Statistical Methods In Economics - I

CONDITIONAL PROBABILITY

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1. Learning Outcomes

After going through this chapter, you should be able to:

- Understand the concept of conditional probability and its application
- Explain the independence of events
- Apply various counting techniques like addition, multiplication, permutation and combination to probability
- Understand the Bayes' theorem and its application

2. Introduction

CONDITIONAL PROBABILITY:

It is defined as the probability of occurrence of an event A, given that another event B has already occurred. We may represent the conditional probability of event A as:

$P(A/B) = P(B \cap A) / P(B)$, provided $P(B) \neq 0$, or
$P(B \cap A) = P(B) * P(A/B)$

Similarly, $P(A \cap B) = P(A) * P(B/A)$ or
$P(B/A) = P(A \cap B) / P(A)$, provided $P(A) \neq 0$

For example, if a bag contains W white balls and B blue balls, and two balls are drawn at random (without replacement). The probability of getting a white ball is $\frac{W}{(W+B)}$. Suppose in the first turn, a white ball was obtained then the conditional probability of getting a white ball in the second turn is: $\frac{W-1}{(W+B-1)}$

Similarly, if the ball obtained in the first draw was blue, the conditional probability of getting a white ball in the second turn is: W/(W+B-1)

Now, Let A denote a white ball in first draw, and B denote a white ball in second draw, then

$P(A) = \frac{W}{(W+B)}$ and

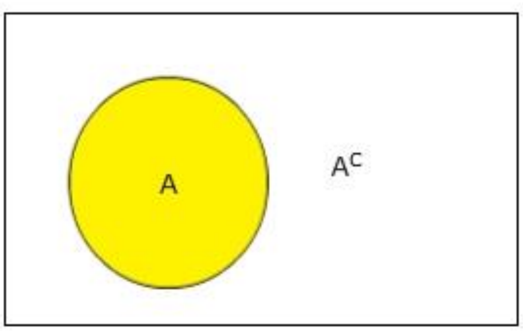
$P(A \cap B) = \frac{W-1}{(W+B-1)}$, given A

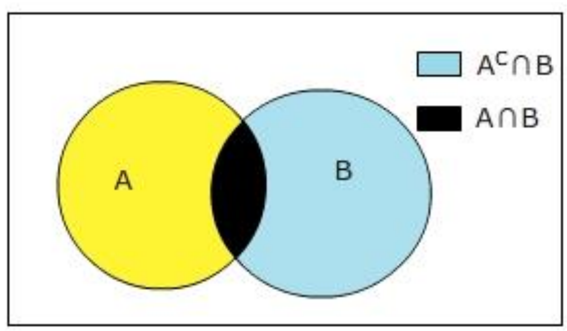
Now, the conditional probability of B =

$P(B/A) = P(A \cap B) * P(A)$

$= \frac{W-1}{(W+B-1)} * \frac{W}{(W+B)}$

We have already studied that, $P(A \cap B) = P(A/B) / P(B)$
Similarly, $P(A^C \cap B) = P(A^C/B) / P(B)$, where A^C = complement of A





From the venn diagrams we can say,

$$P(B) = P(A \cap B) + P(A^c \cap B)$$
$$= P(A)P(B \mid A) + P(A^c)P(B \mid A^c).$$

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3. Theorems Of Probability/ Counting Techniques

3.1 Theorem Of Total Probability/ Addition Theorem:

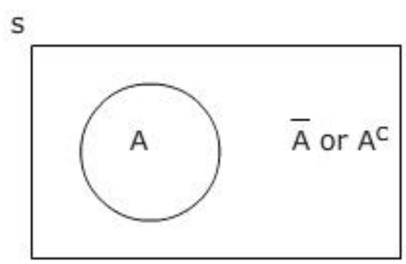
According to this theorem, for any two mutually exclusive i.e. P(A∩B = 0), exhaustive and equally likely events A and B, we add the probabilities of the two events while calculating the probabilities of the occurrence of either A or B i.e. if A and B have no elements in common, then

P(AUB) or P(A or B) = P(A) + P(B)
This is also known as the ***addition rule***.

For example, a class has 15 girls and 20 boys, thus total students = P(GUB) = P(G) + P(B) = 35

If the two events are not mutually exclusive i.e. P(A∩B=0), then the addition rule becomes:
P(AUB) = P(A) + P(B) - P(A∩B) or
P(A or B) = P(A) + P(B) - P(A and B) or
P(A + B) = P(A) + P(B) - P(A X B)

i) **Theorem of complementary event**:
Let A= occurrence of an event A and A^C is its complement, thus
P(A) = 1 – P(A^C), since we know that P(S) = P(A) + P(A^C) = 1



ii) **Extension of Total Probability theorem**:
If E_1, E_2, ………, E_n are mutually exclusive to each other, then

$$p(E_1 \cup E_2 \cup \cdots \cup E_n) = p(E_1) + p(E_2) + \cdots + p(E_n)$$

iii) **Theorem of total probability with mutually Non-Exclusive Events**:

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

For three event A, B and C,
P(AUBUC) = P(A) + P(B) + P(C) – P(A∩B) – P(A∩C) – P(B∩C) + P(A∩B∩C)

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3.2 Theorem Of Compound Probability/Multiplication Theorem

According to this theorem, if we want to find the probability of two event A and B occurring together simultaneously, we can multiply the probability of the event A and the *conditional probability* of event B given that event A has actually occurred, denoted by P(B/A)
P(B/A) is the ratio of number of events favorable to events A and B to the number of events favorable to event A.
i.e. $P(B/A) = P(A \cap B) / P(A)$ or
$P(A \cap B) = P(A) * P(B/A)$
This is also known as the ***multiplication rule***.

i) **Extension of Compound Probability Theorem**:
Suppose there are three events A, B and C, then
$P(A \cap B \cap C) = P(A) * P(B/A) * P(A \cap B)$

For example, if P(AUB) = 6/8, P(A) = 6/16, P(B) = 2/4. Find P(B/A) and P(A/B).
$P(A \cap B) = P(A) + P(B) – P(AUB) = 6/16 + 2/4 – 6/8 = 1/8$
$P(B/A) = P(A \cap B) / P(A) = 1/8 / 6/16 = 1/3$
$P(A/B) = P(A \cap B) / P(B) = 1/8 / 2/4 = ¼$

For example, following results were obtained for three subjects namely Mathematics (M), Economics (E) and Hindi (H) in a class:
25% of the students passed in M
20% of the students passed in E
35% of the students passed in H
7% of the students passed in M and E
5% of the students passed in M and H
2% of the students passed in E and H
1% of the students passed in all the three subjects
Find the probability that a student got passing marks in at least one of the subject:

$P(M) = .25$, $P(E) = .20$, $P(H) = .35$, $P(M \cap E) = .07$, $P(M \cap H) = .05$, $P(E \cap H) = .02$, $P(M \cap E \cap H) = 0.1$.
Therefore, $P(MUEUH) = P(M) + P(E) + P(H) – P(M \cap H) – P(M \cap E) – P(E \cap H) + P(M \cap E \cap H) = .25 + .20 + .35 - .07 - .05 - .02 + .01 = .67$

The Fundamental Principle of Counting

If we have to make a choice among various ‘n’ number of decisions/options that has to be made, then total number of choices is given by:

$C = c_1 \times c_2 \times c_3 \times \ldots \times c_n$
Where c_1 = the number of ways to choose the1st option,
c_2 = the number of ways to choose the2nd option, etc.

Permutation Rule:
If we want to place/draw E elements (out of n elements) in some sequential order, without replacement, we use the rule of permutation. Thus, permutation of E elements out of n, is defined as the number of ways to order E elements from a set of n elements, without repetition ($E \leq n$). Order is what matters in permutation and objects are drawn without replacement and it is denoted as ^nP_E.
Explanation: Suppose the first object is placed in n ways, since this object won't be replaced, the second object can be placed in (n – 1) ways and the process continues till the last object is placed.

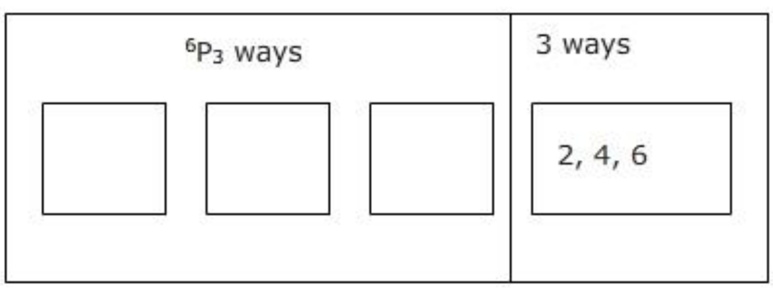
$$^nP_E = \frac{n(n-1)(n-2)\ldots(n-E+1)}{(n-E)!} = \frac{n!}{(n-E)!}$$

For example, there are 7 digits numbered 1,2,3…7. Suppose we have to make a number plate of 4 digits, without replacement. Find the probability that the number plate formed is an even number.

Solution: For the number to be even, the last number should be an even number. Remaining three digits can be any number from the remaining 6.

Total number of ways favorable = $\frac{7!}{3!\ 1!}$

Total number of even numbers = 3 i.e. {2,4,6}
The last digit can be arranged in 3 ways. Since we don't replace the numbers, assuming that the last digit is an even number, the rest of the 3 digits can be arranged in $^6P_3$ ways.



Thus, P(Even) = (3 * 6P3) /($\frac{7!}{3!\ 1!}$) = $\frac{3 * 6! * 3!}{7! * 3!}$ = 3/7

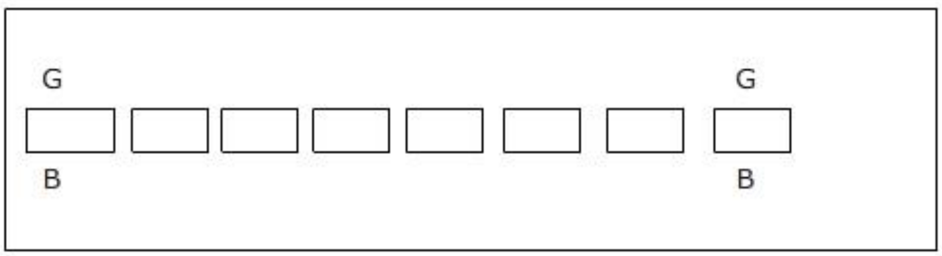
For Example, Suppose 3 boys and 5 girls sit together in a row in a class. Find the probability that: i) all boys sit together, ii) same gender sit in the extreme ends.

Solution: Total number of ways favorable = $\frac{8!}{3!\,5!}$ = 56 ways

i) Assume BBB as 1 object and we arrange 6 objects i.e. 5 girls and 1 BBB. Thus number of arrangements = $\frac{6!}{5!1!}$ = 6 arrangements. Thus,

Probability that boys sit together = 6/56 = 3/28

ii) There can be two ways i.e. one where boys sit at extreme ways and another way in which girls sit at extreme ends.



If girls are at each end, number of distinct arrangements of 6 remaining places in which remaining 3 girls and 3 boys adjust = $\frac{6!}{3!3!}$ = 20

If boys are at each end, number of distinct arrangements of 6 remaining places in which remaining 5 girls and 1 boy adjust = $\frac{6!}{5!1!}$ = 6

Thus, the same gender sit in the extreme ends = $\frac{20+6}{56}$ = 13/28

Combination Rule:

If we want to choose/draw E elements (out of n elements), without replacement, we use the rule of combination. Thus, combination of E elements out of n, is defined as the number of ways to draw/choose E elements from a set of n elements, without repetition ($E \leq n$). Here order does not matter and it is denoted as ^nC_E or $\binom{n}{E}$.

Explanation: Suppose the first object is placed in n ways, since this object won't be replaced, the second object can be placed in (n – 1) ways and the process continues till the last object is placed.

$$^nC_E = \frac{n(n-1)(n-2)...(n-E+1)}{(n-E)!\,E!} = \frac{n!}{(n-E)!\,E!} = \frac{^nP_E}{E!}$$

For Example: Suppose you pick up 13 cards out of a pack of 52 well-shuffled cards. Find the probability that i) it has at least one king, ii) it has 3 queens, iii) 6 clubs, 4 diamonds, 2 hearts and 1 spade.

Solution: Since the order does not matter, we may randomly choose 13 cards out of 52 applying combination rule and thus our sample size becomes ${}^{52}C_{13}$

i) We know that there are four kings in a pack of 52 cards, so at least one king means we can choose either 1 king and 12 other cards $[({}^{4}C_{1})\ ({}^{48}C_{12})]$, or 2 kings and 11 other cards$[({}^{4}C_{2})\ ({}^{48}C_{11})]$, or 3 kings and 10 other cards$[({}^{4}C_{3})\ ({}^{48}C_{10})]$. Thus,

$$P(\text{At least one king}) = \frac{[({}^{4}C_{1})\ ({}^{48}C_{12})] + [({}^{4}C_{2})\ ({}^{48}C_{11})] + [({}^{4}C_{3})\ ({}^{48}C_{10})]}{{}^{52}C_{13}}$$

ii) We know that there are three queens in a pack of 52 cards, thus 3 queens means we can choose 3 queens out of 4 in ${}^{4}C_{3}$ ways and other 10 cards in ${}^{48}C_{10}$ ways. Thus, $P(3 \text{ queens}) = ({}^{4}C_{3} * {}^{48}C_{10}) / {}^{52}C_{13}$

iii) We know there are 13 cards of each suit i.e. spade, diamond, heart and club. We choose 6 clubs in ${}^{13}C_{6}$ ways, 4 diamonds in ${}^{13}C_{4}$ ways, 2 hearts in ${}^{13}C_{2}$ ways and 1 spade in ${}^{13}C_{1}$ ways. These 4 cards can again be arranged in 4! Ways. Thus,

$$P(\text{6 clubs, 4 diamonds, 2 hearts and 1 spade}) = \frac{\binom{13}{6}\binom{13}{4}\binom{13}{2}\binom{13}{1}}{\binom{52}{13}}$$

4. Conditional Probability and Concept Of Independence

From the compound probability theorem we know that,

$P(B/A) = P(A \cap B) / P(A)$ or
$P(A \cap B) = P(A) * P(B/A)$

Example: If you draw a card from a pack of 52 well-shuffled cards, find out the probability that it is a king of hearts, given that the card drawn in red.

Solution: Since the card drawn is red, it could be of hearts or diamonds.
Let A = event that the card is red. Thus, $P(A) = 26/52$
Let B = event that the card is king of heart.
Thus $P(A \cap B) = 1/52$

$P(B/A) = P(A \cap B) / P(A) = (1/52) / (26/52) = 1/26$

Independent Events

We have already learnt about independent events in our last chapter i.e. if occurrence of event A does not affect the occurrence of another event B. This implies that,
$P(B/A) = P(B/A^C) = P(B)$
From the compound probability theorem we get,
$P(A \cap B) = P(A) * P(B/A)$
$= P(A) * P(B)$
Similarly, for three independent events A, B, and C,
$P(A \cap B \cap C) = P(A) * P(B) * P(C)$ and so on.

5. Examples

Example 1: One ticket is drawn at random from 100 tickets numbered 0, 1, 2, 3, …, 99. If sum of the two digits on the ticket is i, such that, $0 \leq i \leq 18$. Let E_i be the event that the sum of the two digit number is i. Let F_j be the event that the product of the two digits is j, given $0 \leq j \leq 9$. For each possible i, find $P(E_i/F_0)$.

Solution: F_0 is the first event out of ten tickets drawn. Thus $P(F_0) = 1/10$
$(E_i \cap F_0)$ = event that the ticket numbered 0 is drawn. Thus, $P(E_i \cap F_0) = 1/100$
Thus, $P(E_i/F_0) = P(E_i \cap F_0) / P(F_0)$

Example 2: Suppose a card is drawn from a pack of 52 well-shuffled cards. Find the probability that the card is a black Ace given it is a spade.

Solution: Number of black aces in a deck = 2, P(BA) = 2/52
Number of spades = 13, P(S) = 13/52
$P(BA \cap S) = 1/52$
$P(BA/S) = P(BA \cap S) / P(S) = 1/13$

Example 3: Suppose there is a Apple i-phone wholesaler who has 20% of the phones duplicate in his showroom. Suppose retailer buys i-phone from him. He has 10% probability of buying a duplicate i-phone. Find out the conditional probability that the retailer buys an original i-phone.

Solution: Let B = event that the retailer buys the phone
O = event that the painting is original
Given, P(O) = 0.8 and $P(B/O^c) = 0.1$
Now assuming, P(B/O) = 1, we apply Bayes' rule here

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Thus, $P(O/B) = \dfrac{P(O) \cdot P(B/O)}{P(O) \cdot P(B/O) + P(O^c) P(B/O^c)}$ = 0.8 / (0.8 + .02) = .80 / .82 = 40/41

Example 4: A manufacturer makes light bulbs and found that 5% of the bulbs have a common defect. Researchers studied that 93% of these defective bulbs show a certain behavioral characteristic, while this characteristic was exhibited in 2% of the non-defective bulbs. A bulb was examined which showed a characteristic symptom. Given this behavioral symptom, find the conditional probability that the bulb has a defect.

Solution: Let A = event that the bulb is defective

B = event that the bulb has a characteristic symptom

Given, P(A) = 0.05, P(B/A) = 0.93, and $P(B/A^c) = 0.02$

Thus, $P(A/B) = \dfrac{P(B/A)P(A)}{P(B/A)P(A) + P(B/A^c) \cdot P(A^c)}$

= (0.93 * 0.05) /[(0.93 * 0.05) + (0.02 * 0.95)

= 93 / 131

Example 5: Suppose according to a survey, life expectancy of women in USA is 70 years with a probability of 0.70 and is 80 years with a probability of 0.55. Suppose a woman in USA is 70 year old, what is the probability that she will survive till 80 years? (Note if $A \subset B$, then P(AB) = P(A))

Solution: Let A= event that the Woman lives till seventy years

B = event that the woman lives till eighty years

If the woman lives for eighty years, she would have already lived for 70 yrs, thus, $B \subset A$.

Thus, P(B/A) = P(AB) / P(A) = P(B) / P(A) = .55 / .70 = 55/70

Example 6: Suppose a wholesaler receives a shipment of 1000 light bulbs. There is an equally likely probability that there are 0, 1, 2, or 3 defective units in the lot. Find the probability that 'no defective' light bulb unit is selected from the lot if one light bulb is selected at random.

Solution: Let G = event that the light bulb is non-defective / good

D_k = Event that there are k number of defective light bulbs

$P(D_0) = P(D_1) = P(D_2) = P(D_3) = ¼$

$P(G|D_0) = 1000/1000 = 1$

$P(G|D_1) = 999/1000$

$P(G|D_2) = 998/1000$

$P(G|D_3) = 997/1000$

$$P(D_0|G) = \frac{P(G|D_0)P(D_0)}{P(G|D_0)P(D_0)+P(G|D_1)P(D_1)+P(G|D_2)P(D_2)+P(G|D_3)P(D_3)}$$

$$=1\cdot 1/4 / [(1/4)(1+999/1000+998/1000+997/1000)] = 1000 / 3994$$

Example 7: In a survey, 85% students say that they obey the rules in the school. Previous experience show that 20% of students who do not obey the rules, say that they obey, out of fear of parents. If a student is picked at random, find the probability that he does obey the rules in the school. (Assume: all who obeys rules says that they obey).

Solution: P(say) = 0.85. P(say/don't obey) = 0.20, we assumed P(say/obey) = 1
P(say) = P(say/obey) + P(say/don't obey) [1 – P(obey)]
Thus, P(obey) = [P(say) - P(say/don't obey)] / [1 - P(say/don't obey)]
= [0.85 – 0.20] / [1 – 0.20]
= 13 / 16

6. Multi-Stage Experiment and Bayes' Theorem

It is not necessary for an event to be single-stage, if it can be broken down into stages, an experiment can become a multi-stage.
Following results may take place when we apply conditional probabilities to events A and B in case of a multi-stage experiment:

- I) Event A and event B may remain in the same stage and not enter another stage of the experiment. Here we find the conditional probability of A given B using: $P(A/B)=P(A\cap B) / P(B)$
- II) Event A is still in the first stage that has already occurred whereas event B is in the next stage that is yet to occur i.e. experiment is not yet complete.
- III) Event A is in the previous stage whereas event B will occur in a later stage and the experiment is still incomplete

We use **the Bayes' theorem/rule/network** in the IInd and IIIrd results.

Bayes' Theorem

Suppose an event B occurs n times and all are mutually exclusive to each other. Thus, B_i's covers the entire sample space. Now, let us assume an event A which may occur if and only if one of the events B_1, B_2, B_3, ….., B_n occurs. This implies that if the unconditional probabilities i.e. $P(B_1)$, $P(B_2)$, $P(B_3)$,….., $P(B_n)$ are known, then the conditional probabilities i.e. $P(A/B_1)$, $P(A/B_2)$, ….., $P(A/B_n)$ will also be known.

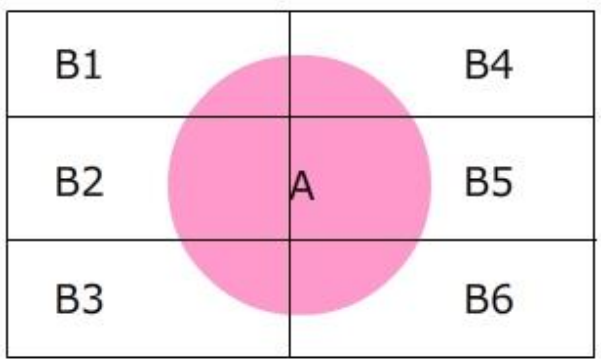
$$P(A) = \sum_{i=1}^{n} P(A \cap Bi) = \sum_{i=1}^{n} P(Bi) \,.\, P(A/Bi)$$

Now taking event A to be given, we can find out the conditional probabilities of events B_1, B_2, B_3, ….., B_n . Thus, given that A has actually occurred, the conditional probability $P(B_i/A)$ can be calculated as:

$$P(Bi/A) = P(Bi \cap A) / P(A) = P(A/Bi) \,.\, P(Bi) / \sum_{i=1}^{n} P(Bi) \,.\, P(A/Bi), \text{ therefore}$$

$$P(Bi/A) = \frac{P(Bi) \,.\, P(A/Bi)}{\sum_{i=1}^{n} P(Bi) \,.\, P(A/Bi)}$$

This is known as Bayes' Theorem.



For example,

First urn contains 2 white and 4 blue marbles and second urn contains 2 white and 2 blue marbles. A marble is transferred from urn 2 to urn 1 and then a marble is picked from urn 1 randomly. If it turns out to be a blue marble, what is the probability that the transferred marble was white?

Solution: Let B1 = transferred marble was white, B2 = transferred marble was blue

Let A = marble drawn from urn 2 is blue

P(B1) = ½, P(B2) = ½,

P(A/B1) = 3/7, P(A/B2) = 5/7

$$P(B1/A) = \frac{P(B1).\, P(A/B1)}{P(B1).\, P(A/B1) + P(B2).\, P(A/B2)}$$

$$= (½*3/7) / (½*3/7 + ½*5/7) = 3/8$$

Or, $P(A/B) = \frac{P(B/A) . P(A)}{P(B/A) . P(A) + P(B/A^c) . P(A^c)}$, where A^c = complement of A.

For Example: An entrepreneur expects his profits to rise over the next four quarters with probability 0.4. As a result, a plan to increase the plant size over the next 12 months was prepared. When profits were analysed for past years data, in 8 out of ten cases, profit occurred. Thus there was profit predicted in 2 out of ten cases but loss occurred. Based on this information, how should the entrepreneur revise his plans of the probability that profit will occur?

Solution: The probabilities as per entrepreneur's expectation:

P(Profit) = 0.4 (profit will occur)

$P(\overline{\text{Profit}}) = 0.6$, where $\overline{\text{Profit}}$ = 'no profit'

Let A be the analysis of the past years profits. The conditional probabilities of this analysis is given by,

P(A/Profit) = 0.8

$P(A/\overline{\text{Profit}}) = 0.2$

By Bayes' formula, the revised probabilities associated with profit/ $\overline{\text{Profit}}$ are computed as:

$$P(\text{Profit}/A) = \frac{P(\text{Profit})\, P(A/\text{Profit})}{P(\text{Profit})\, P(A/\text{Profit}) + P(\overline{\text{Profit}})\, P(A/\overline{\text{Profit}})}$$

$= .32/.44 \approx 0.73$

7. Summary

- If two events A and B are not independent, we use conditional probability which is defined as the probability of occurrence of an event A, given that another event B has already occurred

- Conditional probability of A, given B is $P(A/B) = P(B \cap A) / P(B)$, provided $P(B) \neq 0$

- Addition rule says, P(AUB)/ P(A or B) = P(A) + P(B)

- Multiplication rule says, $P(A \cap B) = P(A) * P(B/A)$

- According to permutation rule, $^nP_E = \frac{n(n-1)(n-2)...(n-E+1)}{(n-E)!} = \frac{n!}{(n-E)!}$

- According to combination rule, $^{n}C_{E} = \frac{n(n-1)(n-2)\ldots(n-E+1)}{(n-E)!\,E!} = \frac{n!}{(n-E)!\,E!} = \frac{^{n}P_{E}}{E!}$

- According to Bayes' rule, $P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B/A) \cdot P(A) + P(B/A^c) \cdot P(A^c)}$

8. Exercise

1. Define conditional probability and its applications
2. What are the counting techniques in probability?
3. What are the theorems of probability?
4. Define addition rule and multiplication rule with examples.
5. Describe and explain Bayes' theorem with example.
6. There are three workers in an industry a, b, and c making shirts. They make 25%, 35% and 40% of the total shirts produced respectively. Out of the total shirts made by them, 5%, 4% and 2% respectively was found to be defective. If we select a shirt at random, find the probability that worker C has made it.
7. Suppose there are two coins. One is a fair coin with probability of head = ½ and the second one is loaded with head i.e. probability of head in the second coin is 2/3. Suppose we toss a coin picking one of the two coins at random and a head turns up, find the probability that it is a fair coin.
8. A has 50% chance of selling goods in his shop. If two customers enter his shop, find the probability that A will sell the goods.
9. If events A and B are mutually exclusive, prove that P(A/AUB) = P(A)/P(A) + P(B).
10. There are two bags containing white and black balls. Suppose bag 1 contains 2 white and 2 black balls and bag 2 contains 2 white and 4 black balls. If one ball is randomly selected, find the probability that theyb will be of the same color.

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9. References

1. Jay L. Devore, Probability and Statistics for Engineers, Cengage Learning, 2010.

2. PH Karmel, M Polasek, Applied Statistics for Economists, 4th edition.

10. MCQs

1. Suppose a medical company needs to find out if a certain drug can or cannot lead to an improvement in symptoms for some patient with a particular medical condition. A study has been done and following results were seen:

	Improvement	**No Improvement**	**Total**
Drug	270	530	800
No Drug	120	280	400
Total	390	810	1200

On the basis of the above table, given that the drug was provided, find out the conditional probability that the patient shows improvement.

i) .3375

ii) .325

iii .225

iv) .275

Ans: i) .3375

Hint: Let I = event that there is improvement

D = event that the patient took the drug

We need to find out $P(I/D) = P(I \cap D) / P(D)$

$= 270 / 800$

$= .3375$ (Ans.)

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2. Taking in reference the study of the table provided in question 1, find the conditional probability that the patient was given the drug, given that the patient shows improvement.

i) .225

ii) .692

iii) .667

iv) .665

Ans. ii) .692

Hint: : Let I = event that there is improvement

D = event that the patient took the drug

We need to find out $P(D/I) = P(D \cap I) / P(I)$

$= 270 / 390$

$= .692$

3. Suppose two cards are drawn without replacement from a deck of 52 cards. Find the probability that both the cards are aces.

i) .0045

ii) .0050

iii) .0065

iv) .0385

Ans. i) .0045

Hint: Probability that the first card is ace = 4/52

Now, assuming that first card drawn is an ace and it is not replaced, the probability that second card is also an ace = 3/51

Thus, probability that both cards are aces = 4/52 * 3/51 = .0045

4. Suppose two balls are drawn at random without replacement from an urn containing 4 red, 2 white and 3 green balls. Find the probability that the balls drawn are same in color.

i) .28

ii) .14

iii) .50

iv) .56

Ans. i) .28

Hint: Probability that two marbles are of same color = P(2R or 2W or 3G)

P(2R) = 4/9 * 3/8 (probability that first ball is red is 4/9 and assuming it to be true and without replacing it, probability that second ball is red is 3/8)

Similarly, P(2W) = 2/9 * 1/8

And P(3G) = 3/9 * 2/8

Thus, P(2R or 2W or 3G) = 4/9*3/8 + 2/9*1/8 + 3/9*2/8 = 20/72 = .28

5. A bulb manufacturing company has three machines A, B and C. Machines A, B and C produces 30%, 50% and 20% respectively of the total bulbs produced. Of their output, 1%, 4% and 3% respectively are defective. If one bulb is selected at random, find the probability that it was produced by machine B and is also defective.

i) .40

ii) .04

iii) .02

iv) .20

Ans. iii) .02

Hint: Probability that machine B produces a bulb P(B) = .50

Probability that the bulb is defective and is produced by machine B = P(D/B) = .04

Probability that bulb was produced by machine B and is defectice = P(B∩D)

P(B∩D) = P(B) * P(D/B)

= .50 * .04 = .02

6. A bulb manufacturing company has three machines A, B and C. Machines A, B and C produces 30%, 50% and 20% respectively of the total bulbs produced. Of their output, 1%, 4% and 3% respectively are defective. If one bulb is selected at random, find the probability that the bulb is defective.

i) .08

ii) .028

iii) .029

iv) .027

Ans. iii) .029

Hint: Probability that the bulb is produced by machine A and is defective = P(D/A) = .01

Probability that the bulb is produced by machine B and is defective = P D/B) = .04

Probability that the bulb is produced by machine c and is defective = P(D/C) = .03

Thus, probability that the bulb is defective is: P(D):

P(D) = P(A)*P(D/A) + P(B)*P(D/B) + P(C)*P(D/C)

= .3*.01 +.5*.04 + .2*.03 = .029 (Ans.)

7. Suppose we again go back to the study shown in the table provided in question 1: Can you say that 'taken drug' and 'improvement are independent events?

i) Yes they are independent

ii) No they are not independent

iii) Can't say

Ans. ii) No

Hint: Let I = event that there is improvement

D = event that the patient took the drug

For independence, we need to prove $P(D \cap I) = P(D) * P(I)$

$P(D \cap I) = 270/1200 = .225$

$P(D) * P(I) = 800/1200 * 390/1200 = .2167$

Thus, $P(D \cap I) \neq P(D) * P(I)$, so they are not independent.

Statistical Methods In Economics - I

8. Suppose you have 5 show pieces, and you want to arrange 3 of them in your show case. In how many different ways can you arrange them?

i) $^5C_3$ = 10 ways

ii) $^5P_3$ = 60 ways

iii) 3/5

iv) None

Ans. ii) $^5P_3$

Hint: This is the case of permutation. Since we want to arrange the show pieces and are concerned with order, number of ways we can arrange the show pieces =

$^5P_3$ = 5! / 2! = 60 ways

9. If a card is drawn from a deck of 52 well shuffled cards, what is the probability that it will be a queen or a king?

i) 1/52

ii) 1/26

iii) 1/2

iv) 2/13

Ans.

Hint: Let A = event that card is a king, probability of a king = 1/52

Let B = event that card is a queen, probability of a queen = 1/52

Now, since both the events are independent i.e. $P(A \cap B) = 0$

Thus, P(A or B) = P(AUB) = P(A) + P(B) = 2/52 = 1/26

10. In how many ways can we choose a sub-committee of 5 members from a club consisting of 10 members?

i) $^{10}P_5$ = 30240 ways

ii) $^{10}C_5$ = 252 ways

iii) 5/10

iv) ½

Ans. ii) $^{10}C_5$

Hint: This is an example of combination where we have to choose 4 members out of 10.

Thus, $^{10}C_5$ = 252 ways