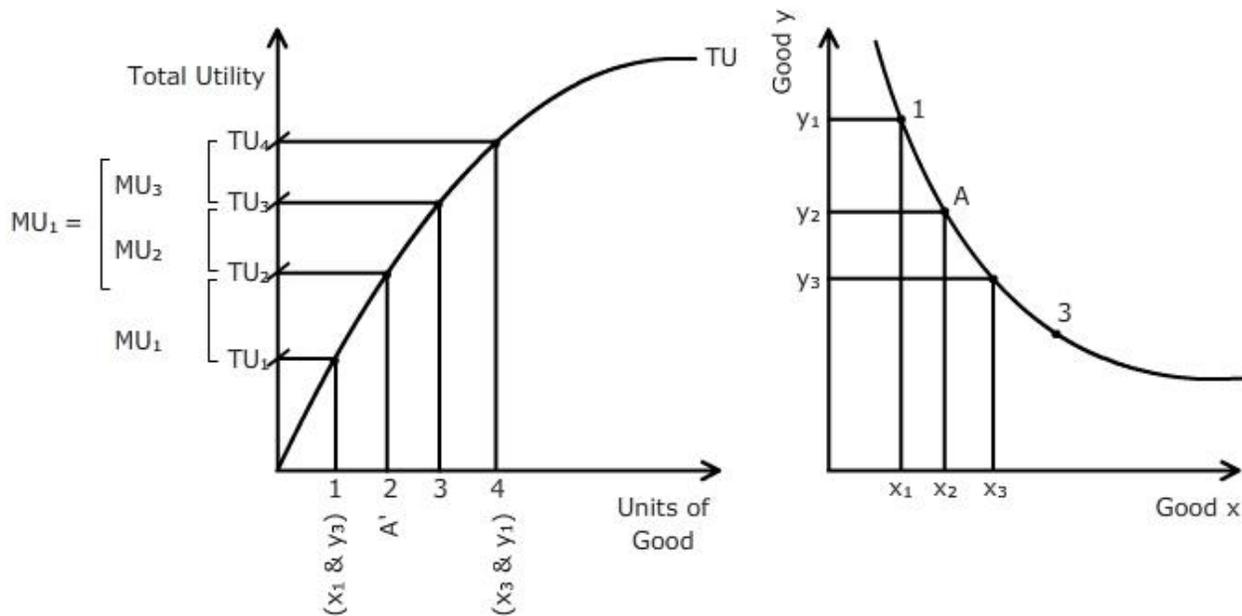


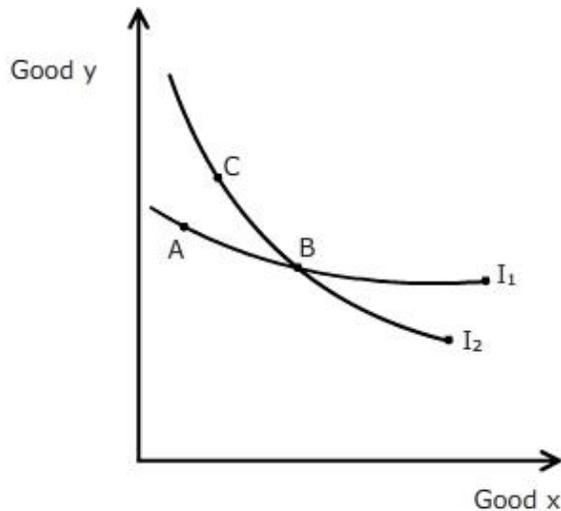
Figure6 (a) total utility curve, (b) Indifference curve

At point 1 on Indifference curve  $I_0$ ,  $x_1$  of x and  $y_1$  of y is consumed that yields utility of level  $I_0$ . Panel B of figure 6 shows corresponding utility achieved from these consumptions. When one moves to point A, consumption of y decrease to  $y_2$  and of x increases to  $x_2$ . This leads to increase in utility from  $TU$  to  $TU_1$  so  $MU_1$  is added. To compensate, there should be greater fall in y (more than one unit) since fall in one unit of y decreases utility by  $MU_3$  (which is less than  $MU_1$ ). So, y must fall by two units (since  $MU_3 + MU_2 = MU_1$ ). Further, if x is added then  $TU_3$  is achieved and change is  $MU_2$  but now y must fall by less than a unit since again fall in utility due to y ( $MU_1$ ) would be greater than rise in utility from x ( $MU_2$ ). A change in y should be brought such that loss in utility equals  $MU_2$ .

#### 4.2.4 Indifference curves do not intersect

Indifference curves should not intersect so as to retain axiom of transitivity. Let us consider that indifference curve intersect as shown in figure 7.

Figure7 intersecting indifference curves



Point C compared to point A yields more of utility as described by monotonicity. A consumer is indifferent between A and B (being on same indifference curve  $I_0$ ) and also, indifferent between B and C (being on same indifference curve  $I_1$ ). So by transitivity consumer should be indifferent between A and C which is not the case. Hence, axiom of transitivity is violated.

### 4.3 Well behaved preferences and Indifference curves

The assumption of monotonicity assumes that more of both the goods are better. Hence, higher the indifference come higher is the utility as is shown in fig. 8. The shaded area in fig 9, shows weakly preferred set to  $(x_1, y_1)$ .

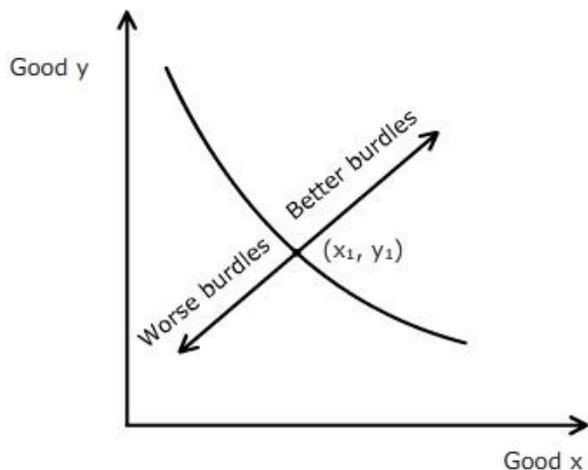


Figure 8 Monotonicity

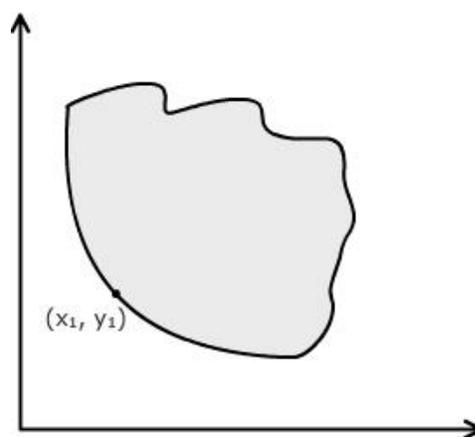
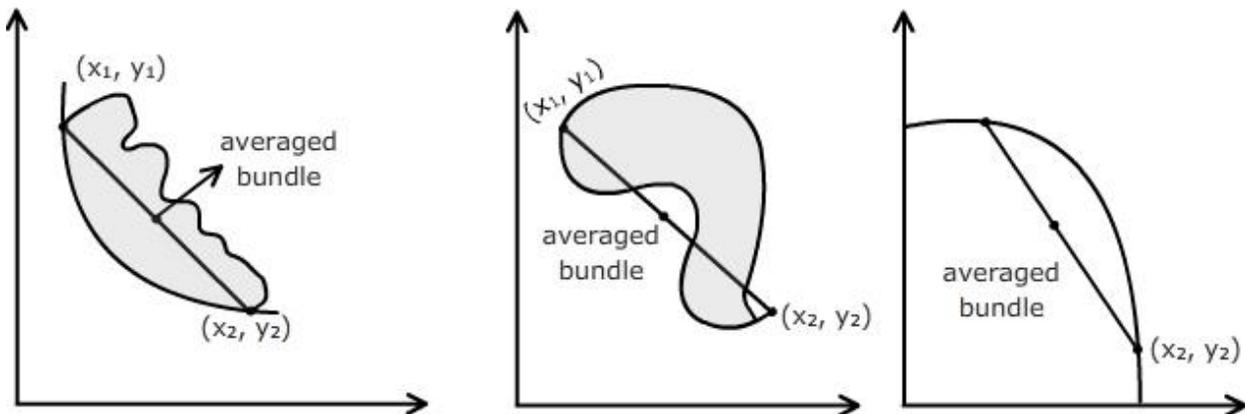


Figure 9 weakly preferred set

The second assumption for well behaved preferences is that averages are preferred to extremes. Let  $(x_1, y_1)$  &  $(x_2, y_2)$  be on the same indifference curve in fig. 10. Then average of these two extreme boundless lie above the curve. And, this average would be in a weakly preferred set iff indifference curve is convex. Consider other (non convex) cases in panel b & c of fig 10.

Figure 10



#### 4.4 Satiation point

The level of consumption beyond which utility attached to an extra unit of the good is zero or negative is known as satiation level or saturation level. Assume two goods, chapatti and vitamin B tablets. Up to a limit, both are needed and add to utility. Say for individuals 6 chapattis and two tablets are required for healthy living, beyond this level utility falls. Let us denote such respective levels by  $x_1$  in fig. 11( it is done for sake brevity).

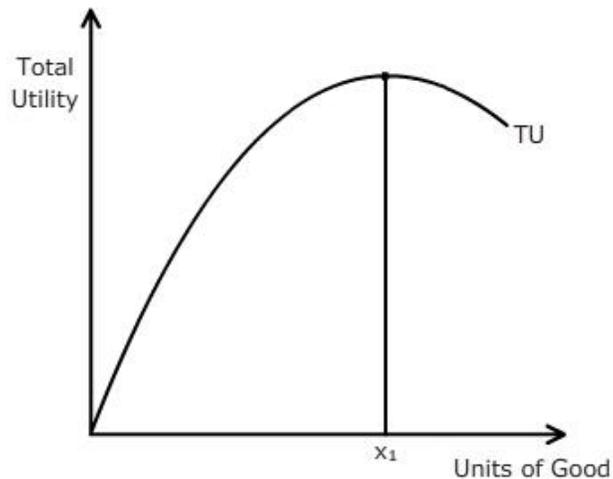


Figure11 total utility declines after  $x_1$

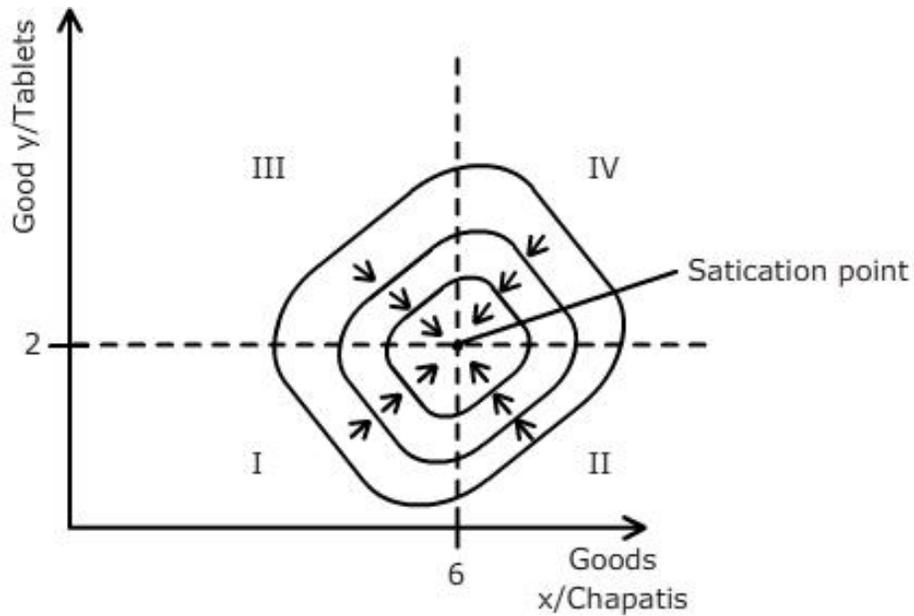


Figure 12 satiation point

The arrows in this figure, show chapatti direction of maximal increase.

Highest level of utility is achieved is at ( 6,2) which is known as bliss point ( fig.12) Indifference curve is downward sloping in quadrant 'I' showing that one good has to be sacrificed to add extra unit of good to keep utility constant

Figure13 both are 'good'

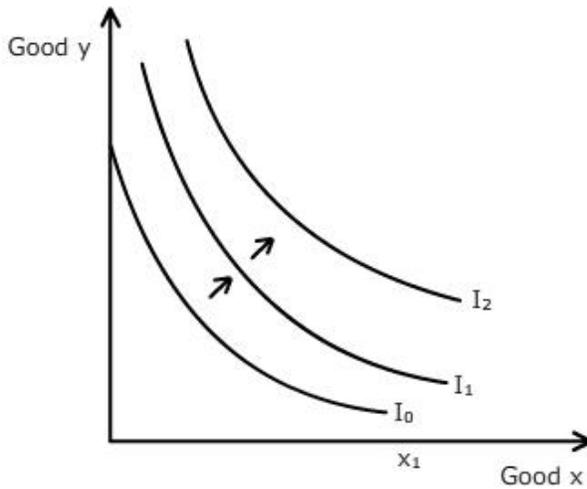
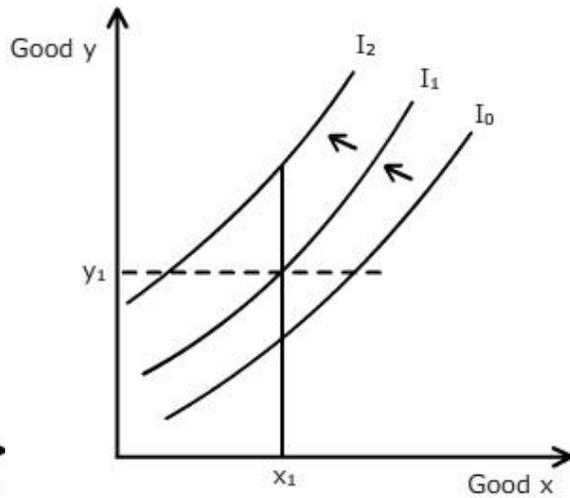


Figure 14 x is 'bad'



In 'II', Good x (chapatti) becomes 'bad' i.e. add to disutility (you can think beyond this he can have digestive order). But good on y – axis, tablets is yet a 'good'. Indifference curve is reproduced again in fig14. The disutility from addition of x is to be compensated by utility from good y. Indifference curve I<sub>2</sub>, denotes higher utility than I<sub>1</sub> and I<sub>0</sub>. At I<sub>2</sub>, y<sub>1</sub> level of y is combined with smaller level of bad x compared to I<sub>0</sub>, hence I<sub>2</sub> shows greater utility.

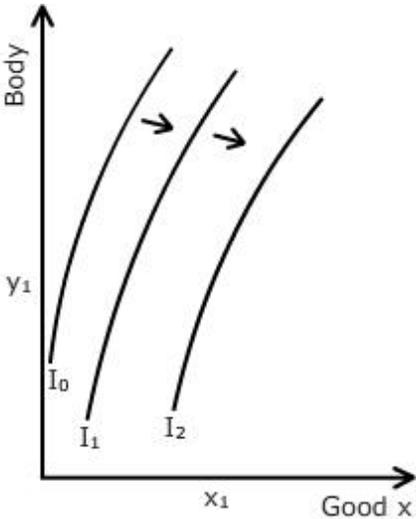
Alternatively, with  $x_2$  level of bad  $x$  higher level of good  $y$  is combined at  $I_2$  indifference curve.



For Scooby and Shaggy facing monster is a bad.... So to keep them at same utility velma needs to offer Scooby Snax(good).

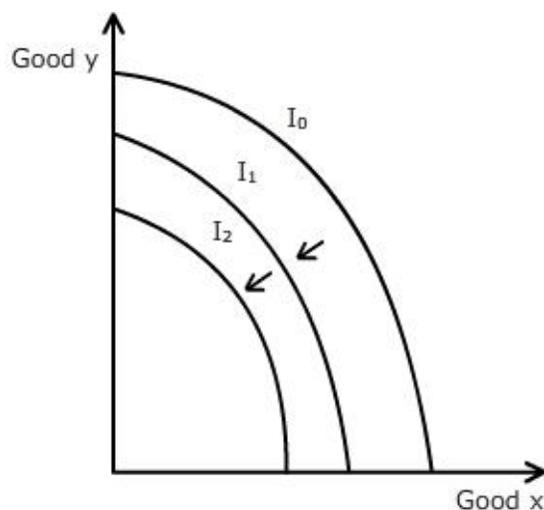
In 'III', good on  $y$  - axis becomes 'bad' & good  $x$  is yet 'good'. The indifference curves are reproduced in fig.15.

Figure15  $y$  is 'bad'



In 'IV', both goods become 'bad'. Indifference curves are negatively sloped but direction of maximal increase is towards origin & indifference curves are concave to the origin as shown in fig 16. The indifference curves are circles with satiation point at the centre.

Figure16 both are 'bad'



## 4.5 Marginal Rate of substitution

The slope of indifference curve<sup>9</sup> is known as marginal rate of substitution. The slope indicates change in y when x changes by 1 unit. For indifference curve, it would mean that y is substituted for x in a way that utility is held constant. And, as we saw in earlier sections; at smaller levels of y, smaller sacrifice would be made as loss in utility from sacrificing 1 unit is high and at greater levels of y, larger sacrifice could be made. Hence, marginal rate of substitution is diminishing<sup>10</sup> along the indifference curve (assuming convexity and monotonicity). Figure17 shows  $\Delta y$  is declining<sup>11</sup> for 1 unit addition in good x.

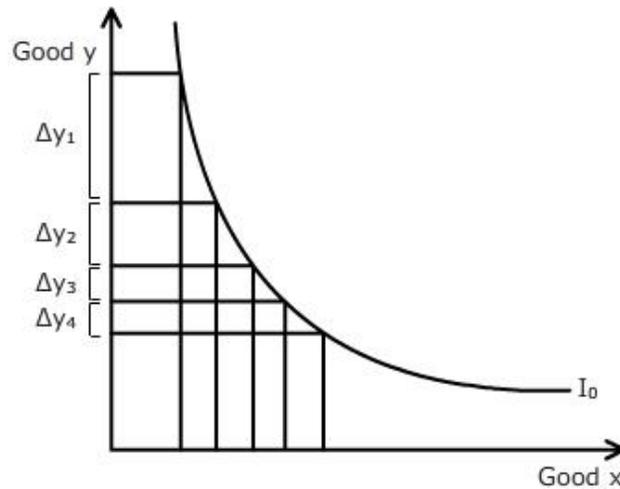
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<sup>9</sup> Slope of indifference is different at at different points of indifference curve. Only for a straight line slope is constant.

<sup>10</sup> It is the absolute number which is referred here.

<sup>11</sup> Also see appendix to this chapter for calculus treatment of it.

Figure17 diminishing MRS



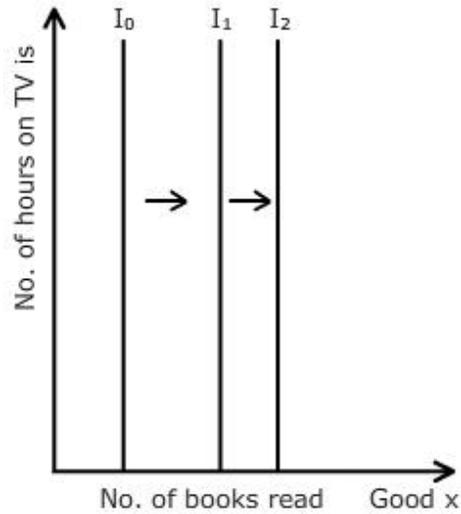
## 4.6 Special cases of preferences & their indifference curves

In this section, we would cover various forms of utility functions and analyze marginal rate of substitution of their respective indifference curves.

### 4.6.1 Neutral Good

A good is said to be neutral if any quantity of a good can be added & no change in utility is brought about. Assuming good y is neutral & good x is good then indifference curves are vertical and utility increases with increase in consumption of x only. Example of such a good ( neutral ) could be that consumer does not care about how many hours T.V runs in his room but he cares how many books he get to read.

Figure18 good on y axis is neutral

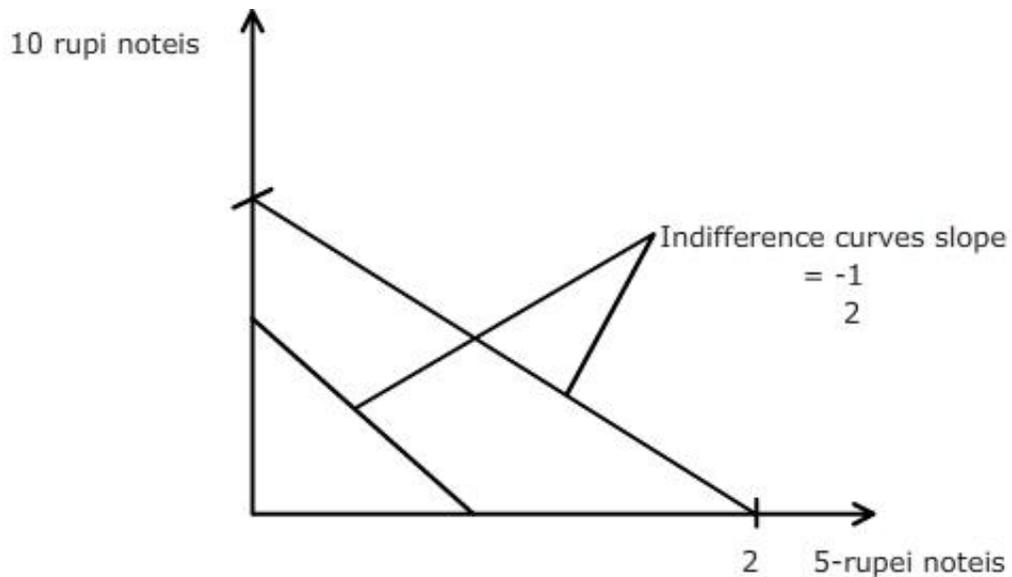


#### 4.6.2 Perfect substitutes

Two goods are substitutes if the consumer is willing to substitute one good for another at a constant rate. Constant marginal rate of substitution means that indifference curves are straight lines.

One 'Ten rupee bank note' is perfect substitute of two 'five rupee bank note'. If ten rupee note is denoted by  $y$  & five rupee note by  $x$ : then utility  $U = x + 2y$ .

Figure19

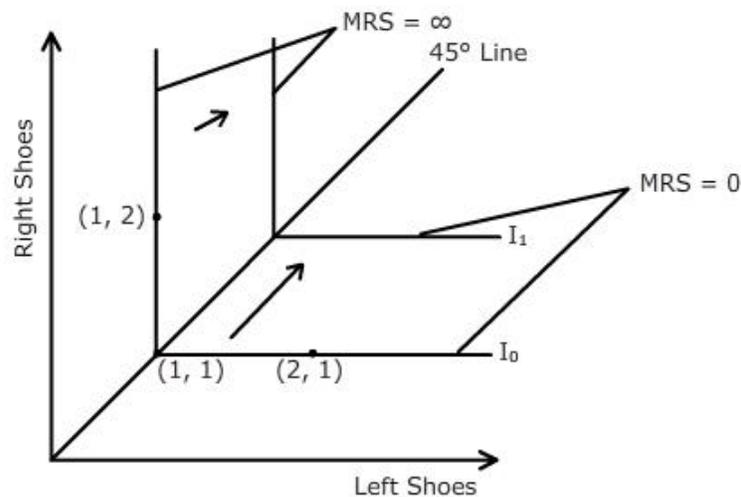


The trick to remember that what should be coefficient of x & y is utility function is: since, 1 unit of y gives twice the utility compared to 1 unit of x. Likewise, if 'a' units of y gives same utility as 'b' units of x then utility function becomes:  $U = ax+by$  and  $MRS = -a /b$

### 4.6.3 Perfect Compliments

Perfect complements are goods that are consumed together in fixed proportions. Best example quoted for this is left shoes & right shoes. Bundle (1,1) gives same utility as (2,1) or (1,2) because extra of either of these would not be useful & hence, no utility is gained.

Figure20 perfect compliments



Indifference curves for perfect complements are hence L-shaped with kink at 45° line (because of 1:1 ratio). Slope at vertical portions is infinity as  $\Delta x$  (denominator) is zero & slope at horizontal portion is zero as  $\Delta y$  (numerator) is zero. If two teaspoons of sugar is added to one cup of tea then indifference curves are as in panel A of fig 21. For a general case, where 'a' units of x are consumed with & 'b' units of y indifference curves are as shown in panel B of fig.21.

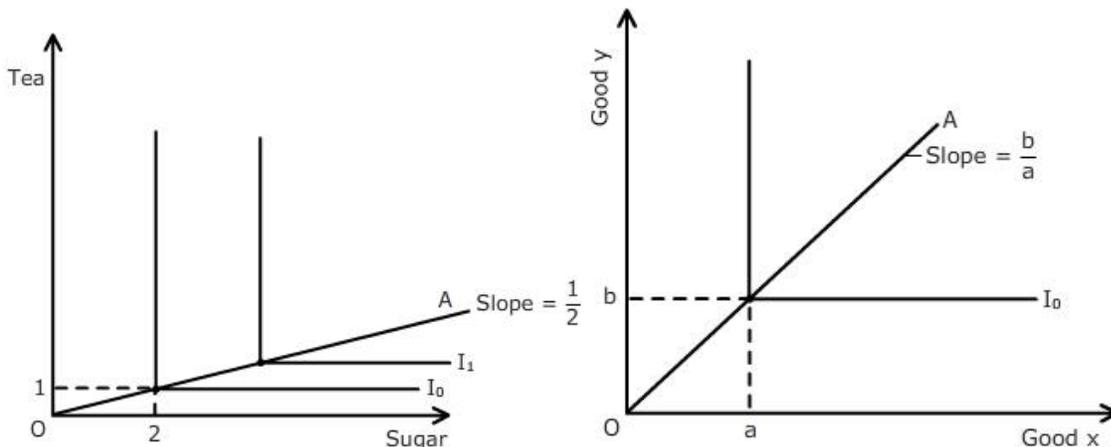


Figure 21: (a) 2 tps of sugar is taken with 1 cup of tea; (b) 'a' units of x is taken with 'b' units of y

The utility function is written as follows:-

$U = \min\{x, y\}$ ; if one unit of good y is consumed with 1 unit of x.

$U = \min\{2y, x\}$ ; for sugar (x) & tea (y) example.

&  $U = \min\{bx, ay\}$ ; if 'a' units of x are consumed with 'b' units of y.

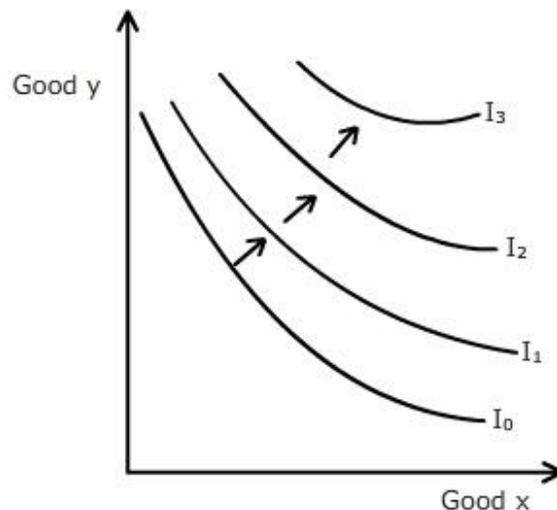
In the form  $U(x_1, x_2) = \min\{x, y\}$  utility would be minimum of two levels of consumption of goods. Consider shoes example:

Utility,  $U=1 = \min[1, 1] = \min[2, 1] = \min[1, 2]$

#### 4.6.4 Cobb Douglas Preferences

Cobb Douglas utility function has convex shaped indifference curves. Utility function takes the form:  $U(x, y) = x^a y^b$ . The indifference curves appear like in figure 22.

Figure 22 Cobb Douglas preferences



Marginal rate of substitution of x for y is given by:

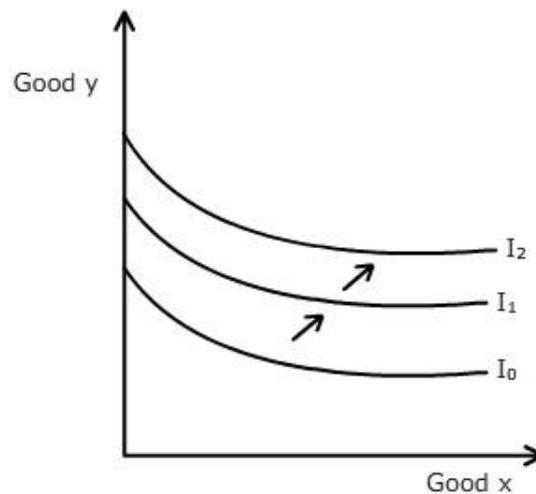
$$MRS = -MU_x / MU_y = -ax^{a-1}y^b / bx^ay^{b-1} = -ay/bx$$

#### 4.6.4 Quasi linear preferences

Quasi linear, as the name suggests utility function is linear but in one good and non-linear in other. Let utility function be linear in good y and non-linear in good x. then, utility function:

$U(x,y) = v(x) + y$ . Here utility is equal to the height of indifference curve along the y-axis i.e. when x is zero, utility is equal to consumption of y.

Figure23 Quasi linear preferences

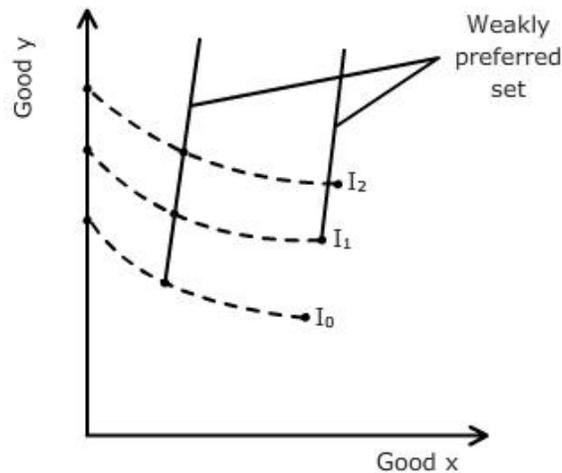


The indifference curves are just vertically shifted versions of one indifference curve.

### 4.6.5 Discrete goods

All the cases considered till now assumed that goods are continuously divisible. There are many goods like milk, rice, wheat that can be consumed in fraction. But there are goods like automobiles, smart-phones, TV sets which are discrete in consumption.

Figure24 discrete goods



The dashed lines connect indifferent bundles (though consumption at other than dots are not possible). Strong lines show weakly preferred to  $(x_1, y_1)$

## Summary:

(1) Three axioms of preferences viz. completeness, reflexivity and transitivity are made about the consistency of consumers' preferences.

(2) Utility function is a mapping of consumption bundles to their respective real values. While ranking bundles, order of preferences only matters and this approach is known as ordinal utility. Positive monotonic transformation of utility function is always consistent with original ranking of bundles.

(3) Properties of indifference curves are as follows:

(a) Negatively sloped, (b) convex to the origin, (c) curves are thin, (d) indifference curves do not intersect and (e) higher the indifference curves higher the utility.

(4) Beyond a point /level of consumption of any good, disutility is generated i.e. too much of a good or negative utility. At this level, utility is maximum and such a point is known as satiation point.

(5) Well behaved preferences exhibit declining marginal rate of substitution i.e. sacrifice of  $y$  each time when  $x$  is increased; keeps on declining.

(6) Indifference curves that are convex (but do not general smooth curve with declining MRS) are for perfect substitutes, perfect compliments, and quasi-linear preferences and even for discrete goods.

(6) Goods can be good, bad or neutral. If a good is good then it adds to utility when its consumption is increased else add to disutility (negative utility) if bad and zero utility if neutral.

## Exercises

Q1. Sumit has Cobb –Douglas preferences and his utility function is given by  $U(x,y) =xy$ . State true or false about the following statements considering sumit’s preferences:

- a)  $(10,5) \sim (5,10)$
- b)  $(20,5) \geq (4,25)$
- c)  $(15,4) \geq (7.5,7.5)$

Q2. The marginal rate of substitution for sumit with utility function is  $U(x,y)=xy$ ; is given by:

- a)  $x/y$
- b)  $y/x$
- c)  $x^2/y^2$
- d)  $y^2/x^2$

Q3. Match the utility functions with their MRS at  $(x,y)=(4,9)$ :

- |                       |        |
|-----------------------|--------|
| i) $x+y$              | a) 1.5 |
| ii) $x+\sqrt{y}$      | b) 1   |
| iii) $x^{0.5}y^{0.5}$ | c) 6   |

Q4. If the utility function is  $U(x,y) = x^2-y^2$ . Then what can you conclude about nature of these goods:

- a)  $x$  and  $y$  both are good
- b)  $x$  is good and  $y$  is bad
- c)  $x$  is good nad  $y$  is neutral
- d)  $x$  is bad and  $y$  is good

Q5. Which of the following pairs of goods are compliments or substitutes:

- a) popular novel and Magazine.
- b) A camera and a film.

Q6. a) Consider a utility function  $U(x,y) = \sqrt{xy}$ . Calculate its MRS.

b) Is  $V(x,y) = x^2y^2$  monotonic transformation of  $U$ ?

c) Calculate MRS of  $V(x,y)$ . Is it different from MRS calculated in part (a)?

Q7. If both good  $x$  and  $y$  are bads, draw and explain consumer's indifference curves.

Q8. a) Consider a utility function of Vaibhav as  $U(x,y) = 4\sqrt{x} + y$ . Vaibhav consumes 9 units of  $x$  and 15 units of  $y$ . His consumption of  $x$  is reduced to 4 units, but is given sufficient of good  $y$  that there is no loss (or no addition to) of utility. How many units of  $y$  does Vaibhav consume?

b) Draw indifference curves showing vaibhav's preferences.

c) Calculate Vaibhav's MRS. Is his preferences homothetic?

Q9. Poonam's utility function is  $U(x,y) = \max\{x,y\}$ .

a) Draw her indifference curves.

b) If Poonam consumes  $(10,20)$ , Calculate her utility at this bundle.

c) State true or false about Poonam's preferences:

i)  $(10,20) \sim (20,10)$

ii)  $(20,10) > (15,15)$

d) Is Poonam's preference convex?

Q10. Consider utility functions:

a)  $U(x,y) = xy$

b)  $U(x,y) = x^2 + y$

Show that both of these has diminishing marginal rate of substitution.

## Glossary

- **Indifference curve:** A curve showing the locus of combinations of the amounts of two goods such the consumer is indifferent between any combinations on that curve.
- **Marginal rate of substitution:** It refers to the amount of one good that is required to compensate the consumer for giving up an amount of another good such that the consumer has same level of utility as before.
- **Perfect substitutes:** Two goods are substitutes if the consumer is willing to substitute one good for another at a constant rate.
- **Perfect compliments:** Perfect complements are goods that are consumed together in fixed proportions.

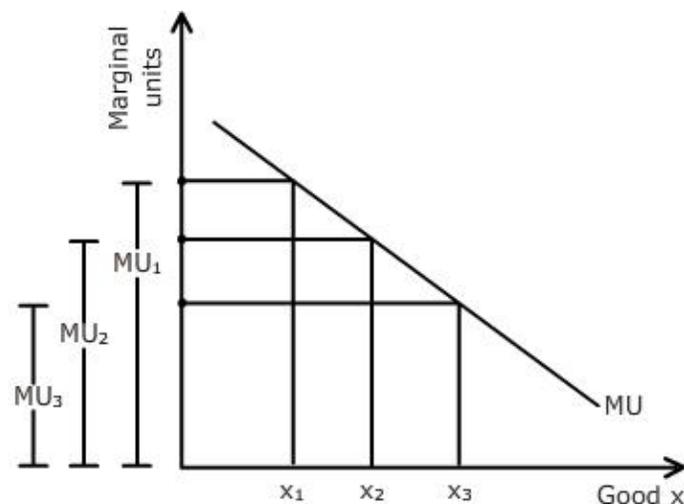
- **Bad good:** A good is said to be bad if addition in its quantity creates disutility even at lower levels of consumption.
- **Neutral good:** A good is said to be neutral if any quantity of a good can be added & no change in utility is brought about.
- **Monotonicity:** Monotonicity principle is 'More is better' and implicitly takes good-good and leaves bad-good.
- **Well behaved preferences:** preferences are well behaved when indifference curves are negatively sloped and convex to the origin.

## Appendix

### A.1 Diminishing marginal utility

Given utility function  $U(x)$ ,  $du/dx$  is positive stating that 'good' adds value to its consumer. Fall in marginal utility as  $x$  increases; refer to the downward slope of marginal utility curve. This is computed by  $d^2u/dx^2$ .  $d^2u/dx^2$  should be negative not  $du/dx$  to fulfill diminishing marginal utility. MU in first quadrant assumes all positive values and but its slope is negative, due to negative relationship between addition made to utility and consumption of units.

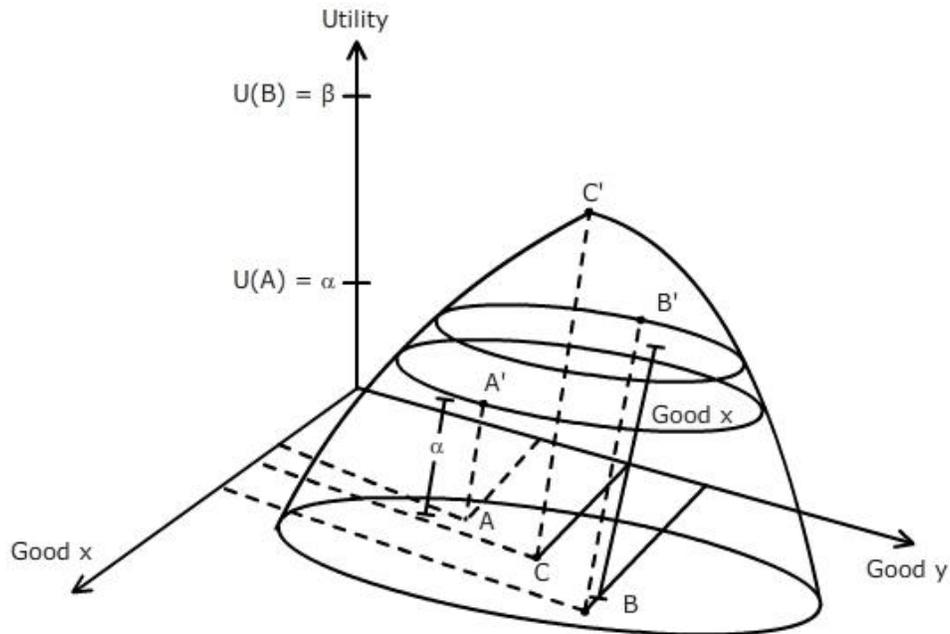
Figure A1 diminishing marginal utility



### A.2 Indifference curves are level curves of utility function

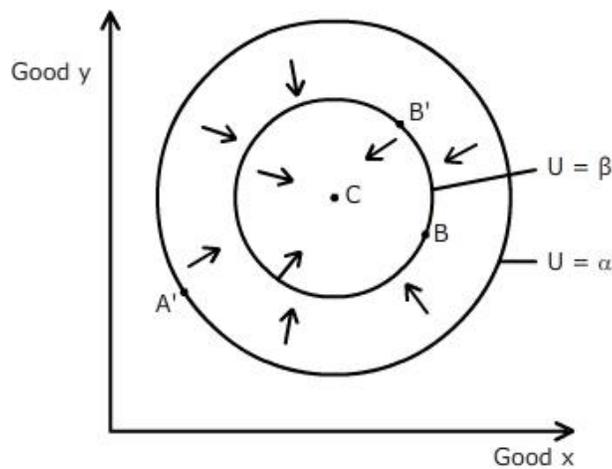
We assume two good  $x$  &  $Y$  on the  $x$  &  $y$  – axis respectively. Various combinations of these goods like  $A$  and  $B$  yields some utility which is represented on  $Z$ -axis. Combination is raised to the height equivalent to utility of bundle  $A$  shown by point  $A'$ . Similarly; for all such bundles, utility curve is drawn (as shown in fig. A2).

FigureA2 utility function



At various heights say  $\alpha$ , a disc is seen (with point  $A'$  on it). Assume, this disc drops down on  $x$   $y$  plane; which then appears like in fig.A3.  $C'$  is the Bliss point where utility is maximum.  $C'$  corresponds to point  $C$  in  $xy$  plane, which is satiation point.

FigureA3 indifference curves as level curves of utility function



### A.3 Diminishing Marginal Rate of Substitution

Utility depends on consumption of good  $x$  & good  $y$ .

$$U = f(x,y)$$

Taking total differential of it, we get :

$$\Delta U = f'_x \Delta x + f'_y \Delta y$$

$$\Delta U = MU_x \Delta x + MU_y \Delta y$$

On an indifference curve, there is no change in level of utility. Therefore,

$$MU_x \Delta x + MU_y \Delta y = 0$$

$$\text{Or, } \Delta y / \Delta x = -MU_x / MU_y$$

As x increases  $MU_x$  falls and as y declines  $MU_y$  increases. Both of which imply that this fraction falls as x increases. Hence we witness diminishing marginal rate of substitution along the indifference curve.

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C. Snyder and W. Nicholson, Fundamentals of Microeconomics, Cengage Learning (India), 2010.

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