LESSON: Revealed Preference Theory

LESSON DEVELOPERS: Rupali A Khanna

UNIVERSITY: Kamala Nehru College, Delhi University

THE REVEALED PREFERENCE THEORY

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2. INTRODUCTION

Paul A Samuelson suggested the economic theory of consumer behavior can be largely built up on the notion of “revealed preference”. Revealed preference is a theory which reverses the process of the indifference curve theory. Indifference curve theory relies on the assumption that preferences can tell a lot about consumer’s behavior. But in reality, it is not possible to directly observe a consumer’s preference. They have to be inferred from consumer’s behavior. The revealed preference theory was originally intended "to develop the theory of consumer's behaviour freed from any vestigial traces of the utility concept ",i.e., as a substitute for the " utility function " and related formulations, but it has tended to become complementary to the latter.

To develop some concepts, we assume that consumer’s preference does not change when we observe his/her behavior. This is truer for shorter periods rather than long ones. However economists deal with shorter time spans while forming the theory of consumer behavior until stated otherwise. The question is, if we know the choices a consumer makes, can we determine his or her preferences? The answer is yes, but for that we need to have information about sufficient number of choices that have been made when prices and income level varied. In the next section, we lay down the assumptions for the theory which will be followed by the theory, its principle and its axioms in the subsequent sections.

3. ASSUMPTIONS

In a two good world, this theory assumes the consumer is rational and he spends all of his income to purchase a combination of both goods. It is also assumed the underlying preferences—whatever they may be—are known to be strictly convex. In other words, the consumer has convex preferences imply that there will be a unique demanded bundle at each level of income M. Another underlying assumption is that of consistency i.e. if one consumption bundle is preferred over the second consumption bundle, then with same price set, the second consumption bundle will not be preferred to the first one in some other state. Moreover, transitivity is assumed which means that if one consumption bundle is

preferred over the second consumption bundle and the second bundle is preferred over the third bundle, then by transitivity, first bundle is preferred over the third bundle.
Keeping in mind all these assumptions, we develop the theory in the next section.

4. THE REVEALED PREFERENCE THEORY

Now, if we confine ourselves to the case of two commodities x and y, we can conceptually observe for any individual a number of price quantity situations. The prices of x and y are P_{x1} and P_{y1} respectively and what matters is the relative prices P_x/P_y. So each observation consists of three numbers, $(P_x/P_y, x, y)$. Convexity assumption says that only one combination of x and y is associated with one price ratio px/py.

Theoretically, for any point (x,y), we can determine a unique P_x/P_y i.e.

$$P_x/P_y = f(x,y) \quad (1)$$

This implies that there are many combinations of x and y which can be bought in preference to what was actually bought. But they weren't. Hence, they are all "revealed" to be inferior to the actually bought consumption bundle. No other line of reasoning is needed.

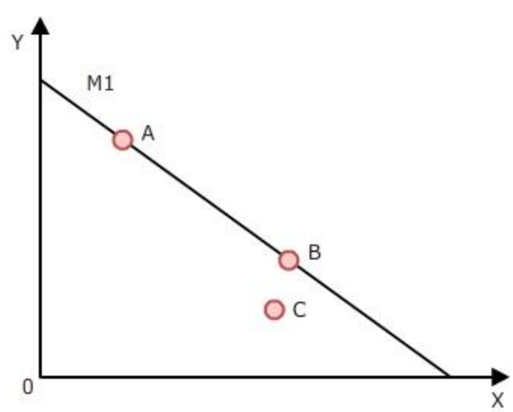
Suppose there are two consumption bundles $A(x_1, y_1)$ and $B(x_2,y_2)$ with P_x & P_y as price of x and y respectively and $A(x_1,y_1)$ lies on the budget line and $B(x_2,y_2)$ lies under the budget line. We assume in the consumer theory that consumer tries to maximizes his utility and hence spends his entire income M (i.e. savings are nil), then what can we infer about the preference of the consumer for these two bundles? We see that both bundles are affordable as they satisfy the condition

$$x{*}P_x + Y{*}p_y \leq M \quad (2)$$

The bundle $B(x_2,y_2)$ is affordable at the given income and the consumer could have preferred $B(x_2,y_2)$ over $A(x_1, y_1)$ if he/she wanted to and could have saved money. However the consumer opted for $A(x_1, y_1)$ over $B(x_2,y_2)$, this shows that $A(x_1, y_1)$ must be a better bundle than $B(x_2,y_2)$ than anything else consumer can afford. Therefore, it must be better than $B(x_2,y_2)$. This holds true for any bundle lying below or on the budget line.

We can see this with help of a diagram. In figure 1, we have a budget line M1 and there are two consumption bundles. One lies on the budget line and the other below it

Figure1: Revealed Preference



There is another consumption bundle $C(x_3,y_3)$ which lies on the budget line. Still the consumer purchases bundle $A(x_1,y_1)$, even though he/she could have purchased bundle $C(x_3,y_3)$ but did no, we say that $A(x_1,y_1)$ is preferred to $B(x_2,y_2)$. Hence the assumption here used is: there is a unique demanded bundle at every level of income. This uniqueness stays because of the assumption of convex preferences.

Mathematically, we can explain these preferences:

At income M, $A(x_1,y_1)$ is purchased at prices (p_{x1},p_{y1}) even though $B(x_2,y_2)$ is available and affordable. This means $B(x_2,y_2)$ satisfies the budget constraint:

$$x_2{*}P_{x1} + y_2{*}P_{y1} \leq M \tag{3}$$

the bundle (x_1, y_1) is actually bought which means

$$x_1{*}P_{x1} + y_1{*}P_{y1} = M \tag{4}$$

Combining 1 and 2, the fact that (x_1, y_1) is preferred over (x_2,y_2) means that

$$x_1{*}P_{x1} + y1{*}P_{y1} \geq x_2{*}P_{x1} + y_2{*}P_{y1} \tag{5}$$

if this inequality is satisfied, then we say that (x_1, y_1) is directly revealed preferred to (x_2,y_2).

Principle of the Revealed Preference theory : ***Let (x_1, y_1) be the chosen consumption bundle when prices are (p_{x1}, p_{y1}), and let (x_2, y_2) be some other consumption bundle such that $x_1{*}P_{x1} + y_1{*}P_{y1} \geq x_2{*}P_{x1} + y_2{*}P_{y1}$, then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1, x_2) \succ (y_1, y_2)$.***

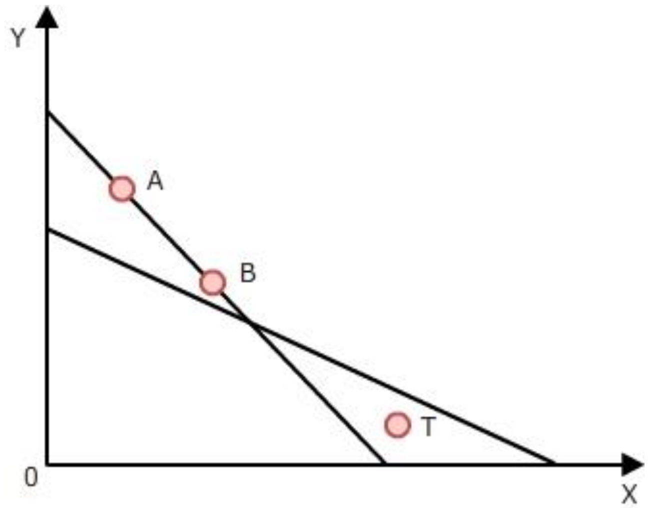
Revealed preferred means that the consumer bought (x_1,y_1) when (x_2,y_2) was available. In other words, we can say (x_1,y_1) is chosen over (x_2,y_2) and hence we can state the principle of revealed preference by saying: "If a bundle X is chosen over a bundle Y , then X must be preferred to Y ."

We take another case where the demanded bundle $B(x_2,y_2)$ at prices (P_{x2},P_{y2}) is itself revealed preferred to another consumption bundle $T(x_4,y_4)$ such that

$$X_2{*}P_{x2} + y_2{*}P_{y2} \geq x_4{*}P_{x2} + y_4{*}P_{y4} \tag{6}$$

Then by transitivity assumption, we can infer that (x_1,y_1) is preferred to (x_4,y_4). Revealed preference theory tells that (x_1,y_1) must be a better consumption bundle than (x_4,y_4). In other words, we say that $A(x_1,y_1)$ is **indirectly revealed preferred** to $T(x_4,y_4)$. The number of consumption bundles in this transitivity chain can be any number. This is an interesting and powerful finding because point $T(x_4,y_4)$ can lie above the budget line on which $A(x_1,y_1)$ is lying. We can see this diagrammatically more clearly:

Figure 2: Indirectly Revealed Preference



In the diagram, point T(x_4,y_4) lies above the budget line on which point A(x_1,y_1) is lying. This is true for any point inside the shaded region. If we generalize this, we can conclude that the indifference curve through point A(x_1,y_1)must lie above the shaded region.

Case: Revealed preference for time spent in entertainment hub

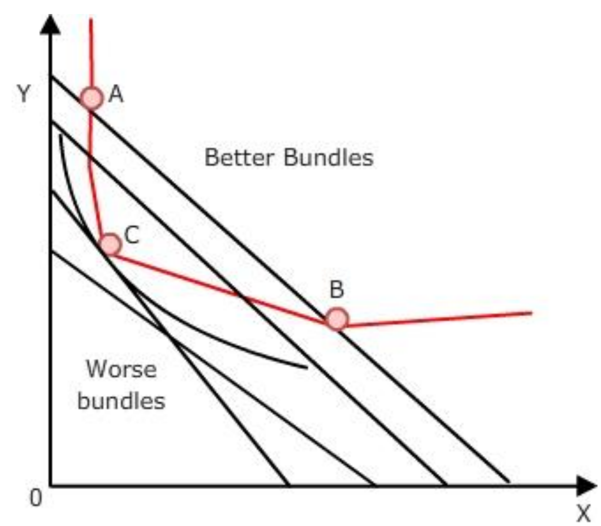
An entertainment hub with eight lane bowling alley, a swimming pool and a re-creation center in a posh area is offering the use of its facilities to everyone who are ready to pay an hourly price. The owner decides to change the price policy. He decided that he will charge an annual charge but will reduce the hourly fee. This new price set up can make consumers better off or worse off in comparison to the old set up. This depends on the consumers' preferences. If Rahul has Rs1000 of income available each week for recreational activities and the hub charges Rs 60 as fee per hour. He used the facility generally for 10 hours per week. Under the new price scheme, he is required to pay Rs 400 per week as fixed charges but now have to give Rs 15 per hour. What can we conclude about this two part tariff? In the first price arrangement, Rahul spent Rs600 on hub activities and rest Rs400 for other recreations like movie, restaurant meals etc. In the new price setting, if he chooses the original combination, then he has to pay Rs400 + Rs150 i.e. Rs550 and still Rs450 left for other recreations. But he does not choose the original basket but goes for another combination of 12 hours per week. This way his total expenditure would be Rs580. He could have chosen initial 10 hrs setting but has not chosen. This means he is better off with 12 hours and is on a higher indifference curve. The new pricing arrangement makes Rahul better off because initially he chooses 10 hours but when prices are altered, he chooses to go for 12 hours (10 hours) were obviously affordable).

5. CONSTRUCTION OF INDIFFERENCE CURVE FROM REVEALED PREFERENCE

The revealed preference theory is an alternative theory to utility theory. It treats the individual's choice behavior as a primitive feature and proceeds by making assumptions about the consumer behavior. It makes assumptions about objects that are directly observable rather than preferences that are not. The theory defines a principle of rationality that is based on observed behavior and then uses this principle to approximate an individual's utility function. Now we have assumed that preferences are convex. If we add more assumptions, we can move towards deriving the indifference curve with the help of revealed preference theory.

Suppose we have 2 consumption bundles A, B & C and A & B are revealed preferred to C. This implies the weighted average of A & B are preferred to C as well. Now, if we add another assumption of monotonicity of preferences, then all the consumption bundles having more of A, B or C will be preferred to C. This holds true for any weighted average of these bundles.

Figure3: Constructing the Indifference Cu



The consumption bundles below point C are "worse bundles" as C is revealed preferred to all these points. All these so called worse bundles have lesser cost than cost of bundle C. Similarly, we can imply that all bundles in the upper shaded area are preferred to C and hence are "better bundles". The indifference curve on which C lie should typically lie between these two areas of "worse bundles" and "better bundles". We can see this more clearly in the figure above. The indifference curve passing through C which shows the utility level at C is positioned between the two shaded areas.

In terms of the functional form $P_x/P_y = f(x,y)$, specifically taking an example of rectangular hyperbola with the form $P_x/P_y = y/x$

We identify a slope, d_y/d_x, with each price ratio, $- P_x/P_y$. We have the simplest differential equation.

$$d_y/d_x = -P_x/P_y$$

It is known mathematically that this defines a unique curve through any given point, and a (one-parameter) family of 'curves throughout the surrounding (x, y) plane. These solution curves (or "integral solutions" as they are often called) are such that when any one of them is substituted into the above differential equation, it will be found to satisfy that equation. These solution curves are the conventional "Indifference curves".

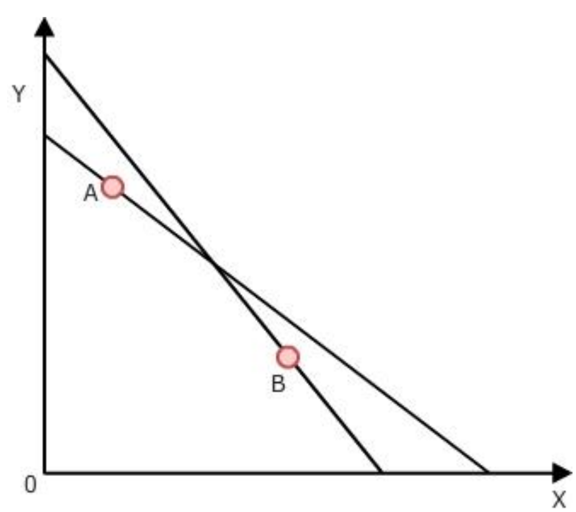
6. THE TWO AXIOMS

The above theory developed relies on the fact that consumer buys the best consumption bundle he/she can afford. The following axioms reflect that a consumer's observed choices will display a certain amount of consistency and that a consumer is following a maximizing model.

6.1 WEAK AXIOM OF REVEALED PREFERENCE THEORY

The axiom is developed with a help of an illustration. Suppose there are two consumption bundles $A(x_1,y_1)$ and $B(x_2,y_2)$. They lie on different income constraints as seen in the following figure and revealed preference theory may lead to strange conclusions.

Figure4: WARP: the Violation



The figure says that a certain income M1, consumer chooses $A(x_1,y_1)$ over $B(x_2,y_2)$, when $B(x_2,y_2)$ was available and with income M2, consumer chooses $B(x_2,y_2)$ over $A(x_1,y_1)$ which is clearly contradicting to the consumer's maximizing behavior. This displays two possibilities: a) Either the consumer is not a rational consumer, or b) there is some aspect of choice which is not observed, i.e. a possibility of a change in tastes of the consumer or a change in the economic environment. However, in an unchanged relatively shorter term environment, the above illustration is a violation of the preference theory. The theory says such inconsistency will not arise. The consumption bundle not chosen when available must be a 'worse bundle' or an inferior bundle to what is chosen. This important assumption is stated formally in the **Weak Axiom of Revealed Preference**.

WEAK AXIOM OF REVEALED PREFERENCE (WARP): If a consumption bundle A(x1,y1) is preferred over another consumption bundle B(x2,y2) and A(x1,y1) ≠ B(x2,y2), then there can be no budget set containing both alternative choices A(x1,y1) and B(x2,y2), where B(x2,y2) is preferred over A((x1,y1).

In other words, if bundle A is directly revealed preferred to bundle B, then B cannot be directly revealed preferred to bundle A. The assumption that the choice behavior satisfies the WARP captures the consistency idea. In the above figure, the WARP is violated/not satisfied and hence the behavior of the consumer is not utility maximizing behavior. If the bundle B is affordable when the bundle A is purchased, then when the bundle B is purchased, the bundle A must not be affordable.

Mathematically, if $A(x_1,y_1)$ is chosen at prices (p_{x1},p_{y1}) and $B(x_2,y_2)$ is chosen at prices (p_{x2},p_{y2}), then if

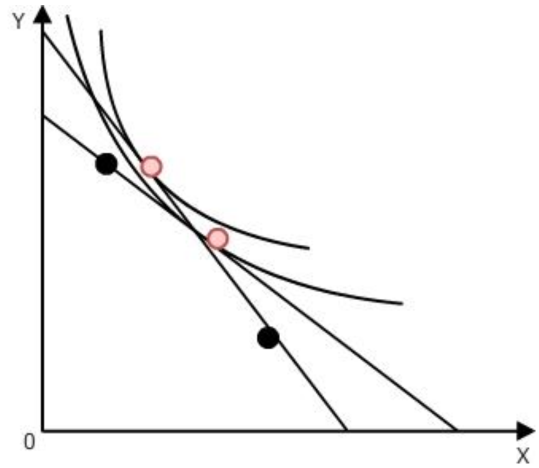
$$p_{x1}*x_1 + p_{y1}*y_1 \geq p_{x1}*x_2 + p_{y1}*y_2, \quad (7)$$

then, the following is not possible:

$$P_{x2}*x_2 + P_{y2}*y_2 \geq P_{x2}*x_1 + P_{y2}*y_1 \quad (8)$$

There is no set of indifference curves in the same indifference map which can show both consumption bundles as maximizing bundles.

Figure5: Possible Indifference curves



In the above figure, we look at two other bundles A′(x_1′,y_1′) and B′(x_2′,y_2′) and if, instead of A and B, the bundles are A′(x_1′,y_1′) and B′(x_2′,y_2′), then we can identify a set of indifference curves which satisfies WARP.

How to check WARP:

State	Bundle	Px	Py	x	y
State I	Bundle I	1	2	2	2
State II	Bundle II	3	1	3	1

Expenditure on bundle I at prices (1,2)= 2*1 + 2*2 = 6 ; Expenditure on bundle II at prices (1,2)= 3*1 + 2*1 = 5; Bundle I is purchased when bundle II was affordable.

Expenditure on bundle I at prices (3,1) = 3*2 + 1*2 = 8 ; Expenditure on bundle II at prices (3,1) = 3*3 + 1*1 = 10; Bundle II is chosen when bundle I is available. Hence WARP is violated.

The procedure is: we calculate the cost of the consumption bundles at each set of prices, and form a matrix of expenditures. The diagonal of the matrix shows the actual expenditure in each state. The other entries in a row show the possible expenditure if the other bundle is purchased. We check for non diagonal entries, whether it is more or less than the actual expenditure and if it is less, then we mark it with *. If the box of column m and row n is marked with a *and if column n and row m is also marked with a *, then WARP is violated.

Price	**Bundle I**	**Bundle II**
State I	**6**	**5***
State II	**8***	**10**

In the above example, m,n is marked with a * and n,m is also marked. Hence WARP is not satisfied. This simply says that bundle II could have been purchased with state I prices but consumer chose bundle I and vice versa hence violating WARP.

However, WARP is a necessary condition of choices to be consistent with the utility theory. However, it is not sufficient condition. Consumer choice may satisfy the weak axiom but cannot be generated by a rational preference relation. [1]For this we need a necessary and sufficient condition on consumer behavior on the same lines as WARP. This led to the Strong Axiom of Revealed Preference (SARP). It was developed by Houthakker in 1950 and it is discussed in the next subsection.

6.2 STRONG AXIOM OF REVEALED PREFERENCE THEORY

We have already noted that if A(x_1,y_1) is revealed preferred to B(x_2,y_2) and B(x_2,y_2) is revealed preferred to C(x_3,y_3)., then A(x_1,y_1) must be revealed preferred to C(x_3,y_3). Then by transitivity, a rational consumer would never reveal his preference for C(x_3,y_3) over A(x_1,y_1), if the latter is available. WARP talks about one consumption bundle being directly revealed to another implying the impossibility of the vice versa. Whereas, SARP is if a consumption bundle is indirectly revealed preferred to another consumption bundle, then the opposite cannot be true.

STRONG AXIOM OF REVEALED PREFERENCE (SARP): If a consumption bundle A(x1,y1) is directly or indirectly revealed preferred to C(x3,y3), then C(x3,y3) cannot be directly or indirectly revealed preferred to A(x1,y1), if A(x1,y1) is affordable/available.

This clearly says that preferences are transitive. It follows that the revealed preference must be transitive. Thus SARP is a necessary implication of optimizing behavior i.e. if a consumer is always choosing the best things that he can afford, then his observed behavior must satisfy SARP. SARP is also a sufficient condition for optimizing behavior i.e. if the observed choices satisfy SARP, then it is always possible to find preferences for which the observed behavior is optimizing behavior. Thus SARP is both a necessary and a sufficient condition for observed choices to be compatible with the economic model of consumer choice.

To check SARP: we take a numerical 3X3 matrix of expenditures. We need 3 set of prices (px,py) with 3 consumption bundles of x and y bought at each set of prices. The matrix is as follows:

	Bundle I	Bundle II	Bundle III
Price set I	40	20*	44**
Price set II	42	40	30*
Price set III	24	30	20

We can observe that bundle I is directly revealed preferred to bundle II as bundle II is affordable but still consumer has chosen bundle I (indicated by * sign). in the second state, consumer chooses bundle II because bundle I is not affordable. Similarly, bundle II is directly revealed preferred to bundle III (indicated by * sign). In the third state, the consumer chooses bundle III as bundle II is not affordable. All this implies that bundle I is indirectly revealed preferred to bundle III. This can be indicated by double star sign **. Now testing for SARP requires that we should check whether bundle III is revealed preferred to bundle I or not in any way, when both are affordable. We find that bundle III is purchased

[1] See Mascollel- whinston & Greene

in state III when bundle I is not available (not affordable). If bundle I is affordable and still bundle III is chosen, then this is a violation of SARP.

To check both WARP and SARP:
There can be a case where WARP is not violated but SARP is violated. We can see this with the help of an numerical example:

	Bundle I	Bundle II	Bundle III
Price set I	46	47	46
Price set II	39	41	46
Price set III	24	22	23

In state I, bundle I is directly revealed preferred to bundle III. In second state, bundle II is directly revealed preferred t bundle I. in the third state, bundle III is directly revealed preferred to bundle II. Hence WARP is not violated. This shows, bundle I is preferred to bundle III; bundle II is preferred to bundle I and bundle III is preferred to bundle II. So by transitivity, bundle I is preferred to bundle II which is inconsistent with bundle II being preferred to bundle I. Hence SARP is violated. Hence this data set is not rationalized by well behaved preferences.

7. COST OF LIVING INDEX NUMBERS

Consumption generally changes from one time period to another i.e. the consumer consumes a different consumption bundle at different times. The reasons can be different. It can be change in prices, change in income or it may be change in tastes of a consumer. We need to examine the change in consumption from one period to another.

So if we take a base period "b" and a current period/state as "t", we want to see how consumption changes from b to t and whether the consumer is in a better state today or in a worse state. Suppose there are two goods x and y and a consumer G consumes (x_t, y_t) in period "t" at prices (p_{tx}, p_{ty}). This means he has chosen the bundle (x_t, y_t) over any other bundle available and affordable at prices (p_{tx}, p_{ty}). Similarly, in period "b", G consumes a certain consumption bundle (x_b, y_b) at prices (p_{bx}, p_{by}). We are interested in knowing how average consumption changes from one period to the other. ***Cost of Living Index*** exactly does that. These indices measure the cost of a typical consumer's consumption bundle in each period and compare the change in cost.

For this purpose, we attach weights wx and wy to each good in each period, which will enable us to form some kind of ***cost of living index***. Before forming an index with weights, we define ***cost of living Index***, which is ratio of the present cost of a consumption bundle of consumer goods and services compared with the cost during a base period.

With given weights, we can form a quantity index:

$$I_q = \frac{w_x * x_t + w_y * y_t}{w_x * xb + w_y * y_b} \qquad (9)$$

The value of Iq will tell whether the consumer is better off or worse off in the present period. If it is more than one, then we can infer that the consumption in the present period is more than the consumption in the base period. If the value is less than one, then we say that the consumption has reduced and the consumer is worse off in the current period "t".

7.1 Quantity indices

Now the question arises is, what should be taken as weights? The answer is, a variable which can truly represent the significance of the good in the total consumption. A very obvious choice would be its own "price" as it would quantify the relative importance to the good. We can use either of the two sets of prices i.e. either base year prices (p_{bx},p_{by}) or the current period prices (p_{tx},p_{ty}). Depending upon, which set of prices we use, we get a different index.

If we use base period prices, then we get the following:

$$I_{bq} = \frac{w_{bx}*x_t + w_{by}*y_t}{w_{bx}*x_b + w_{by}*y_b} \qquad (10)$$

where (w_{bx},w_{by})=(p_{bx},p_{by})

So substituting (w_{bx},x_{by}) with (p_{bx},p_{by}), we obtain

$$L_{bq} = \frac{p_{bx}*x_t+ p_{by}*y_t}{p_{bx}*x_b + p_{by}*y_b} \qquad (11)$$

This kind of index where we use base year prices as weights, is called ***Laspayres Quantity Index***. However, if we use current year's weights, the index becomes

$$I_{tq} = \frac{w_{tx}*x_t + w_{ty}*y_t}{w_{tx}*x_b + w_{ty}*y_b} \qquad (12)$$

where (w_{tx},w_{ty}) = (p_{tx},p_{ty})

so substituting (w_{tx},w_{ty}) with (p_{tx},p_{ty}), we obtain

$$P_{tq} = \frac{p_{tx}*x_t + pty*y_t}{ptx*x_b + pty*y_b} \qquad (13)$$

This index with current year prices as weights is called ***Paasche's Quantity Index***.

Both of these indices answer the question of what has happened to consumption, but they just use different weights in the process. We need to interpret these indices and know what do they reveal about the consumer's welfare.

If Laspayres quanity index is less than one, i.e.

$$L_{bq} = \frac{p_{bx}*x_t + p_{by}*y_t}{p_{bx}*x_b + p_{by}*y_b} <1 \qquad (14)$$

we cross multiply and obtain the following:

$$p_{bx}*x_{\underline{t}} + p_{by}*y_{\underline{t}} \;<\; p_{bx}*x_b + p_{by}*y_b \qquad (15)$$

this clearly shows that the consumer is worse off in the present period as compared to the base period. The consumption bundle (x_b, y_b) was revealed preferred to the consumption bundle (x_t,y_t) in base period but in the current period, the consumer is not able to buy (x_b, y_b) and hence bought (x_t,y_t). Hence he/she is worse off in the present period.
But if the index is greater than one, then we would have

$$p_{bx}*x_{\underline{t}} + p_{by}*y_{\underline{t}} \;>\; p_{bx}*x_b + p_{by}*y_b \qquad (16)$$

which says that when the consumer chose bundle (x_b, y_b) in the base period, bundle (x_t,y_t)was not affordable. But that doesn't say anything about the consumer's ranking of the bundles. Just because something costs more than the consumer can afford doesn't mean that he prefers it to what he is consuming now. Therefore, this result stays inconclusive about the consumer's welfare.

In case of Paasche's quantity index, we use current period prices (ptx,pty) as weights and hence the index becomes

$$P_{tq} = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{tx}*x_b + p_{ty}*y_b} \qquad (17)$$

If Ptq is greater than one, then cross multiplying implies

$$p_{tx}*x_{\underline{t}} + p_{ty}*y_{\underline{t}} \;>\; p_{tx}*x_b + p_{ty}*w_b \qquad (18)$$

Here the consumer is revealing his preferences for (x_t,y_t) over (x_b, y_b), which immediately proves that the consumer must be better off in the current period t as compared to the base period b, since he/she could have consumed the base period consumption bundle in the current period but chose not to do so. Therefore, the consumer's welfare has gone up from the base period to the current period.

However, if the case is that index is less than one, then it means

$$p_{tx}*x_{\underline{t}} + p_{ty}*y_{\underline{t}} \;<\; p_{tx}*x_b + p_{ty}*w_b \qquad (19)$$

which says, that the consumer bought (x_t,y_t) in the current period because the consumption bundle (x_b, y_b) is not available. This does not tell conclusively about the consumer's preferences. (x_b, y_b) was not purchased in the present scenario because it is not affordable with current prices and not because the preferences have changed.

In summary, when have conclusive implications about a consumer's welfare, when Ltq<1 or Pqb>1. But we cannot conclude anything about the consumer's change in wellbeing if we face the other two cases i.e. $L_{tq}>1$ or $P_{bq}<1$.

7.2 Price Indices

Till now, we looked at quantity indices in which we have consumption bundles and we look for appropriate weights and the most obvious ones are prices. Another case is, when we have set of prices and form price indices with some appropriate weights.
A price index with weighted average of prices is as follows:

$$I_p = \frac{w_x*p_{xt} + w_y*p_{yt}}{w_x*p_{xb} + w_y*p_{yb}} \qquad (20)$$

in this case, we choose quantity as weights for calculating the averages. Again, we get two types of indices depending on the weights. If we take base year b's quantity as weights, then we get ***Laspayre's Price Index***, which is as follows:

$$L_p = \frac{p_{tx}*x_b + p_{ty}*y_b}{p_{bx}*x_b + p_{by}*y_b} \qquad (21)$$

similarly, if we take current period t's quantities as weights, we get ***Paasche's Price Index***

$$P_p = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_t + p_{by}*y_t} \qquad (22)$$

Therefore, we got two types of price indices. To use these price indices, we introduce one more index i.e. expenditure index Ie. This index tell us about the change in total expenditure, which is defined as the ratio of total spending in current period t to total spending in the base period b. this will give the change in spending from period b to period t.

Total spending in period t = $ptx*x_t + pty*y_t$

Total spending in period b = $pbx*x_b + pby*y_b$

Hence the expenditure index Ie would be:

$$I_e = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_b + p_{by}*y_b} \qquad (23)$$

we have introduced this expenditure index because the price indices with quantity as weights have current year prices in the numerator and base year prices in the denominator. When the prices are different in these indices, comparisons cannot be made. Comparisons can be made between a) Laspayre's index and the expenditure index and b) Paasche's index and the expenditure index. There are four possible cases which are discussed below:

7.2.1) If the Laspayre's price index is less than the expenditure index, then what does `it reveal about the consumer's welfare?

$$Lp = \frac{ptx*x_b + pty*y_b}{pbx*x_b + pby*y_b} < Ie = \frac{ptx*x_t + pty*y_t}{pbx*x_b + pby*y_b}$$

The denominator gets cancelled from both sides and we get

$$ptx*x_b + pty*y_b < ptx*x_t + pty*y_t$$

This says, that if the Laspeyres price index is less than expenditure index Ie, then the consumer must be better off in year t than in year b because the chosen consumption bundle in current period t is revealed preferred to the consumption bundle chosen in period b. this is because (x_b, y_b) is available in the current period but the consumer chooses (x_t, y_t).

this simply confirms the intuitive idea that if prices rise less than income, the consumer would become better off.

7.2.2) If the Laspayre's price index is more than the expenditure index, then what does it reveal about the consumer's behavior?

$$Lp = \frac{ptx*x_b + pty*y_b}{pbx*x_b + pby*y_b} > Ie = \frac{ptx*x_t + pty*y_t}{pbx*x_b + pby*y_b}$$

This implies that

$$ptx*x_b + pty*y_b > ptx*x_t + pty*y_t$$

This is simply saying that because the consumption bundle (x_b, y_b) is not affordable in the current period thence the consumer goes for (x_t,y_t). We are unable to compare the consumer's welfare, conclude about bundles' affordability and hence consumer's preferences.

7.2.3) If the Paasche's index is more than the expenditure index Ie, then the inferences would be somewhat different i.e. if

$$P_p = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_t + p_{by}*y_t} > I_e = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_b + p_{by}*y_b}$$

Canceling the numerators from each side of this expression and cross multiplying, we get

$$p_{bx}*x_b + p_{by}*y_b > p_{bx}*x_t + p_{by}*y_t$$

this inequality says that the consumption bundle chosen in year *b* is revealed preferred to the consumption bundle chosen in year *t*. This implies that if the Paasche's price index is greater than the expenditure index, then the consumer must be better off in year *b* than in year *t*. he didn't choose (x_t,y_t) when it was available, so when the consumer chooses (x_t,y_t), it is not his/her first choice.

7.2.4) On the other hand, if the Paasche's price index is less than the expenditure index, then what can we conclude?

$$P_p = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_t + p_{by}*y_t} < Ie = \frac{p_{tx}*x_t + p_{ty}*y_t}{p_{bx}*x_b + p_{by}*y_b}$$

$$p_{bx}*x_b + p_{by}*y_b < p_{bx}*x_t + p_{by}*y_t$$

then we see that the consumer chooses consumption bundle (x_b, y_b) in the base period b when (x_t,y_t) was not affordable. Hence we cannot conclude anything about consumer's welfare when he selects (x_t,y_t) in the current period t. we are not able to compare the two situations in period b and period t.

Overall, we can make inferences about the consumer's wellbeing with the help of price indices by comparing it with the expenditure index.

8. SUMMARY

- The Revealed Preference Theory reverses the process of the indifference curve theory.
- There is a unique demanded bundle at every level of income. This uniqueness stays because of the assumption of convex preferences.
- Principle of the Revealed Preference theory: Let (x_1,y_1) be the chosen consumption bundle when prices are (p_{x1}, p_{y1}), and let (x_2,y_2) be some other consumption bundle such that $x_1*P_{x1} + y_1*P_{y1} \geq x_2*P_{x1} + y_2*P_{y1}$, then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1,y_1) \succ (x_2, y_2)$.
- Observing the choices of consumers can allow us to estimate the preferences that lie behind those choices. The more choices we observe, the more precisely we can estimate the underlying preferences that generated those choices.
- Weak Axiom Of Revealed Preference (WARP): If a consumption bundle $A(x_1,y_1)$ is preferred over another consumption bundle $B(x_2,y_2)$ and $A(x_1,y_1) \neq B(x_2,y_2)$, then there can be no budget set containing both alternative choices $A(x_1,y_1)$ and $B(x_2,y_2)$, where $B(x_2,y_2)$ is preferred over $A(x_1,y_1)$.
- Strong Axiom Of Revealed Preference (SARP): If a consumption bundle $A(x_1,y_1)$ is directly or indirectly revealed preferred to $C(x_3,y_3)$, then $C(x_3,y_3)$ cannot be directly or indirectly revealed preferred to $A(x_1,y_1)$, if $A(x_1,y_1)$ is affordable/available.
- Revealed preference theory helps us to compare the consumer's welfare with the help of cost of living indices.

EXERCISE

1. The utility that Ravi receives by consuming food F and clothing C is given by U(F,C)=FC. Suppose Ravi's income in 2005 was Rs12000 and that the prices of food and clothing are Rs10 per unit for both. By 2013, the price of food increased to Rs20 and price of clothing increased to Rs30. Let 100 represent the cost of living index for Ravi for 2005. Calculate the Laspayre's and Paasche's cost of living index for Ravi, given the assumption that Ravi spends equal amounts on both commodities.

2. Which index is better, quantity index or price index? Explain your answer.

3. Given the following information about consumer's preferences, tell whether the consumer is a rational consumer or not:

	Px	Py	X	Y
State 1	2	6	10	15
State II	2.5	5	8	18
State III	2.25	5.5	9	16

4. State whether true or false? Explain why?
"The strong axiom of revealed preference requires that if a consumer chooses x when he can afford y, and chooses y when he can afford z, then he will not choose z when he can afford x."

REFERENCES

1. Paul A. Samuelson (1948): Consumption Theory in Terms of Revealed Preference. Economica, New Series, Vol. 15, No. 60, pp. 243-253

2. Richter, Marcel K (1966): Revealed Preference Theory. Econometrica, Vol. 34, No. 3, pp. 635-645

3. Houthakker, H S (1950): Revealed Preference and the Utility Function. Economica, New Series, Vol. 17, No. 66, pp. 159-174

4. Varian, Hal - Intermediate Microeconomics- A modern approach (8^{th} edition)

5. Rubinfeld, R S and Pindyck, D L – Microeconomics (6^{th} edition)

6. Federico Echenique, Sangmok Lee, Matthew Shum And M. Bumin Yenmez M. Bumin Yenmez (2013): The Revealed Preference Theory Of Stable And Extremal Stable Matchings. Econometrics, Vol. 81, No. 1, 153–171

7. Bandhopydhyay, T and Sengupta, K (1991): Revealed Preference Axioms and Rational Choice. The Economic Journal (101), pp 202-213.