DECOMPOSITION OF PRICE EFFECT

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Learning Objectives

After reading this chapter, you will able to understand the following concepts

Price effect

Substitution effect

Income effect

Signs of Substitution and Income effect

Slutsky price decomposition

Hicksian price decomposition

Slutsky equation

The Law of demand

Compensated demand curves

Uncompensated demand curves

1. Introduction

Mr Ajay is a graduate in economics honours from Delhi University. He currently owns a restaurant at Kamlanagar offering variety of eatables like Paranthas, Momos, Chowmiens, Cutlets etc. For quite some time, he observed that the demand for paranthas was low compared to the demand for other items. Using his knowledge in economics, he decided to offer 50% discount for Paranthas. The question now is will the sales for Paranthas increase as a result of price discount? Or in other words, will the law of demand works in this case?

Let us guess some possibilities. First, since the price of Paranthas has come down, it may attract new customers. Old customers may also started consuming more paranthas. Second, customers may use their savings from lower price of paranthas to purchase other items. Third, some customers may stop consuming Paranthas as its price has fallen due to prestige issue. So, it is not necessary that price discount offer will actually increase the demand for Paranthas.

Therefore, while the Law of demand states that there is inverse relationship between price and quantity demand of a good, there could be some exceptions. The main objective of this chapter is to understand the relationship between price and quantity demand of a good in much greater detail. Understanding the concept of substitution effect and income effect will be crucial to know how the law of demand operates.

1.1 Price Effect

The change in demand due to change in price of a commodity is known as price effect. To understand how changes in price lead to changes in demand, we need to know substitution effect and income effect.

1.2 Substitution Effect

When the price of a commodity changes, the relative price of that commodity with respect to other commodity changes. The change in demand due to change in relative price is known as substitution effect. For instance when the price of good X decrease, it has become relatively cheaper than other goods. So the demand for good x will increase. When we study substitution effect, we assume that the real income of the consumer remains constant.

$$\textbf{Substitution effect}: \boldsymbol{Change\ in\ price\ \Rightarrow change\ in\ relative\ prices\ \Rightarrow change\ in\ demand}$$

The real income of the consumer remaining constant can be interpreted in two different ways:

Slutsky Interpretation: According to Prof. Slutsky holding real income constant implies that the consumer is able to purchase the original commodity bundle even after the price change. Since real income is equal to money income divided by price of the good, a rise (or fall) in price of the good will mean an increase (or decrease) in money income so that the real income remains constant.

Hicksian interpretation: According to Prof. Hicks holding real income constant implies that the consumer's satisfaction level or utility remain unchanged even after the price

change. The means that the consumer will continue to remain on the same indifference curve even after the price change.

1.3 Income Effect

When the price of a commodity changes then the purchasing power of consumer changes. This changes in purchasing power in turn changes the quantity of goods demanded. This is known as income effect. When we study income effect we assume that the relative price of a commodity is constant.

$$\textbf{Income effect}: \boldsymbol{Change\ in\ price\ \Rightarrow change\ in\ purchasing\ power\ \Rightarrow change\ in\ demand}$$

The effect of price changes on quantity demanded due to income effect will depend on the nature of the commodity. For a normal good, a decrease in price of a good will increase the purchasing power which will enable the consumer to consume more of that commodity. For inferior and Giffen good, when price of a commodity decrease leading to increase purchasing power for the consumer, then the quantity demanded of a commodity will decrease.

1.4 Signs of Substitution and Income effects

The sign of substitution effect is always negative. That means when the price of a good goes down (up), the demand goes up (down) due to substitution effect . The change in price and change in demand due to substitution effect always move in opposite direction.

To prove that substitution effect is always negative consider figure 1. Where will the consumer remain on the hypothetical budget line G'H'? First we consider the points on the line G'H' left to the initial equilibrium point. These bundles were all affordable at old prices but were not purchased. Instead the bundle at E_1 was purchased. So the bundle at E_1 is preferred to all the bundle that lies inside the original budget line GH.

Second, since we rule out all the bundles to the left of point E_1, the optimal bundle must be either at E_1 or some point to the right of E_1. All points to the right of E_1 on the line G'H' involves consuming atleast as much of good X as originally. So we get the optimal bundle at point E_2which certainly involves consuming more of good X than at the original equilibrium E_1. Therefore the substitution effect always moves opposite to the price movement. So we say that the Substitution effect is negative.

The sign of income effect can be negative or positive depending on the nature of good. For normal goods, the sign of income effect is negative. That means when the price of a good goes down (up) the demand of a good goes up (down) due to income effect. For inferior goods and Giffen goods, the sign of income effect is positive. That is, when the price of a good goes down (up) the demand of a good goes down (up) due to income effect. So for inferior and Giffen goods the change in price and change in demand due to income effect move in the same direction. However the relative strength of income effect is more for Giffen good than for inferior goods.

The sign of price effect will depend on the sign of both substitution effect and income effect. The sign of price effect for normal good is negative since both the substitution effect and income effect is negative for normal goods. That is, when the price of a good goes down (up), the demand increase (decrease) for normal good. The sign of price effect for inferior good is also negative. This is because the negative substitution effect

outweighs the positive income effect for inferior goods. For Giffen goods the sign of price effect is positive since the positive income effect is stronger than the negative substitution effect for Giffen goods.The summary of the results are provided as below:

Table 1: Signs of substitution effect and income effect

Nature of Commodity	**Price Change**	**Substitution effect**	**Income effect**	**Price effect**
Normal good	Decrease	Negative (Quantity demand increase)	Negative (Quantity demand increase)	Negative (Quantity demand increase)
	Increase	Negative (Quantity demand decrease)	Negative (Quantity demand decrease)	Negative (Quantity demand decrease)
Inferior good	Decrease	Negative (Quantity demand increase)	Positive (Quantity demand decrease)	Negative (Quantity demand increase as SE >IE)
	Increase	Negative (Quantity demand decrease)	Positive (Quantity demand increase)	Negative (Quantity demand decrease as SE>IE)
Giffen good	Decrease	Negative (Quantity demand increase)	Positive (Quantity demand decrease)	Positive (Quantity demand decrease as IE > SE)
	Increase	Negative (Quantity demand decrease)	Positive (Quantity demand increase)	Positive (Quantity demand increase as IE >SE)

2. Graphical analysis of decomposition of Price effect

We have seen that when the price of good changes it leads to changes in quantity demanded of the good through two effects namely the substitution effect and income effect. Substitution effect measures the change in demand due to change in the relative price of the good. Income effect measures the change in demand due to change in purchasing power.

The effect of price change on quantity demand can be studied in two steps:

(1) First we study the change in relative price and hold the purchasing power constant by adjusting money income

(2) Second we study the change in purchasing power and hold the relative prices constant.

2.1 Normal Goods

In figure 1 the original budget line is GH which is tangent to indifference curve I_1 at point E_1.The point E_1 is the initial consumer's equilibrium. The consume buys OX_1 units of good X. Now suppose that the price of good X falls.This will shift the budget line to GH" and the consumer will be at the higher indifference curve I_3. The point E_3 is the final equilibrium and the consumer will now consume OX_3 units of commodity X. The increase in demand of X due to fall in the price of X is X_1X_3. Hence the price effect is X_1X_3. We now decompose this price effect into substitution effect and income effect.

Graphically we can analyse the movement of budget line from GH to GH" into two steps:

(1) We construct a hypothetical budget line which passes though the original consumption bundle at E_1 and which is also parallel to the new budget line. Accordingly the line G'H' is constructed.

(2) Second we shift this hypothetical budget line to the new budget line GH''.

In the first step where we construct a hypothetical budget line, the slope of the budget line changes but the purchasing power remains constant. In the second step, the purchasing power changes while the slope of the budget line remains constant. This two-step exercise is simply a hypothetical construction to analyse the effect of price change on demand. What we actually observed is that there is a change in price and the consumer responded by buying new commodity bundle.

Next we try to explain the economic meanings behind this two-step exercise. In the first step, we constructed a hypothetical budget line which has the same slope as the final budget line. Since the slope of the two budget lines are the same, they also have the same relative prices. However, the two budget lines have different money income as they have different vertical intercept. The hypothetical budget line pass through the initial equilibrium point E_1 implying that the consumer can still afford to purchase the original consumption bundle. This means that the purchasing power of the consumer has remain constant at the hypothetical budget line.

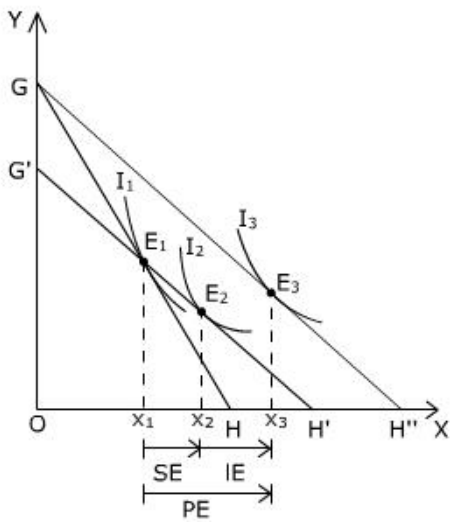
When the price of good X decrease, the purchasing power of the consumer increase. So to keep the purchasing power constant we need to decrease the consumer's income. Similarly we need to increase the consumer's income when the price of good X increase

in order to keep the consumer's purchasing as constant. Therefore the adjustment in income will be negative when the price falls and positive when the price rise.

The original consumption bundle at E_1 is still affordable at the hypothetical budget line. However, it is no longer the optimal consumption bundle. The consumer can now remain at higher indifference curve I_2 and the optimal consumption bundle at the hypothetical budget line is given by E_2. The bundle of goods given by E_2 is the optimal bundle as it the purchase made after the change in price and after adjusting the money income which enable the consumer to just afford the original bundle of goods. The Substitution effect is indicated by the movement from E_1 to E_2. It shows how the consumer substitutes the cheaper good X for good Y when the price of good X falls while holding the purchasing power as constant. The increase in demand for good X when its price decrease due to substitution effect is X_1X_3

The income effect can be seen by studying the parallel shift of the hypothetical budget line to the new budget line. When we compared the hypothetical budget line and the new budget line, we find that the money income of the consumer changes but the relative prices remain constant. The movement from E_2 to E_3 is the income effect. This is because if GG' amount of income is given back to the consumer he will shift to higher indifference curve I_3 and the consumer final equilibrium will be at E_3. The increase in quantity demand of X due to the increase in purchasing power associated with the fall in price of good X is X_2X_3. Hence income effect is X_2X_3

When price change, Income effect can increase or decrease the demand for good depending on the nature of good. The purchasing power of the consumer increases When the price of good decreases. To keep the purchasing power constant we need to decrease the money income of the consumer when the price of good falls. For a normal good, this decrease in income will lead to fall in demand. However for inferior and Giffen good, this decrease in income will lead to increase in demand.

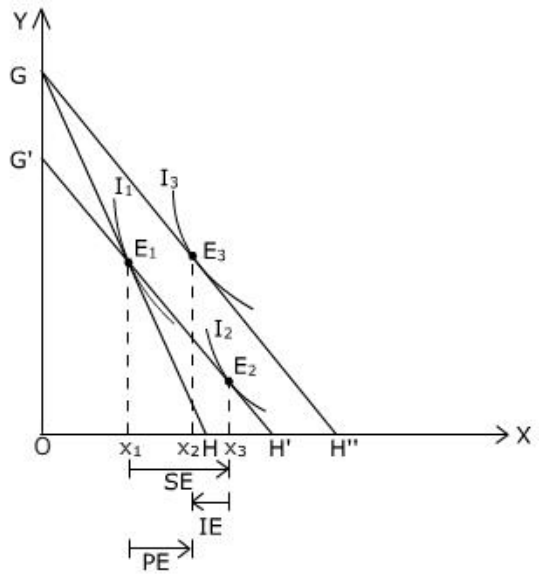


- ✓ Price effect: X_1X_3
- ✓ Substitution effect: X_1X_2
- ✓ Income effect: X_2X_3

Figure 1: Slutsky Decomposition of Price effect for normal goods when price of good X falls.

2.2 Inferior Good

The graphical analysis for inferior good is shown in figure 2. We consider the case of fall in price of good X as in figure 1. When price of good X falls, the increase in demand due to substitution effect is X_1X_3. However, the income effect will lead to decrease in demand by X_3X_2. The substitution effect is greater than the income effect .Hence the Price effect is X_1X_2. The increase in demand of good X due to fall in the price is greater for normal goods than for inferior goods.



- ✓ Price effect: X_1X_2
- ✓ Substitution effect: X_1X_3
- ✓ Income effect: X_3X_2

Figure 2: Slutsky's price decomposition for inferior goods.

2.3 Giffen goods

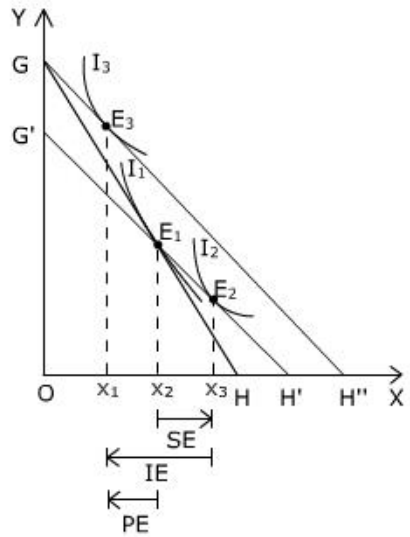
<u>The story of Irish Potatoes</u>

In the beginning of the 17th century, potatoes became increasing popular as a staple food for the people of Ireland. The abundance of potato as a food helps the Irish population to increase from 3 millions in the late 1500s to almost 8 million by 1840. However, in the 1850s fungus appears causing a blight that destroyed most of the crop. The price of potatoes skyrocketed and the great Irish Potato famine lead to death of millions of people.

Sir Robert Giffen as a boy observed that some Irish families increased their consumption of potatoes during the famine. The poor families gave up their consumption of expensive food and consume more potatoes to maintain adequate calories intake. Therefore the demand for potatoes increase even as its price increases. Hence, the term Giffen goods.

The graphical analysis for Giffen good is shown in figure 3. As in figure (1) and figure (2), we again consider the case of the fall in price of good X. When price of good X falls, the Substitution effect leads to increase in demand by X_2X_3.However the income effect will lead to decline in the quantity demand byX_3X_1. The income effect outweighs the

substitution effect in case of Giffen goods. Hence, the decrease in price leads to decrease in quantity demand of good X. The price effect is X_2X_1



- ✓ Price effect: X_2X_1
- ✓ Substitution effect: X_2X_3
- ✓ Income effect: X_3X_1

Figure 3: Slutsky's price decomposition for Giffen goods

3. Special cases of Price decomposition

3.1 Perfect Complements

When two goods are perfect complements, they are consumed jointly and the indifference curves are L-shaped. The initial equilibrium of the consumer is at E_1 where the indifference curve I_1 is tangent to the original budget line GH. Now when the price of good X decrease, the budget line becomes GH''. The final equilibrium of the consumer is at E_2 where indifference curve I_2 is tangent to final budget line GH". The price effect is measured by the movement from E_1 to E_2.

To measure the substitution effect the consumer must be at the hypothetical budget line G'H'. The only possible optimal bundle that the consumer can chose on the hypothetical budget line is E_1. All other points are not possible as indifference curves cannot intersect with each other. The substitution effect is thus zero in this case. The change in demand is captured entirely by the Income effect. Hence price effect is equal to income effect in case of perfect complements.

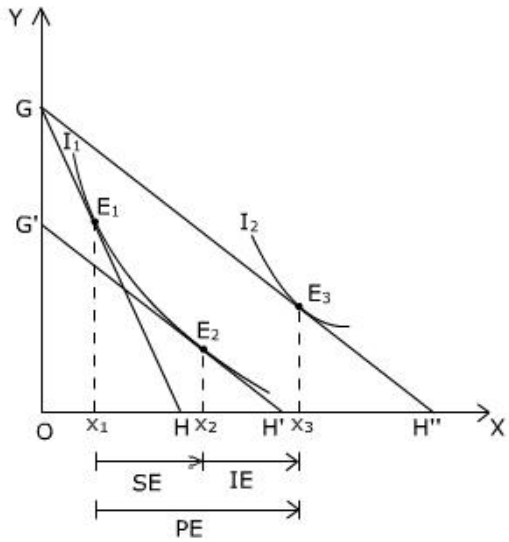


Figure 4: Slutsky's price decomposition when goods are perfect complements

3.2 Perfect Substitutes

When goods are perfect substitutes, the indifference curves are straight lines and the consumer equilibrium is possible only at the corner. The initial equilibrium of the consumer is at E_1 where the indifference curve I_1 meets the original budget line. When the price of good X falls, the consumer's equilibrium jump from the vertical axis to the horizontal axis. The final equilibrium of the consumer is at E_2.The change in demand is entirely due the substitution effect. The income effect here is zero.

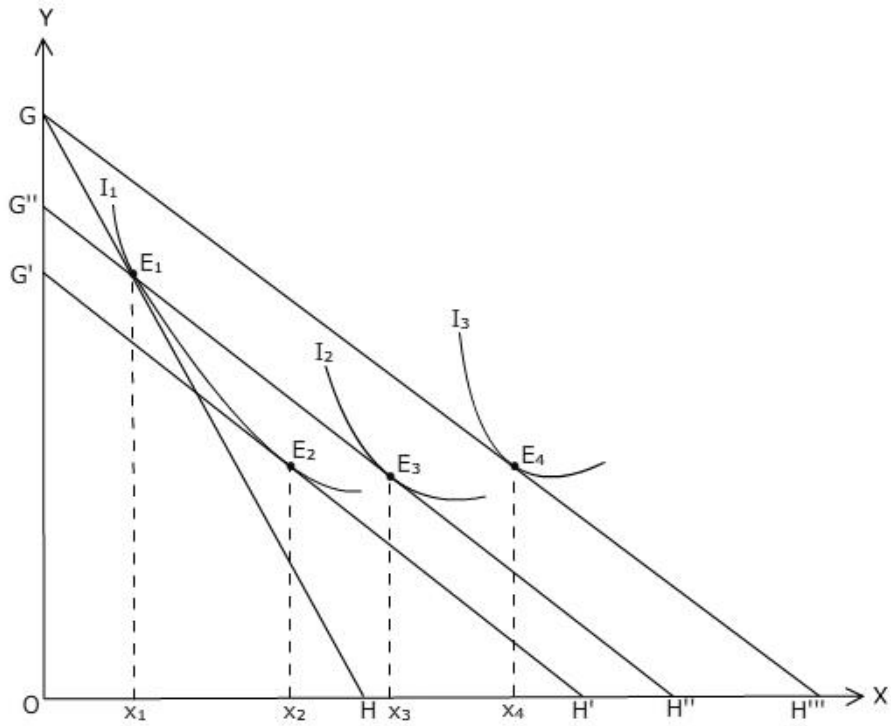


Figure 5: Slutsky's price decomposition when goods are perfect substitutes

3.3 Quasi Linear Preferences

When consumer's preferences are quasilinear, a change in income did not cause any change in demand for good X. So income effect is zero. This means when the price of good X falls, the change in demand for good X is entirely due to substitution effect.

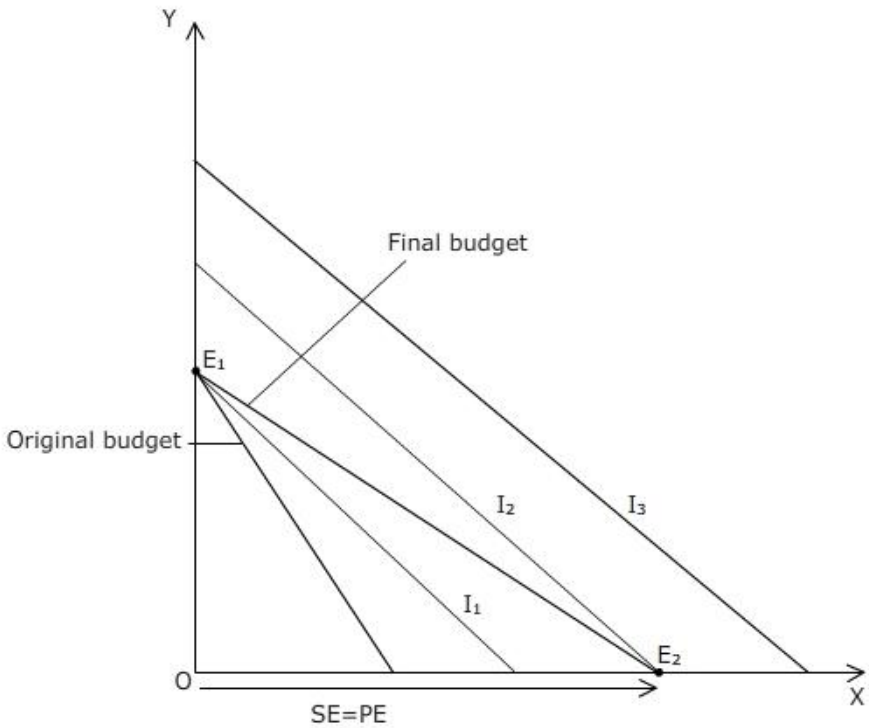


Figure 6: Slutsky's price decomposition when consumer's preferences are quasilinear

4. Hicksian decomposition of Price effects

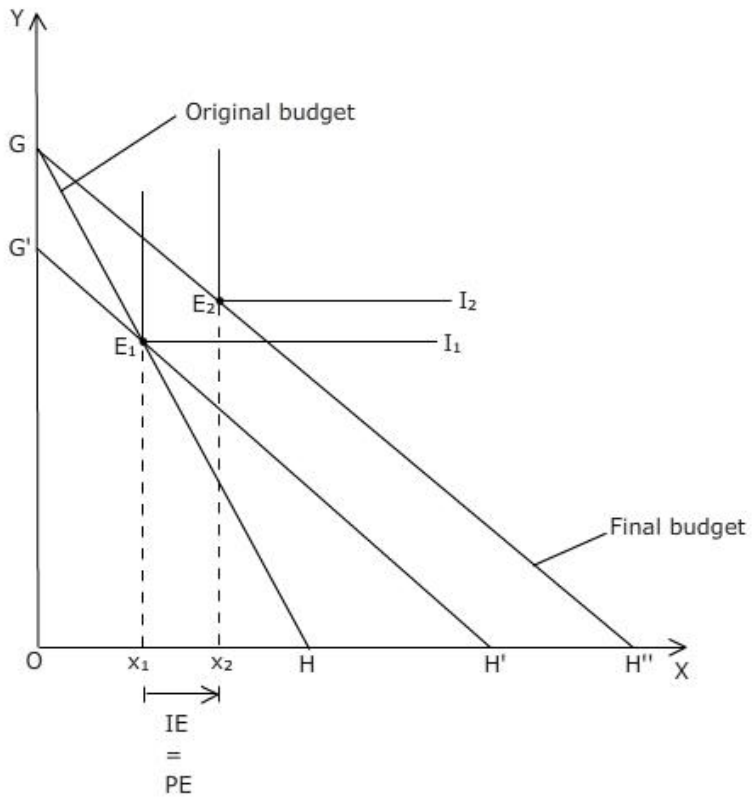
We have already analysed the decomposition of Price effect using Slutsky's method. Now we will do so using Hicksian's method. We will analyse the effect of price decrease for normal goods only.

4.1 Normal goods

The Hicksian price effect decomposition is for normal goods are shown in figure 4. The original budget line is GH which is tangent to indifference curve I_1 at point E_1. The point E_1 is the initial consumer's equilibrium. The consume buys OX_1 units of good X. Now suppose that the price of good X decrease. This will shift the budget line to GH'' and the consumer will be at the higher indifference curve I_2. The point E_3 is the final equilibrium and the consumer will now consume OX_3 units of commodity X. The increase in demand of X due to fall in the price of good X is X_1X_3. Hence the price effect is X_1X_3. We now decompose this price effect into substitution effect and income effect.

The Hicksian substitution effect can be seen graphically by constructing a hypothetical budget line parallel to the new budget line and yet tangent to the original indifference curve I_1. Accordingly the line G'H' is constructed. In this line the consumer's equilibrium is at E_2 where indifference curve I_2 is tangent to the hypothetical budget line G'H'. The movement from E_1 to E_2 is the substitution effect. The increase in quantity demand when price decrease due to substitution effect is X_1X_2.

The movement from E_2 to E_3 is the income effect. This is because if GG' amount of income is given back to the consumer he will shift to higher indifference curve I_3 and the consumer final equilibrium will be at E_3.The increase in quantity demand of good X due to the income effect is X_2X_3.Hence income effect is X_2X_3.



- ✓ Price effect: X_1X_3
- ✓ Substitution effect: X_1X_2
- ✓ Income effect: X_2X_3

Figure 7: Hicksian decomposition of Price effect for normal goods.

4.2 Comparison of Slutsky and Hicksian Substitution effects

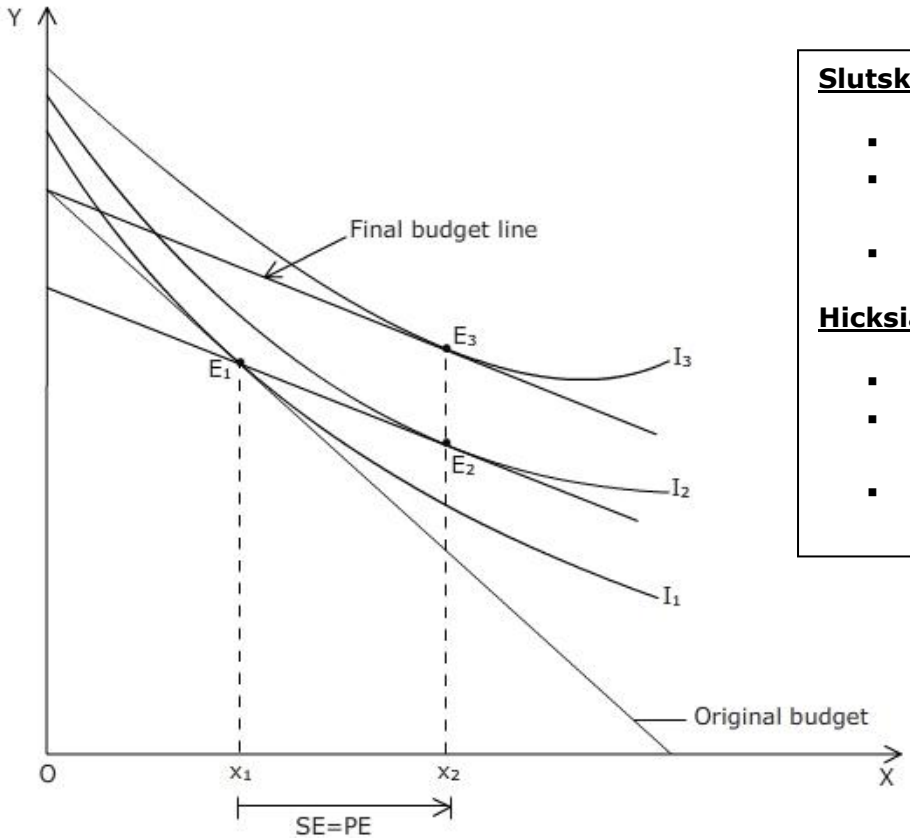
The Slutsky substitution effect differs from the Hicksian substitution effect. The differences in measurement arise due to the varying interpretation of real income of the consumer. In the Slutsky case, the real income of the consumer is held constant by enabling the consumer to purchase the original consumption bundle after the price change. In the Hicksian analysis, the real income of the consumer is held constant by allowing the consumer to remain on the same indifference curve after the price change.

The comparision of Slutsky and Hicksian Substitution effect is shown in figure 8.The initial consumer equilibrium is at E_1 where the indifference curve I_1 is tangent to the original budget line GH. Now suppose the price of good X falls. The new budget line is GH''' and the consumer is at higher indifference curve I_3. The final consumer equilibrium is at E_4 where budget line GH''' and the the indifference curve I_3 are tangent to each other. The price effect due to fall in price of good X is X_1X_4.

The Slutsky's Substitution effect is shown graphically by drawing a hypothetical budget line G''H'' which is parallel to the new budget line GH''' and passes through the original equilibrium E_1. At the hypothetical budget line G''H'', the consumer can still afford the old

commodity bundle at E_1. However, the consumer can remain at higher indifference curve I_2. We get the new consumer's equilibrium at E_3. The increase in quantity demand of good X due to Slutsky Substitution effect is X_1X_3

The Hicksian's Substitution effect is shown graphically by drawing another hypothetical budget line G'H' which is parallel to the new budget line GH''' and yet is tangent to the original indifference curve I_1. As the price of good X falls, the consumer moves down along the original indifference curve I_1 and consume more of good X. The consumer is now at new equilibrium E_2. The increase in demand of good X due to Hicksian Substitution effect is X_1X_2



Slutsky method

- Price effect: X_1X_4
- Substitution effect: X_1X_3
- Income effect: X_3X_4

Hicksian Method:

- Price effect: X_1X_4
- Substitution effect: X_1X_2
- Income effect: X_2X_4

Figure 8: Comparasion of Slutsky and Hicksian method of price decomposition

Let us now compare Slutsky substitution effect and Hicksian Substitution effect. Slutsky substitution effect is given by X_1X_3 where the Hicksian Substitution effect is given by X_1X_2. We find that the Slutsky's substitution effect exceeds Hicksian Substitution effect.

5. The Law of Demand

The inverse relationship between price and demand of a commodity as envisage by the Law of demand leads to downward sloping demand curve. While most goods obey the Law of demand there are few exceptions. To understand this we need to know how substitution effect and income effect works.

The substitution effect leads the consumer to buy more of a cheaper commodity and less of a dearer commodity. Hence they are in conformity with law of demand. The income

effect depends on the nature of goods. The violation of the Law of demand comes from the income effect

For normal goods, both the substitution effect and income effect works in the same direction. So the Law of demand always hold for the normal goods.

For Inferior goods, the substitution effect and income effect works in opposite direction. However substitution effect dominates income effect and the Law of demand still holds. In case of Gifffen goods, income effect is stronger than the substitution effect and hence we get upward sloping demand curve.

6. Demand Curves

We can draw the relationship between price and quantity demanded of a good either by holding income fixed or holding utility fixed. The first case gives rise to uncompensated demand curves while the second case give rise to compensated demand curves.

6.1 Uncompensated demand curve

A Uncompensated demand curve gives us the relationship between the price and quantity demanded of a good under the assumption that all other determinants of demand are held constant. There are three basic determinants which are held constant namely income, price of other good (here price of good Y) and the individual's preference. Any change in these factors will shift the entire demand curve to new position.

The derivation of uncompensated demand curve is shown in figure 9. In the upper panel of figure (9), the initial consumer's equilibrium is shown by E_1. As price of good X falls from P_x' to P''_x ,the budget line changes from GH to GH'. The consumer's equilibrium also changes from E_1 to E_2. When the price of good X falls further to P_x''', the budget line becomes GH" and the consumer's final equilibrium is at E_3. With the fall in price of good X, the consumer demand for good X changes from X_1 to X_2 and finally toX_3.

The relationship between the change in price and demand of good x is captured in the lower panel of figure 9. Accordingly this relationship between price and demand of good X is used to construct the uncompensated demand curve as shown in the figure.

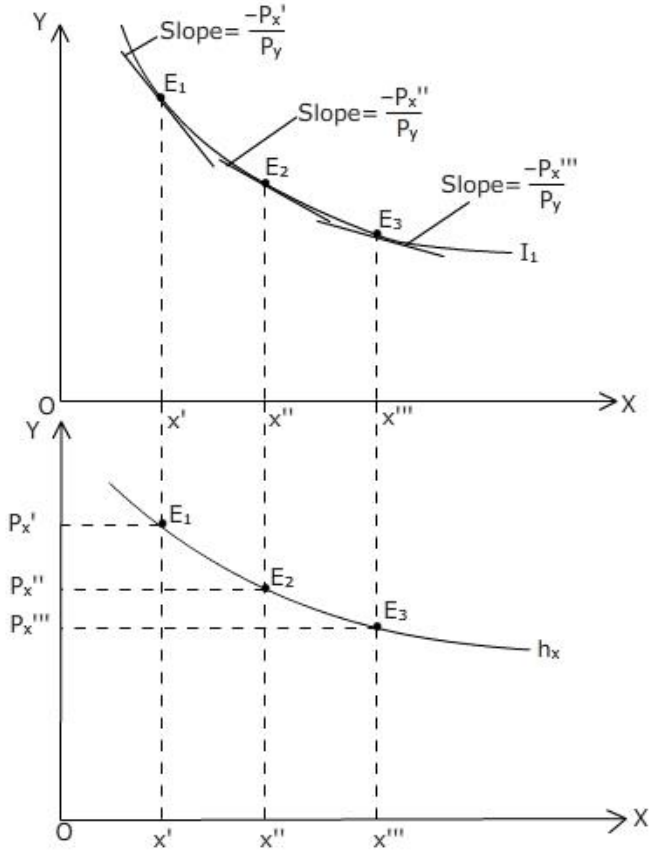


Figure 9: Derivation of uncompensated demand curve

6.2 Compensated demand curve

A compensated (or Hicksian) demand curve gives us the relationship between price and quantity demanded of a good under the assumption that other prices and utility remains constant. Hence the curve only shows the substitution effects of changing prices.

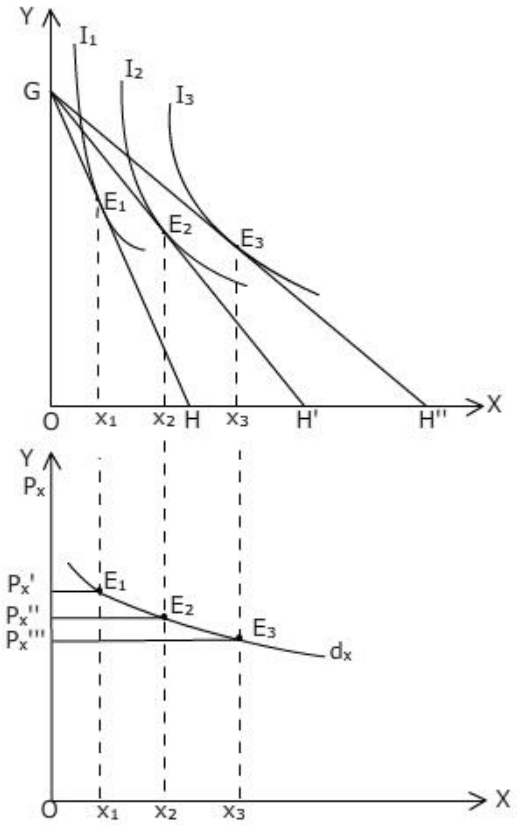


Figure 10: Derivation of compensated demand curve

The derivation of compensated demand curve is shown in figure 10. In the upper panel of the figure, we successively reduce the price of good X while holding the utility fixed at I_1 .As price of good X decreases, we need to reduce the nominal income of the consumer to prevent any increase in utility. The consumer is constrained to remain at I_1 by compensating for the effect of price changes on purchasing power. If the price of good X rises, the consumer's income would have to be increase to enable him/her to remain on indifference curve I_1.The results are shown in figure 10.

6.3 Comparison of Compensated and Uncompensated demand curves

The relationships between the two demand curves are given in figure 11. Both the curves intersect at price P_x'' because the consumer's income is just sufficient to attain utility I_1 at this price. So X'' is demanded under both the demand curves. When prices are above P_x'', the consumer's income is increase to enable him/her to remain at I_1. So for prices above P_x'', more of good X is demanded under compensated demand curves. Conversely for prices below P_x'', less of good X is demanded under compensated demand curves. The uncompensated demand curve is flatter than compensated demand curves as it incorporates both substitution and income effect whereas the compensated curve reflects only substitution effects.

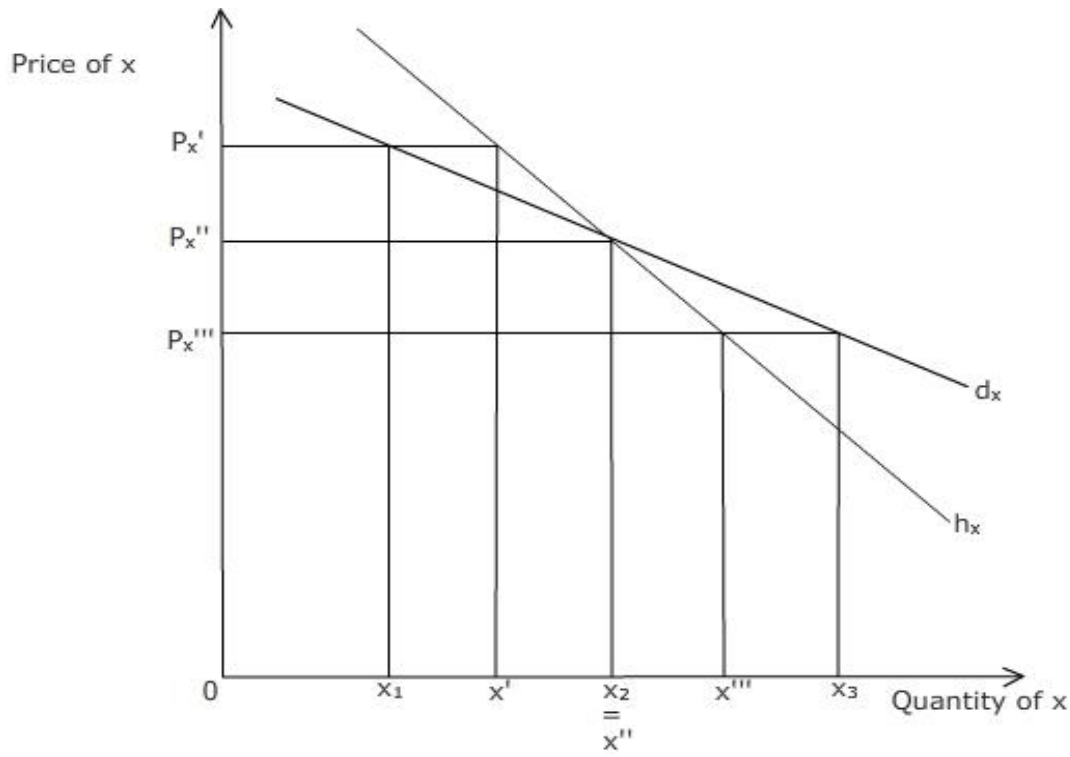


Figure 11: Comparison of compensated and uncompensated demand curve

7. Algebraic treatment of Price effect

Substitution effect measures the change in the demand for good X when its price changes from P_1 to P_2 and at the same time money income changes from M_1 to M_2

$\Delta X^{SE} = X(P_2, M_2) - X(P_1, M_1)$

The income effect measures the change in the demand for good X when we change income from M_2 to M_1 while holding the price of good X fixed at P_2

$\Delta X^{IE} = X(P_2, M_1) - X(P_2, M_2)$

The Price effect measures the total change in demand due to the change in price from P_1 to P_2 while holding income constant at M_1

$\Delta X^{PE} = X(P_2, M_1) - X(P_1, M_1)$

It can be shown that the price effect can be broken up into two things: the Substitution effect and the Income effect.

$X(P_2, M_1) - X(P_1, M_1) = [X(P_2, M_2) - X(P_1, M_1)] + [X(P_2, M_1) - X(P_2, M_2)]$.

Therefore, $\Delta X^{PE} = \Delta X^{SE} + \Delta X^{IE}$

The above equation says that the total change in demand (i.e the Price effect) equals the substitution effect plus the income effect. The RHS of the equation is identically equal to the LHS. So it is an identity, meaning that it is true for all the values of P_1, P_2, M_1 and M_2. This equation is also known as Slutsky identity.

The sign of Price effect is determine by the signs of both the substitution effect and income effect. The sign of substitution effect is always negative. It means that when price increase (decrease), the demand will go down (up). The sign of income effect can be negative or positive depending upon the nature of the goods. For normal goods, an increase (decrease) in income leads to increase (decrease) demand. For Inferior and Giffen goods an increase (decrease) in income will lead to fall (rise) in demand. Therefore the price effect can be negative or positive depending upon the signs of both the substitution effect and the income effect.

When we have normal goods, both the substitution and income effect work in the same direction. An increase (decrease) in price means that demand will go down (up) due to the substitution effect. When price rise (falls) it means a decrease (increase) in purchasing power which leads to fall (rise) in demand for normal goods.

The total change in demand for normal goods due to change in price can be represented by:

$\Delta X^{PE} = \Delta X^{SE} + \Delta X^{IE}$

(-ve) (-ve) (-ve)

For normal goods both the substitution effect and the income effect have the same sign and reinforce each other. Therefore an increase (decrease) in price will lead to fall (rise) in demand for normal goods.

When we have inferior goods, the substitution effect is negative while the income effect is positive. So an increase (decrease) in price leads to increase (decrease) in demand due to income effect. The sign of price effect will depend on the relative strength of the two opposing effects. For inferior goods, the substitution effect outweighs the income effect and the sign of price effect is negative. It can be shown as below:

$\Delta X^{PE} = \Delta X^{SE} + \Delta X^{IE}$

(-ve) (-ve) (+ve)

When we have Giffen goods, the positive income effect is stronger than the negative substitution effect. The price effect is thus positive for Giffen goods meaning an increase (decrease) in price leads to rise (fall) in demand.

$\Delta X^{PE} = \Delta X^{SE} + \Delta X^{IE}$

(+ve) (-ve) (+ve)

Rates of Change

We have already described the Slutsky identity algebraically in terms of absolute changes as given below:

$\Delta X^{PE} = \Delta X^{SE} + \Delta X^{IE}$

However it is sometime useful to express it in terms of rate of change. In such a case, we define

ΔX^{M} to be the negative of the Income effect

$\Delta X^M = X\,(P_2, M_2) - X\,(P_2, M_1) = -\Delta X^{IE}$

The Slutsky identity then becomes

$\Delta\, X^{PE} = \Delta\, X^{SE} - \Delta\, X^M$

Dividing each side of the identity by $\Delta\, P$ we get

$$\frac{\Delta\, X^{PE}}{\Delta\, P} = \frac{\Delta\, X^{SE}}{\Delta\, P} - \frac{\Delta\, X^M}{\Delta\, P}$$

The relationship between the change in income $\Delta\, M$ and the change in price $\Delta\, P$ are given by: $\Delta\, M = X\,\Delta\, P$

Which implies that $\Delta\, P = \frac{\Delta M}{X}$

Putting this formula we get

$$\frac{\Delta\, X^{PE}}{\Delta\, P} = \frac{\Delta\, X^{SE}}{\Delta\, P} - \frac{\Delta\, X^M}{\Delta M}\, X$$

Finally we get the Slutsky identity in terms of rates of change. We now interpret each terms as follows:

$$\frac{\Delta\, X^{PE}}{\Delta\, P} = \frac{X\,(P_2, M_1) - X\,(P_1, M_1)}{\Delta\, P}$$

gives us the rate of change in demand as price changes while holding income as constant.

$$\frac{\Delta\, X^{SE}}{\Delta\, P} = \frac{X\,(P_2, M_2) - X\,(P_1, M_1)}{\Delta\, P}$$

gives us the rate of change in demand as the price change while adjusting income so as to enable the consumer to be able to afford the old bundle, that is the substitution effect.

$$\frac{\Delta\, X^M}{\Delta\, M}\, X = \frac{X\,(P_2, M_2) - X\,(P_2, M_1)}{M_2 - M_1}$$

gives us the rate of change of demand income changes while holding prices as constant, that is the Income effect.

The change in demand due to the IE can also be stated as below:

$$\Delta\, X^M = \frac{X\,(P_2, M_2) - X\,(P_2, M_1)}{\Delta\, M}\, X\,\Delta\, P$$

since $X\,\Delta\, P = \Delta M$

Therefore, income effect can be seen in two parts: the change in demand as income changes multiply by the original level of demand when the price changes by $\Delta\, P$.

Relationship between change in income and change in price

Suppose the consumer consume a commodity bundle (X,Y) at price- money combination of (P_x, P_y, M). So we have

$M = P_x X + P_y Y$

Now assume that the price of X changes from P_x to $P_x{}'$. To enable the consumer to afford the old bundle (X,Y) we need to adjust money income from M to M'. so we have

$M' = P_x' X + P_y Y$

Now we subtract the second equation from the first

$M' - M = X\,[P_x' - P_x]$

Let $P_x' - P_x = \Delta P$ and $M' - M = \Delta M$. so the above equation becomes

$\Delta M = X\,\Delta P$

Therefore if price of good X goes up (down), we need to increase (decrease) the money income to enable the consumer to afford the old bundle (X,Y)

8. Numericals

Question 1:
Ajay whose weekly income is Rs 100 has the following demand function for candy bar:

X = 10 + M/2P where X is the demand for Candy bar, M is his income and P is the price of candy bar.

Calculate the price effect, substitution effect and income effect when the price of Candy bar fall from Rs 4 to Rs 2.

Solution:

We are given that initial income = Rs 100, initial price of candy bar is Rs 4 and final price of candy bar is Rs 2. So we have

$M_1 = 100\; P_1 = 4\; P_2 = 2$

The demand for candy bar when price is Rs 4 and income is Rs 100 is given by:

$$X\,(4, 100) = 10 + \frac{100}{2\,X\,4} = 22.5$$

The demand for candy bar when price is Rs 2 and income is Rs 100 is given by:

$$X\,(4, 100) = 10 + \frac{100}{2\,X\,2} = 35$$

The change in demand for candy bar when price fall from Rs 4 to Rs 2 is given by:

$Price\ effect = \Delta X^{PE} = X\,(P_2, M_1) - X\,(P_1, M_1) = 35 - 22.5 = 12.5$

When price of candy bar falls from Rs 4 to Rs 2, we need to adjust the money income to enable the consumer to afford the old bundle. The change in income is given by:

$\Delta M = X \Delta P = 22.5 X (2 - 4) = -45$

The new income which enables the consumer to afford the old bundle is given by

$M_2 = M_1 + \Delta M = 100 - 45 = 55$

The change in demand due to substitution effect is given by:

$\Delta X^{SE} = X (P_2, M_2) - X (P_1, M_1) = \left(10 + \frac{55}{2 X 2}\right) - \left(10 + \frac{100}{2 X 4}\right) = 1.25$

The change in income due to income effect is given by:

$\Delta X^{IE} = X (P_2, M_1) - X (P_2, M_2) = \left(10 + \frac{100}{2 X 2}\right) - \left(10 + \frac{55}{2 X 2}\right) = 11.25$

Note: Price effect = substitution effect + income effect

Question 2:

In which of the following case (s) is the Law of demand being violated?

(a) Normal goods (b) Inferior goods (c) Giffen goods

Solution:

The Law of demand basically states that there is inverse relationship between price and quantity demand of a good. For normal goods when there is a decrease (increase) in price, both substitution and effect ensures that that quantity demand goes up (down). For inferior good, substitution effect and income effect have opposing effect. However since substitution effect is stronger than income effect, a decrease (increase) in price will still lead to an increase (decrease) in demand. The Law of demand will be violated in case of Giffen good since income effect is stronger than substitution effect

9. Summary

1. The change in price of a good has two types of effects: the relative price of a good changes and the total purchasing power of income also changes.

2. The Substitution effect measures the change in demand of a good due to the change in relative prices assuming that the purchasing power remains constant.

3. The income effect measures the change in demand due to the change in purchasing power of income assuming that the relative prices are held constant

4. The Slutsky Substitution effect allows the original consumption bundle to remain affordable while the Hicksian case allows the utility of the consumer to remain constant after the change in price.

5. The Slutsky equation says that the price effect is the sum total of the substitution effect and income effect.

6. The uncompensated demand curves draw the relationship between price and quantity demanded by holding income as fixed while the compensated demand curves holds the utility of the consumer as constant.

7. The Law of demand says that we must have downward sloping demand curves for normal goods.

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