Discipline Courses-I
Semester-I
Paper : Mathematics & Statistics for Business (Statistics)
Lesson: Mathematics For Finance
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Lesson: Mathematics of Finance
Table of Contents

1. Learning Outcomes:

After studying this lesson, you should be able to:

- understand the concept of continuous compounding,
- distinguish between nominal and effective rates of interest,
- understand the relationship between nominal and effective rates of interest,
- understand the concept of force of interest,
- learn about various kinds of annuities,
- compute present value and future values of different types of annuities,
- calculate annual depreciation using various methods,
- understand the concept of Sinking Fund and solve problems involving sinking funds,
- understand amortization and solve problems based on it,
- using the concept of present value to solve the problems related to bonds.

2. Introduction:

Money has a time value. Individuals, entrepreneurs and industrialists borrow money from friends, banks, financial institutions or fellow businessmen. These borrowings have to be repaid with interest. Just as any resource is available to a person at a price, similarly interest is the price for acquiring money from others. It is the reward for benefit to the person who is parting with it and not expending it herself/himself. Such a person has made an investment of money in order to get a reward. To her/him this interest is nothing but the return on investment.

The study of mathematics of finance helps a person (borrower) in ascertaining whether a proposal of borrowing on certain terms, say at a certain rate for a specified period of time should be accepted or not. It provides him with what is called as the cost of obtaining a loan. Similarly, it helps a person (lender) in ascertaining if a proposal to lend on certain terms, say at a certain rate for a specified period of time is attractive enough to be accepted or should it be rejected.

There are various models that describe the mathematics of finance. In some models, the amount that is borrowed or lent is one lump sum that is exchanged in the current period and is to be repaid after some periods. In other cases, a fixed amount may be invested at the beginning or end of a fixed period of time during a relatively long period. Such fixed amount is referred to as an annuity. In other cases, periodical payments may differ in their denominations. Also the rate of interest may either be simple or compound. The concepts of present value and future value of an ordinary annuity, a deferred annuity, and the concept of perpetuity come in handy while dealing with borrowing and lending money. Different software and inbuilt functions in Ms-Excel help a person compute the desired values for analysis and decision-making with great ease. However, before applying those functions in such applications or using sophisticated software, a theoretical base must exist that justifies what is to be computed and how is it to be interpreted. This lesson intends to build such a foundation for analyzing the mathematics involved in financial transactions.

We all know that the assets we possess decrease in their value over time. This may happen due to normal wear and tear, obsolescence over time, poor maintenance of the asset, improper handling and servicing, over-use of the asset, etc. The understanding of the concept of depreciation, its difference from amortization, and the methods of computing it are also of immense importance to any businessperson. The knowledge of such concepts enables her/him to value the asset correctly and facilitate in decisions regarding repair or replace; lease or use for own production.

The concept of investing in bonds or debentures also requires the knowledge of the manner of valuing them. It is of interest to a person investing in bonds to know as to how they are valued. Such concepts are discussed in the following sections of this lesson that is expected to equip the readers with the terminology and mathematical application in areas of finance.

3. Nominal and Effective Rates of Interest

Usually, rates of interest are expressed in terms of percentage per annum. The stated annual rate of interest is nothing but nominal rate of interest (r). Thus, the rate of interest, compounded a given number of times per annum is referred to as the nominal rate of interest.

When interest is charged semi annually (two times in a year), quarterly (four times in a year) or monthly (twelve times in a year), the lender earns more than the nominal rate of interest because the compound is being done more than once (annual). The actual percentage by which an investment grows during a year is called an effective rate of interest.

Money invested at an annual rate of 10% will increase in value by more than 10% in a year if the interest is compounded more than once. It happens because interest compound during one period will itself earn interest during subsequent periods. The effective rate of interest is the interest rate that is equivalent to the nominal compounded rate of interest.

The rate of interest, if compounded annually, would result in the same percentage of interest being earned actually, is called the effective rate of interest. That is, if the conversion period is a year (compounded annually) then the nominal rate of interest and effective rate of interest would be exactly the same or equal.

Suppose a person invests Rs. 100 at 10% per annum compounded annually, then the stated annual nominal rate of interest would be 10% per annum and effective rate of interest would also be 10% per annum. The amount at the end of one year will be Rs. 110. Thus, the given rate of interest and the rate of interest actually earned are exactly the same.

If he invests Rs. 100 at 10% per annum compounded semi-annually, then rate of interest for a six-monthly period will be 5% = $\frac{10}{2}\%$, where 2 is the number of conversions in a year.

Figure 1: Interest Compounded Bi-annually

At the end of the year, the amount will be Rs. 110.25. Thus, 10% per annum is the nominal rate of interest whereas interest actually earned is Rs. 10.25, which is more than the stated annual rate of interest. Thus, 10.25% will be the effective rate of interest.

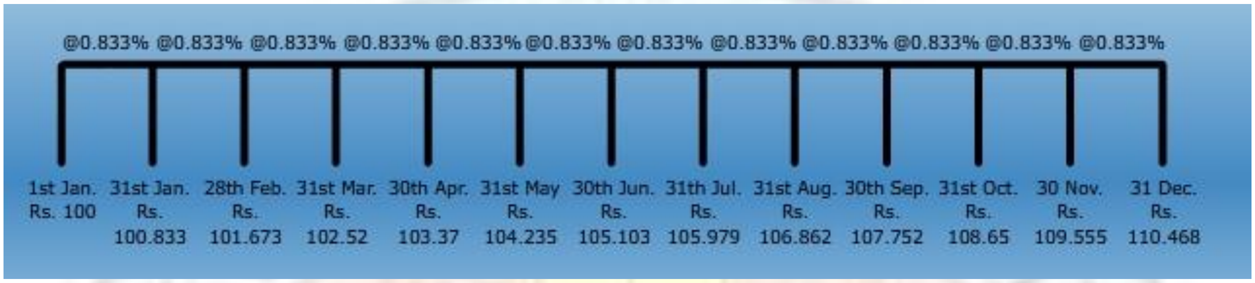
If he invests Rs. 100 at 10% per annum compounded quarterly. Nominal rate of interest remains same at 10% per annum. Rate of interest remains same at 10% per annum. Rate of interest per quarter will be 2.5% = $\frac{10}{4}\%$ where four is the number of conversions in a year.

Figure 2: Interest Compounded Quarterly

At the end of one year, the amount will be Rs. 110.381. The nominal rate of interest remains same at 10% per annum. The effective rate of interest will be 10.381%.

However, if he invests Rs. 100 at 10% per annum compounded monthly. Nominal rate of interest remains same at 10% per annum. Rate of interest remains same at 10% per annum. Rate of interest per month will be 0.833 % = $\frac{10}{12}\%$ where twelve is the number of conversions in a year.

Figure 3: Interest Compounded Monthly



At the end of one year, the amount will be Rs. 110.468. The nominal rate of interest remains same at 10% per annum. The effective rate of interest will be 10.468%.

By making a comparative analysis of the three compounded interests, we find that the effective rate of interest increases as the number of periods of compounding in a year increases from 1 to 12 and is highest at 10.468% for 12 periods in an year.

3.1 Relationship Between Nominal and Effective Rate of Interest

Let r_e be the effective rate of interest corresponding to the nominal rate of interest r, converted m times a year. 'i' is the rate of interest per conversion period. Thus, i = r/m. At the rate of interest i, the principal P in one year amounts to $P(1 + i)^m$. An effective rate is the actual rate compounded annually, therefore at the effective rate re, the principal P in one year will amount to P(1 + re). Therefore,

$$P(1 + r_e) = P(1 + i)^m$$

$$(1 + r_e) = (1 + i)^m$$

$$r_e = (1 + i)^m - 1$$

Hence,

$$r_e = \lim_{m\to\alpha}(1 + r/m)^m - 1 \; r_e$$

$$= (1 + i)^m - 1$$

or Effective Rate of interest $(r_e) = (1 = i)^m - 1$

or $r_e = (1+r/m)^m - 1)$

3.2 Force of Interest

If the nominal rate of interest r is compounded continuously, the equivalent effective rate, can be expressed as

$$re = \lim_{m\to\alpha}\left[(1 + r/m)^m - 1\right]$$

$$= \lim_{m \to \alpha} \left[(1 + r/m)^{m/r} \right]^r - 1$$

Suppose x = r/m, then as m → α, x → 0.

Thus, $r_e = \left[\lim_{x \to 0} (1 + x)^{1/x} \right]^r - 1$

$r_e = e^r - 1$ $\left[\because \lim_{x \to 0} (1 + x)^{1/x} = e \right]$

Force of Interest – The nominal rate of interest compounded continuously and equivalent to a given effective rate is known as force of interest.

Question 1: Calculate the force of interest corresponding to the effective rate 10%.

Solution 1: r_e = 0.10 (Given)

Using the relationship $r_e = e^r - 1$

$0.10 = e^r - 1$ $\Rightarrow 0.10 + 1 = e^r$

r log e = log 1.1 log e

r (0.4343) = 0.0414 ⇒ log 2.7183

$r = \frac{0.0414}{0.4343}$ ⇒ 0.4343

r = 0.09533

or r = 9.533% Force of interest = 9.533%

Thus 9.533%, compounded continuously and 10% effective are equivalent rates.

4. Annuity

It means equal annual payments. It is usually a sequence of payments, equal in size and made at equal intervals of time. So, it has two important features :

(1) Equal in size
(2) Equal intervals of time

Examples:
(1) Life Insurance premiums
(2) EMIs
(3) Creation of Endowment Fund
(4) Recurring Deposits
(5) School/ College Fees

4.1 Important Terms

(1) Periodic payment – It is the size of each payment.
(2) Payment interval – It is time gap between two successive payments.
(3) Term of an Annuity – It is the time gap from the beginning of the first payment period till the end of the last payment period.

(4) Amount/Future Value of an Annuity – It is the sum of the compound amounts of all the payments accumulated at the end of the term, i.e. at the last day of the last payment period.

(5) Present Value/Capital Value of an Annuity – It is sum of the present value of all the payments on the first day of the first payment period.

4.2 Types of Annuities

(i) Ordinary annuity – It is also known as immediate annuity. The payments are made at the end of each payment period.

(ii) Annuity Due – The payments are made at the beginning of each payment period.

(iii) Deferred Annuity – Here, the first payment is postponed until the expiration of a period of time i.e. after specified number of payment periods. Series of payments start after the expiry of a certain specified period.

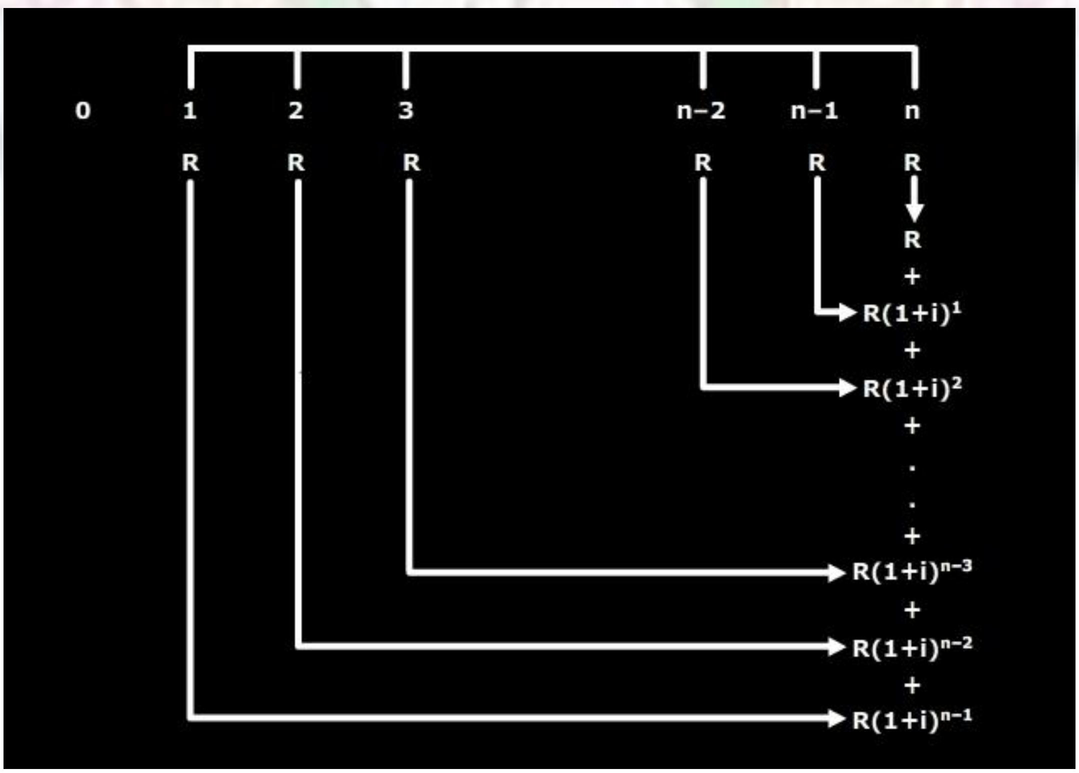
(iv) Perpetual Annuity – Payment continue forever or indefinitely.

(v) Continuous Annuity or Annuity Certain – Payments are made over a fixed period of time.

4.3 The Amount of Present Value of an Ordinary Annuity

The amount or future value is calculated at the end of the term, i.e. on the last day of the last payment period. Each payment is made at the end of each payment period.
Suppose, there is an annuity of 'n' payments of Rs. R each and i is the interest rate per conversion period. The first payment will be made on the last day of the first payment period. For e.g. if its monthly payment starting from the month of January, then the first payment should be made on 31st January. If its quarterly payment, then the first payment should be made on the last day of first quarter i.e. 31st March.

Figure 4: Future Value of investing an amount 'R' periodically at 'i' interest for 'n' periods



The amount or future value is given by

$$S = R + R(1+i)^1 + R(1+i)^2 + \ldots + R(1+i)^{n-2} + R(1+i)^{n-1} \quad (1)$$

Multiplying both sides of the equation (1) by (1 + i), the resultant equation will be

$$S(R+i) = R(1+i) + R(1+i)^2 + \ldots + R(1+i)^{n-1} + R(1+i)^n \quad (2)$$

Subtracting equation (1) from (2) from both the sides. As all other terms will be cancelled

$$S(1+i) - S = R(1+i)^n - R$$

$$s + Si - S = R\left[(1+i)^n - 1\right]$$

$$S = \frac{R\left[(1+i)^n - 1\right]}{i}$$

This is the formula used to calculate an amount or future value of an ordinary annuity consisting of n payments of Rs. R each at the rate of i per period.

In the case of continuous compounding, the amount S of an annuity in which Rs. R is paid each year for N years at the rate of interest of r% per annum compounded continuously is given by

$$S = \int_0^N Re^{rt}\, dt$$

Question 2: Find the amount of an annuity of Rs. 1800 payable at the end of each year for 10 years if the rate of interest is 8% effective.

Solution 2: $S = \frac{R\left([1+i]^n - 1\right)}{i}$

$$S = 1800 \frac{\left[(1+0.08)^{10} - 1\right]}{008}$$

$$S = 1800(14.4865)$$ $\Rightarrow$ S = Rs.260.76

Question 3: A bank pays interest at the rate of 6% per annum compounded continuously. Find how much should be deposited in the bank each year in order to accumulate Rs. 5,000 in 4 years.

Solution 3: $S = \int_0^N Re^{rt}\, dt$

$$S = \int_0^4 Re^{0.06t}\, dt$$

$$\Rightarrow \quad 5000 = R\int_0^4 Re^{0.06t}\, dt$$

⇒ $5000 = \frac{R}{0.06}\left|e^{0.06}\right|_0^4$

⇒ $5000 = \frac{R}{0.06}\left(e^{0.24}-1\right)$

⇒ $5000 = \frac{R}{0.06}\left(1.2712-1\right)$

⇒ $5000 = \frac{R}{0.06}\left(0.2712\right)$

⇒ 300 = R (0.2712)

⇒ $R = \frac{300}{0.2712}$

⇒ R = Rs. 1106.20

Question 4: Find the amount of an annuity consisting of payments of Rs. 500 at the end of every 3 months for 4 years at 8% compounded quarterly.

Solution 4: R = Rs. 500 n= 4(4) = 16 $i = \frac{8\%}{4} = 2\%$ or 0.02

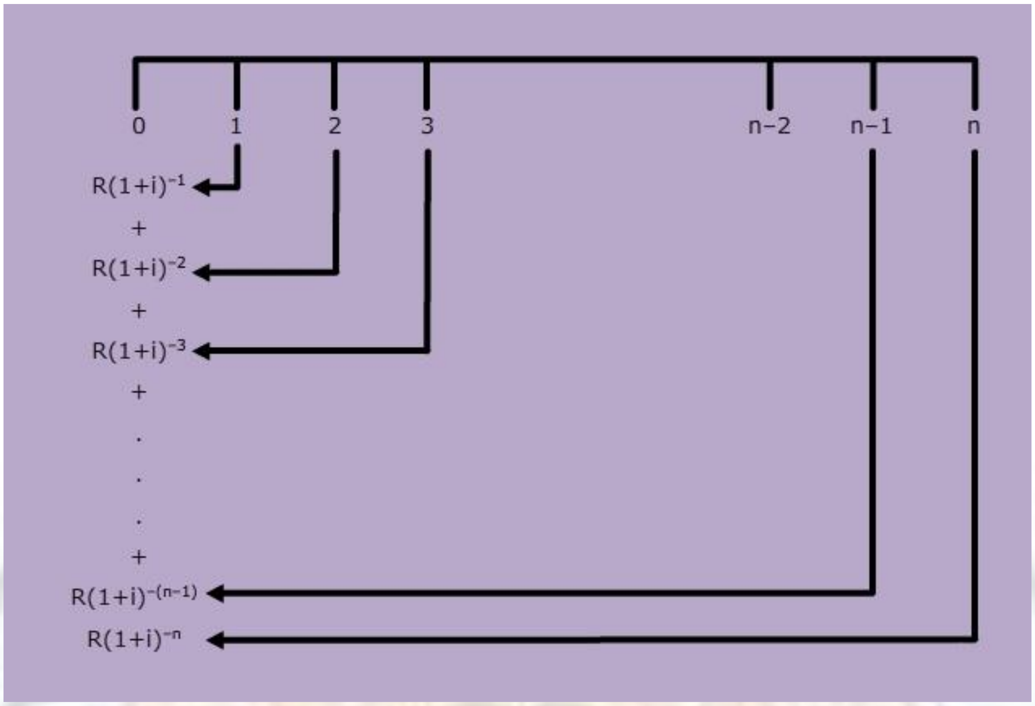
$$S = \frac{500\left[(1+0.02)^{16}-1\right]}{0.02}$$

S = 500 (18.6392) S = Rs. 9320

Present Value or Capital Value

It is the sum total of the present values of all the payments made at the end of each payment period. It is always calculated on the first day of the first payment period.

Figure 5: Present Value of an amount 'R' invested periodically at 'i' interest at the end of each period for 'n' periods



Present value (P) = $R(1+i)^{-1} + R(1+i)^{-2} + \ldots + R(1+i)^{-n}$

It is a geometric series with first term = $R(1+i)^{-1}$ and common ratio $(1+i)^{-1}$

Thus, sum is given by

$$P = R(1+i)^{-1}\left[\frac{1-(1+i)^{-n}}{1-(1+i)^{-1}}\right]$$

or $$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

The present value /capital value of an ordinary annuity consisting of payments of Rs. 'R' each at the rate 'i' per period.

In the case of continuous compounding

$$P = \int_0^N Re^{-rt}$$

where R = annual payment

r is the rate of interest per annum compounded continuously.

Question 5: Find the present value of an annuity of Rs. 1000 per annum assumed to be payment continuously for 10 years, at the rate of interest 4% per annum compounded continuously.

Solution 5: R = 1000 n = 10 i = 0.04

$$P = \int_0^{10} 1000\, e^{-0.04t} dt$$

$$P = \frac{-1000}{0.4} \left| e^{-0.04t} \right|_0^{10}$$

$$P = \frac{-1000}{0.04} \left(e^{-0.4} - 1 \right)$$

or $P = \dfrac{1000}{0.4}\left(1 - e^{-0.4}\right)$ $\qquad (\because\ e^{-0.4} = 0.67032)$

$$P = \frac{1000}{0.04}(1 - 0.67032)$$

P = 25000

P = Rs. 8242

Question 6: Find the present value of an annuity of Rs. 2500 for 5 years at 8% per annum.

Solution 6: $P = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$P = 2500\left[\frac{1-(1+.08)^{-5}}{.08}\right]$$

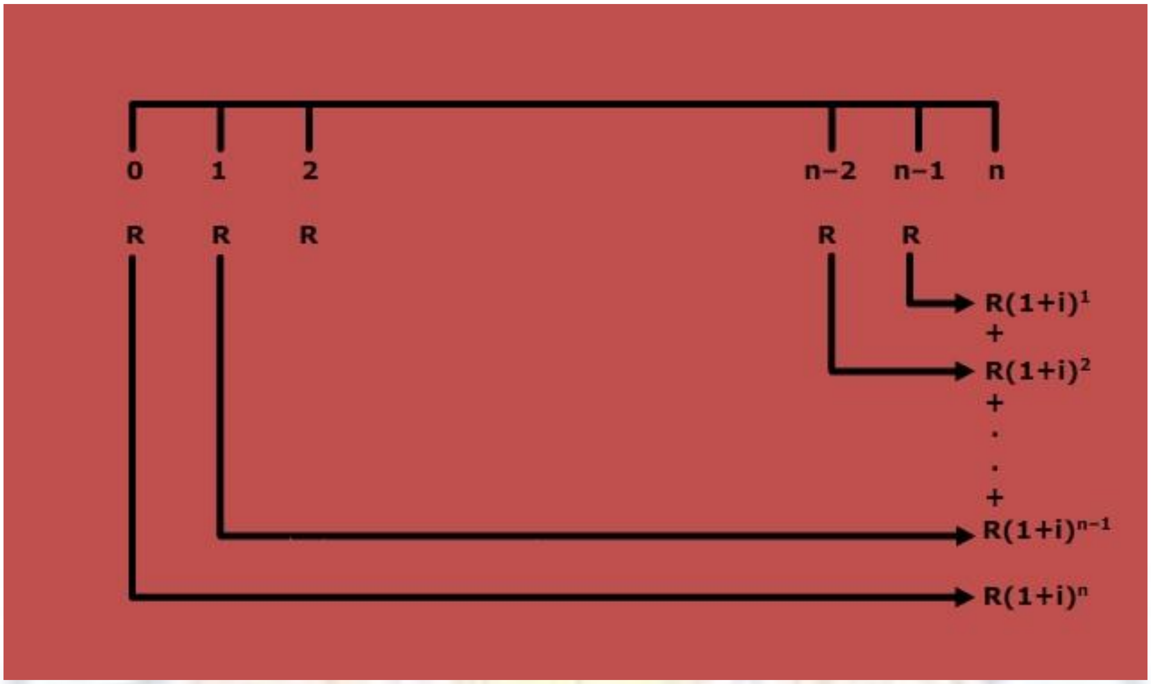
P = 2500 (3.99271004)

P = Rs. 9981.775

4.4 Future Value and Present value of Annuity Due

The payment is made at the beginning of each payment period. The first payment earns interest for n periods, second for (n – 1) periods and nth payment for only one period.

Figure 6: Future Value of Annuity Due after 'n' periods



Amount (S) = $R(1+i)^1 + R(1+i)^2 + \ldots + R(1+i)^{n-1} + R(1+i)^n$

$$S = R(1+i)\,[(1+i)^{n-1} + R(1+i)^{n-2} + \ldots + (1+i)^1 + 1]$$

$$S = R(1+i)\,\frac{[(1+i)n-1]}{(1+i)-1}$$

or $$S = R(1+i)\left[\frac{(1+i)^n-1}{i}\right]$$

It is the formula for the amount of an annuity due consisting of n payments of Rs.R each at the rate i per period is :

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right]$$

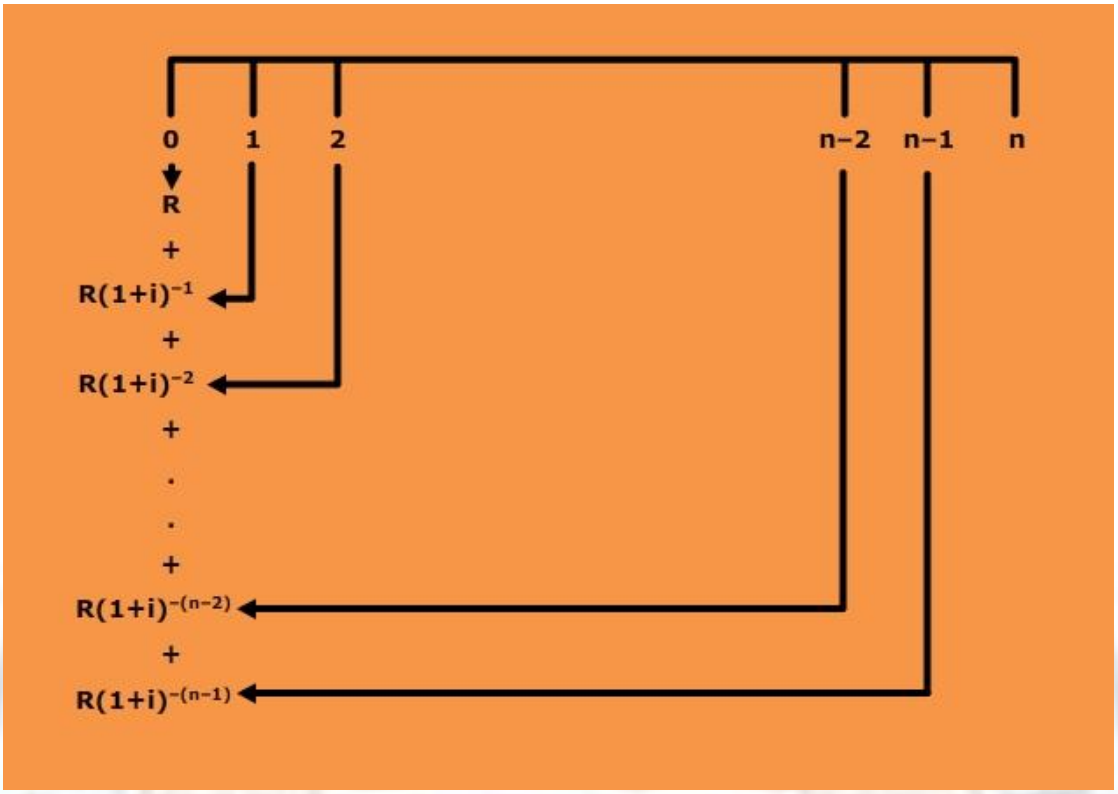
Question 7: At the beginning of each quarter, Rs. 500 is deposited into a saving accounts that pays 8% quarterly compounded. Find the balance at the end of 2 years.

Solution 7: $S = 500\left[\frac{(1+0.02)^9-1}{0.02}\right]$

$S = 500\,(9.7546)$ $\Rightarrow S = \text{Rs. } 487730$

Present Value: It is the sum of the present value of all payments calculated on the first day of the first payment period. Suppose, there is an annuity consisting of n payments of Rs. R each and i be the rate of interest per conversion period.

Figure 7: Present value of Annuity Due after 'n' Periods



$P = R + R\,(1+i)^{-1} + R(1+i)^{-2} + \ldots + R(1+i)^{-(n-2)} + R(1+i)^{-(n-1)}$

$$P = R + R\left[\frac{1-(1+i)^{-(n-1)}}{i}\right] \quad \text{or} \quad P = R + Ran - 1$$

Question 8: What is the present value, at 10% compounded semi-annually, of an annuity due of Rs. 500 payment semi-annually for 5 years.

Solution 8: R = Rs. 500 $\quad$ n = 5(2) = 10 $\quad$ $i = \frac{10}{200} = 0.05$

$$P = 500\left[1+\left(\frac{1-(1+0.05)^{9}}{0.05}\right)\right]$$

P = 500 (1 + 7.1078)

P = 500 (8.1078)

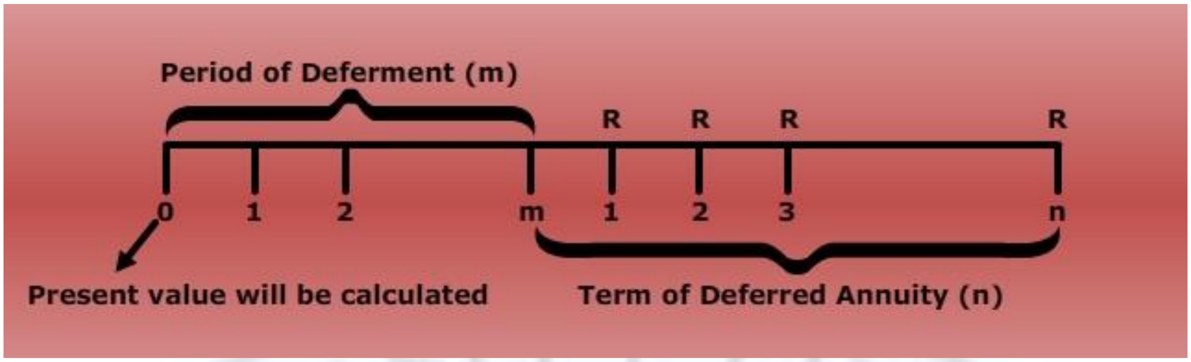
P = Rs. 4054

4.5 Future Value and Present Value of Deferred Annuity

Suppose, a man borrows Rs. 10,000 to be repaid in ten equal annual installments and the repayment will start after certain period of time, called 'm' periods. Thus, it is an ordinary annuity starting after specified period of time. Thus, its future value is the same as the future value of an ordinary annuity.

$$\text{Future Value } (S) = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

Figure 8: Future value of a Deferred Annuity for 'n' periods after deferment of 'm' periods



The present value of deferred annuity will always be calculated on the first day of the first period of deferment.

$$P = \frac{R}{i}\left[\frac{1}{(1+i)^{m}} - \frac{1}{(1+i)^{m+n}}\right]$$

P = [Present value of (m + n) payments
- Present value of (m) payments]

$$P = R\left[\left(\frac{1-(1+i)^{-(m+n)}}{i}\right) - \left(\frac{1-(1+i)^{-m}}{i}\right)\right]$$

$$P = R\left[\frac{\gamma - (1+i)^{-(m+n)}\lambda + (1+i)^{-m}}{i}\right]$$

$$P = \frac{R}{i}\left[(1+i)^{-m} - 1 + i\right]^{-(m+n)}$$

Question 9: Find present value of an annuity of Rs. 500 per quarter for 8 years deferred for 3½ years if money is worth 12% converted quarterly.

Solution 9: R = Rs. 500

m = 14 = (3.5 x 4)

n = 18 = (4.5 x 4)

$$i = \frac{12\%}{4} = 3\% \quad \text{or} \quad 0.03$$

$$P = \frac{500}{0.03}\left[\frac{1}{(1.03)^{14}} - \frac{1}{(1.03)^{32}}\right]$$

$$P = \frac{500}{0.03}\left[0.6611 - 0.3883\right]$$

$$P = \frac{500}{0.03}(0.2728)$$

P = 500 (9.09333)

P = Rs.4547

4.6 Perpetuity

It is an annuity where payments continue forever. One can't calculate the amount of a perpetuity.

Present Value – There are two cases:

(i) Case 1- A perpetuity where payment of Rs. R is made at the end of each payment period and i be the rate of interest per conversion period.

Then present value is given by

$$P = R(1+i)^{-1} + R(1+i)^{-2} + \ldots + R(1+i)^{-n}$$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Let put limits $n \to \alpha$

$$P = R\lim_{n\to\alpha}\left[\frac{1-(1+i)^{-n}}{i}\right]$$

As $n \to \alpha$ $(1 + i)^{-n} \to 0$

Thus $P_\alpha = R\left[\frac{1-0}{i}\right]$

or $P_\alpha = \frac{R}{i}$

(ii) Case 2 – A perpetuity, where payment of Rs. R is payable at the beginning of each period.

Then $P_\alpha = R + \frac{R}{i}$

Question 10:At what rate converted semi-annually, will the present value of a perpetuity of Rs. 500 payable at end of each 6 months be Rs. 30,000?

Solution 10: $P_\alpha = \frac{R}{i}$

$30{,}000 = \frac{500}{i}$

or $i = \frac{500}{30{,}000} = 0.01667$

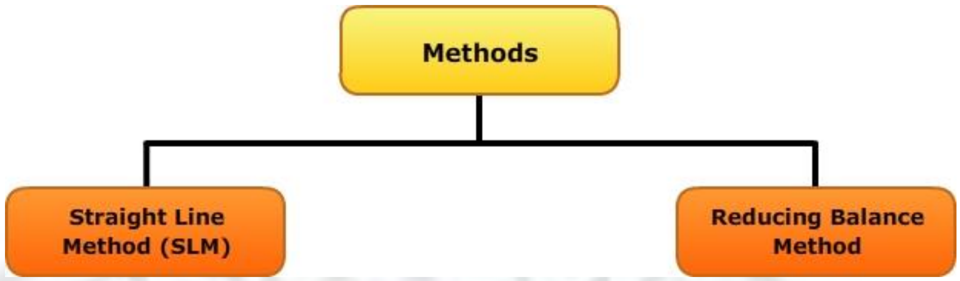
or i = 1.67%

5. Depreciation of Assets

As time goes on, due to wear and tear, exposure, etc., fixed assets like machinery, plants etc. diminish in value. This decrease in value is called depreciation. The rate at which the value of the asset declines is called the rate of depreciation. Scrap value is the value of the asset at the end of its useful life.

5.1 Methods of Computing Depreciation

Figure 9: Methods of Calculating Depreciation



- **Straight Line Method (SLM)**

  The scrap value is deducted from the original cost of the asset. Depreciation is computed by dividing the total cost of the asset by the life of the asset.

  Depreciation = $\frac{\text{Original Cost - Scrap Value}}{\text{Life of an asset}}$

Question 11:The initial cost of a machine is Rs. 40,000 and the scrap value is 4000. If the life is 8 years, find the annual depreciation using straight line method.

Solution 11: C = 40,000 S = 4000 n = 8

D = $\frac{40{,}000 - 4000}{8} = \frac{36{,}000}{8} = \text{Rs.}4{,}500$

5.2 Reducing Balance Method

This method is also known as diminishing balance method (DB) or written down value method (WDV). The depreciation is calculated for each year at a constant rate of the book value of the asset. If C is the 'value' of the asset at the beginning and 'd' is the rate of depreciation, then the depreciated value of the asset, D can be computed as

$D = A (1 – d)$ after a year

After two years, D would be

$D = A (1 – d)^2$

After 'n' years, D would be given as

$D = (1- d)^n$

Question 12: A machine is purchased for Rs. 70,000 and depreciates at a constant rate of 8%. What is the depreciated value at the end of its life, if life is 10 years. What is depreciation for the 5th year?

Solution 12: C = Rs. 70,000 d = 8%

Value at the end of 10 years $= 70{,}000 (1 – 0.08)^{10}$

$= 70{,}000 (0.43439)$

= Rs. 30,407.30

Depreciation for fifth year will be=value at the end of 4th year - value at the end of 5th year

$=70,000(1-0.08)^4 - 70,000\ (1-0.08)^5$

= 70,000 (0.71639) – 70,000 (0.65908)

= 50,147 – 46,136

Rs. 4011

6. Sinking Fund

It is a fund that is created by making equal periodic payments with an objective to have accumulated fund to meet some financial obligation in future. For example, a firm may create a fund for replacing some plant or machinery at some point of time in future or for redemption of debentures.

If (S/T) is the targeted amount and i be the rate of interest per period then R is the size of the periodic payment. Using the formula for computing the future value of an ordinary annuity.

$$S = R\ \frac{\left[(1+i)^n - 1\right]}{i}$$

Or $$R = \frac{Si}{\left[(1+i)^n - 1\right]}$$

Question 13: A sinking fund is required or created for having Rs. 25,000 in 6 years. How much amount should be set aside each semi-annual period into an account paying 6% compounded semi-annually?

Solution 13: S = Rs. 25,000 $\quad i = \frac{6\%}{2} = 3\% \quad \text{or} \quad 0.03$

n = 6(2) = 12

$$S = R\ \frac{\left[(1+i)^n - 1\right]}{i}$$

$$25{,}000 = R\ \frac{\left[(1+0.03)^{12} - 1\right]}{0.03}$$

$$25{,}000 = R\ \frac{\left[1.4257 - 1\right]}{0.03}$$

$$25{,}000 = R\ \frac{(0.4257)}{0.03}$$

25,000 = R (14.19)

$$R = \frac{25000}{14.19}$$

R = Rs. 1762 approx.

Question 14:A firm decided to set aside Rs. 1500 every quarter, and invest the same at the compound rate of 6% p.a. compounded quarterly. Find the sum after 6 years.

Solution 14: R = 1500 i = $\frac{6\%}{4} = 1.5\%$ or $0.015 \quad n = 6(4) = 24$

S = R $\frac{\left[(1+i)^n - 1\right]}{i}$

S = 1500 $\frac{\left[(1.015)^{24} - 1\right]}{0.015}$

S = 1500 $\left(\frac{1.4295 - 1}{0.015}\right)$

S = $\frac{1500}{0.015}$

S = 1500 (28. 6333)

Rs. 42950

7. Amortization

When the periodic payments are made against a debt that covers the outstanding interest and the principal. A loan is said to be amortized if both principal and interests are paid by a sequence of equal payments made over equal periods of time.

For e.g. Buying a house or a car or a consumer durable by making a series of periodic payments is an example of loan that is amortized. The formula used for computing the periodic payment is the same formula used for computing present value of an ordinary annuity.

P = R $\left[\frac{1-(1+i)^{-n}}{i}\right]$

There are P, R, i & n. Given any of three variables, fourth can be calculated.

Question 15:What monthly payment is required to pay-off a loan of Rs. 2000 at 8% per month in 1 year? in 2 years?

Solution 15: N = 12 P = Rs. 2000 i = 8% or 0.08

(i) 1 year using the formula

P = R $\left[\frac{1-(1+i)^{-n}}{i}\right]$

or 2000 = R $\frac{\left[\left(1-(1.08)^{-12}\right)\right]}{0.08}$

or $$2000 = R\,\frac{[1-0.3971]}{0.08}$$

$$2000 = R\,\frac{(0.6029)}{0.08}$$

2000 = R (7.53625) $\Rightarrow$ R = Rs. 265.40

(ii) 2 years P = Rs. 2000, i = n = 2 (12) = 24)

$$2000 = R\,\frac{\left[1-(1.08)^{-24}\right]}{0.08}$$

$$2000 = R\,\frac{[1-0.1576]}{0.08}$$

$$2000 = R\,\frac{(0.8424)}{0.08}$$

2000 = R (10.53)

R = Rs. 190

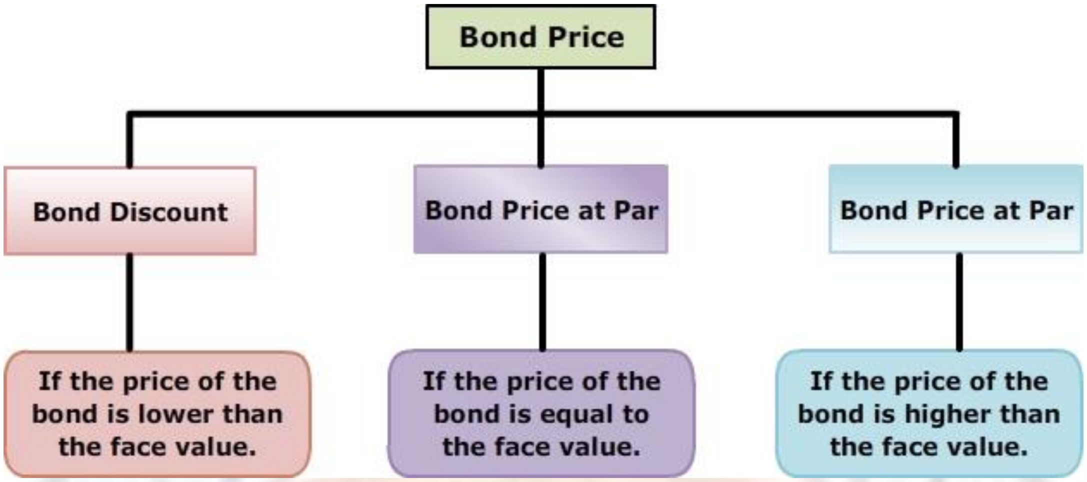
8. Bonds/Debentures

(i) Face Value or Par Value: The face value of the bond is the amount paid to the bondholder at the time of the maturity.

(ii) Nominal interest or Coupon interest: These are generally semi-annual. Nominal rate of interest is always calculated on the face value of the bond.

(iii) Bond price is the amount paid by the bondholder to the firm at the time of original issuance of the bond.

Figure 10: Types of Bond Prices



(iii) Redemption of a bond:

At the end of the specified period, the bondholder returns the bonds to the firm and gets back his original payment. A bond might be redeemed at

- At par $\Rightarrow$ when the bondholder gets the same amount back which he had invested.
- At premium $\Rightarrow$ When he gets more than what he paid.
- At discount $\Rightarrow$ When he gets less than what he paid.

Bondholder gets two benefits

(1) Interest payments which are usually six monthly.

(2) Principal repayment at the time of redemption.

Thus, the purchase price of a bond should be equal to the present value of interest payments and the present value of the principal repayment at the time of maturity or redemption.

Question 16: A bond has a face value of Rs. 1500 and maturity period of 6 years. The nominal interest rate is 6%. What should be the price of the bond to yield an effective rate of 8%?

Solution 16: Annual Interest Payment = $\frac{6}{100}\times 1500 = \text{Rs.}90$

$$= \text{Rs. } 90 \left[\frac{\left(1-(1+0.08)^{-6}\right)}{0.08}\right]$$

$$= \text{Rs. } 90 \ \frac{\left[1-(1.08)^{-6}\right]}{0.08}$$

$$= \text{Rs. } 90 \ \frac{(1-0.6301)}{0.08}$$

$$= \text{Rs. } 90\ \frac{(0.3699)}{0.08}$$

$$= \text{Rs. } 90\ (4.62375) \Rightarrow \text{Rs. } 416.14$$

Present Value of Maturity Payment

$= 1500\ (1 + 0.08)^{-6}$

$= 1500\ (0.6301)$

= Rs. 945.15

Price of the Bond = Present Value of Interest + Present Value of Principal

= Rs. 416.14 + Rs. 945.15

= Rs. 1361.29

Summary:

- Rates of interest are expressed in terms of percentage per annum.
- The rate of interest, compounded a given number of times per annum is called the nominal rate of interest.
- The actual percentage by which an investment grows during a year is called an effective rate of interest.
- The effective rate of interest is the interest rate that is equivalent to the nominal compounded rate of interest.
- If the conversion period is a year (compounded annually) then the nominal rate of interest and effective rate of interest would be exactly the same or equal.
- At the effective rate r_e, the principal P in one year will amount to $P(1 + r_e)$. Therefore,
  $r_e = (1 + i)^m - 1$
  or Effective rate of interest (r_e) = $(1 = i)^m - 1$
  or $r_e = (1+r/m)^m - 1)$
- The nominal rate of interest compounded continuously and equivalent to a given effective rate is known as force of interest.
- Annuity means equal annual payments. It has two important features: Equal in size & Equal intervals of time.
- The payment of future value is made at the beginning of each payment period. The first payment earns interest for n periods, second for (n – 1) periods and nth payment for only one period.
- The decrease in value of fixed assets due to wear and tear, exposure etc. is called depreciation. It can be of two types: Straight Line Method and Reducing Balance Method.
- Sinking fund is a fund that is created by making equal periodic payments with an objective to have accumulated fund to meet some financial obligation in future.
- A loan is said to be amortized if both principal and interests are paid by a sequence of equal payments over equal periods of time.
- Nominal rate of interest is always calculated on the face value of the bond.

Glossary:

- **Bond:** Bonds are fixed interest earning financial assets.
- **Compound Interest**: Interest that accrues on initial principal and accumulated interest of a principal deposit.

- **Conversion period**: The time gap between two successive payments or two successive conversions.
- **Debentures:** These are also fixed interest earning financial assets. For companies they are the part of liabilities.
- **Simple Interest**: It is the interest calculated on the principal for the entire period for which it is borrowed.
- **Principal**: The amount of money that is either invested initially or given on loan.
- **EMI:** Equal monthly installments.

Exercise:

A. Objective Type Questions:

1. **Fill in the blanks:**

a) The nominal and effective rates of interest are exactly the same when compounding is done ________.
b) The nominal rate 'r' convertible continuously, and equivalent to a given effective interest rate r_e is known as _________.
c) Future value of any annuity is always calculated on the last day of the _____ payment period.
d) In the case of _________, periodic payments are made at the beginning of each payment period.
e) Amortization of loan uses the concept of _____ value of an annuity.
f) Life insurance premium is an example of ________.

2. State 'True' or 'False':

a) Money has a place value.
b) Present value or capital value is always calculated on the first day of the first payment period.
c) The difference between nominal and effective rate of interest decreases with the increase in the frequency of conversions being done in a year.
d) The purchasing power of money reduces with time.
e) In the case of ordinary annuity, payments are made at the end of each payment period.
f) Payments continue forever or indefinitely in the case of annuity certain.

B. Short Answer Type Questions:

1) Distinguish between nominal and effective rates of interest.
2) Derive the relationship between nominal and effective rate of interest.
3) Define force of interest. Derive the relationship between nominal and effective rate of interest when compounding is done continuously.
4) What is an annuity? Give few examples.
5) What are various types of annuities? Explain each one in brief.
6) What are the various methods of depreciation?

C. Solve the given problems:

1) Find the compound amount of Rs. 5000 for 4 years at 6% converted:
(i) annually, (ii) semiannually, (iii) quarterly, and (iv) monthly.
2) Find the effective rate that is equivalent to nominal rate of 5% compounded

(i) continuously, (ii) monthly, (iii) quarterly, (iv) semiannually, and (v) annually.

3) Find the force of interest corresponding to the effective rate
(i) 4%, (ii) 6%, and (iii) 8%.

4) Which is better from the point of view of investor: 9.2% compounded semiannually or 9% compounded monthly?

5) Mr. A decides to deposit Rs 3500 at the end of every month for 5 years in a savings bank account that pays interest at 7% compounded monthly. Find the amount in his account at the end of 5 years.

6) At the beginning of each quarter, Rs. 5000 is deposited in a bank that pays 9% compounded quarterly. Find the amount in the account at the end of 4 years.

7) An asset costing Rs 5400 will depreciate to a scrap value of Rs. 600 in 10 years. Find the rate of depreciation.

8) A machine costing Rs. 7,00,000 depreciates at a rate of 10% for first two years, at 15% for next two years and then at 20%. Find the value of machinery at end of 10 years.

9) A loan of Rs. 50,000 is to be repaid in equal installments of principal and interest in 10 years. Find the annual installment given that rate of interest is 5% effective.

10) Face value of a bond is Rs. 2000. It matures in 10 years. The nominal rate of interest is 5%. What should be the price of bond that will yield 8% interest?

Answers to Objective Type Questions:

1. a) annual, b) force of interest, c) last, d) Annuity due, e) Future, and f) Annuity.
2. a) False, b) True, c) False, d) True, e) True, and f) False.

Answers to Problems:

1. i) Rs 6,312; ii) Rs 6,333.50; iii) Rs 6344.50; iv) Rs 6,352;
2. i) 5.13%, ii) 5.11%, iii) 5.0904%, iv) 5.0625 %, v) 5%;
3. i) 3.91%, ii) 5.825%, iii) 7.69%;
4. 9.2% compounded semi annually;
5. Rs. 250575.16;
6. Rs 64,988 .42;
7. 19.72%;
8. Rs. 1,07,389.26;
9. Rs. 6,475 approx;
10. Rs 5,766.48.

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2. Suggested Readings:

- Wikes, F.M., Mathematics for Business, Finance and Economics, Thomas Learning.
- Mizrahi and John Sullivan, Mathematics for Business and Social Sciences, Wiley and Sons.

3. Web Links:

- www.investopedia.com

- Visit the link http://highered.mcgraw-hill.com/sites/0072994029/student_view0/chapter6/multiple_choice_quiz.html to take a quiz on the concepts of present value and future value.
- Visit the link http://highered.mcgraw-hill.com/sites/0070890579/student_view0/chapter15/quiz_questions.html to take an online quiz on bonds and sinking fund.
- Visit the link http://web.utk.edu/~jwachowi/annuity3.html to test yourself on the concept of annuity.
- Visit the link http://highered.mcgraw-hill.com/sites/0073282146/student_view0/chapter6/self_test_quiz.html to test yourself on the fundamentals of finance.