Centre of Mass Frame of Reference

Discipline Course-I
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Paper: Mechanics IB
Lesson: Centre of Mass Frame of Reference

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Centre of Mass Frame of Reference

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Objective

After studying this chapter you will understand:

- The concept of Centre of mass frames and how it helps us to simplify the Collision problem
- Why Centre of mass frame is known as the Zero-Momentum Frame
- Two-body Elastic Collision in the Centre of mass Frame
- Two-Dimensional collision problem in Centre of mass frame
- The graphical way of representation of final momenta and its use for solving the Collision problem

Centre of Mass Frame of Reference

1. Centre of mass frame of reference

In the last chapter, we saw that the Centre of mass frame has total linear momentum about the Centre of mass zero, so we can utilize this to choose the Centre of mass as our frame of reference for the collision problem. Now we define the Centre of mass frame as:

"A frame of reference fixed to the Centre of mass of an isolated (no external force is acting) system of N-particles, is called as the Centre of mass or C.M. frame of reference."

In this frame the position vector of the Centre of mass, $\mathbf{R}_{\mathbf{C.M}}$, is taken to be at the origin of the axes, so the $\mathbf{R}_{\mathbf{C.M}}=\mathbf{0}$. Similarly the Velocity of the Centre of mass, $\mathbf{V}_{\mathbf{C.M}}=\mathbf{0}$.

Hence consequently the total linear momentum of the system, **P**, which is equal to the linear momentum of the Centre of mass, $\mathbf{P}_{\mathbf{C.M}} = M\,\mathbf{V}_{\mathbf{C.M}}=\mathbf{0}$ (as we have seen in earlier chapter), is also zero.

So we see that in the C.M. frame of reference, the total linear momentum is zero, which is why this frame is also known as the *ZERO-MOMENTUM FRAME OF REFRENCE*.

Also in this frame of reference, no external force is acting, so the linear momentum of the system remains constant which in turn implies that the linear momentum of the Centre of mass remains constant, hence the Centre of mass moves with a constant velocity. So the frame associated with the C.M. also moves with the constant velocity. So C.M. frame of reference is an *INERTIAL FRAME OF REFERENCE*.

Now our usual frame of reference ,which is attached with the earth and assumed to be inertial as long as the motion is confined only on the earth, is called the *LABORATORY FRAME OF REFERENCE.* The following figure shows the two type of frame of references:

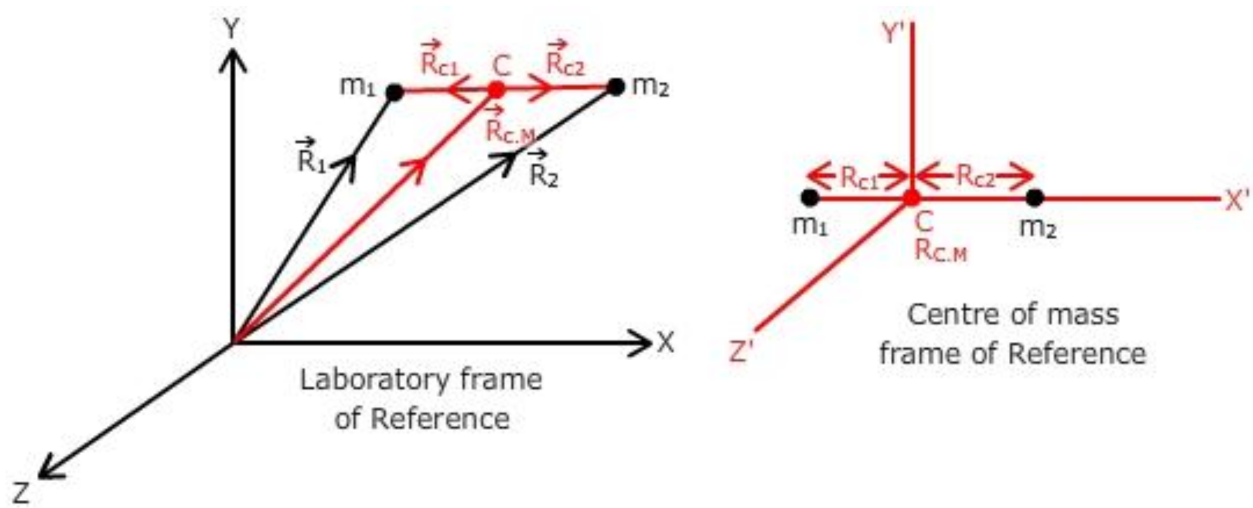

Fig 8.1

Centre of Mass Frame of Reference

2. Two particles elastic collision in Centre of mass frame

As said earlier the C.M. frame of reference provides great simplicity as compared to the laboratory frame of reference. Now we study the elastic collision problem of two particles to elaborate this point. We also assume that the velocities of the two particles are very small as compared to the velocity of light, so the treatment is totally non-relativistic.

We shall adopt the notation as follows. The mass is denoted by m, so for two particles system, the masses are m_1 and m_2. The initial velocities are represented by $\mathbf{u_i}$ and the final velocities are represented as $\mathbf{v_i}$. Here i=1 and 2. Now the initial and final linear momenta of the particles are represented as $\mathbf{k_i}$ and $\mathbf{p_i}$, respectively. The kinetic energies are represented as K_i (for initial) and T_i (for final). Now when we consider the general case of 2or 3-Dimensional collision we have scattering angles represented by θ_i and φ_i. Now to differentiate the laboratory and Centre of mass frame of reference we use the primed symbols for the C.M. frame and unprimed symbols for the Lab frame.

Now the two conservation laws can be written, using above notations, as

(1) Law of linear momentum conservation: $\mathbf{k_1+k_2=p_1+p_2}$
(2) Law of kinetic energy conservation : $K_1+K_2=T_1+T_2$

Or using $\mathbf{k_i}=m_i\mathbf{u_i}$, $\mathbf{p_i}=m_i\mathbf{v_i}$ and $K_i=k_i^2/2m_i$, $T_i=p_i^2/2m_i$, so we have the two equations as:

$$m_1\mathbf{u_1}+m_2\mathbf{u_2}=m_1\mathbf{v_1}+m_2\mathbf{v_2} \qquad (1)$$

and

$$(k_1^2/2m_1)+(k_2^2/2m_2)=(p_1^2/2m_1)+(p_2^2/2m_2) \qquad (2)$$

In the collision problems the initial conditions of the two particles, namely, the masses, the magnitudes of momenta and the trajectories are given. So in 3-Dimensional space, we know the six components of initial momenta $\mathbf{k_1}$ and $\mathbf{k_2}$. We have to find six components of final momenta $\mathbf{p_1}$ and $\mathbf{p_2}$. But we have only four equations (3 of each momenta equations and one of energy equation), so we require further information to solve the problem. Hence some additional information, say of direction (spherical angles θ and φ) of one of the particle is necessary.

3. Two-dimensional elastic collision

Let us study 2-Dimensional elastic collision of two particles as shown in the following figure. Here laboratory frame and Centre of mass frame of reference are both shown. Here we assume that the second particle m_2 is at rest, for simplicity of the problem.

Centre of Mass Frame of Reference

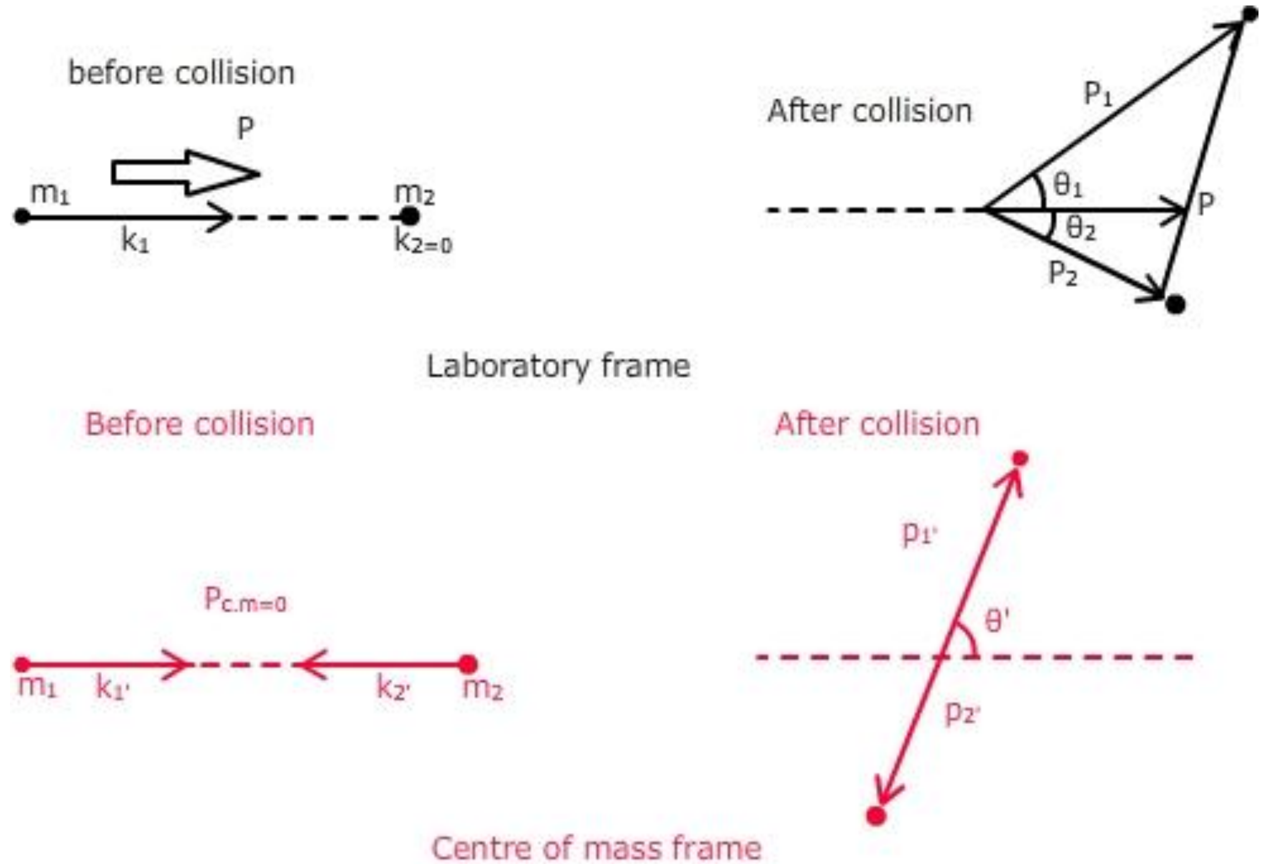

Fig 8.2

Here it is obvious from the figure that in lab frame the linear momentum of Centre of mass of two-particle system is non-zero, while it is zero in C.M. frame. Let us now consider the after collision condition with utmost care. It is redrawn below again separately.

Centre of Mass Frame of Reference

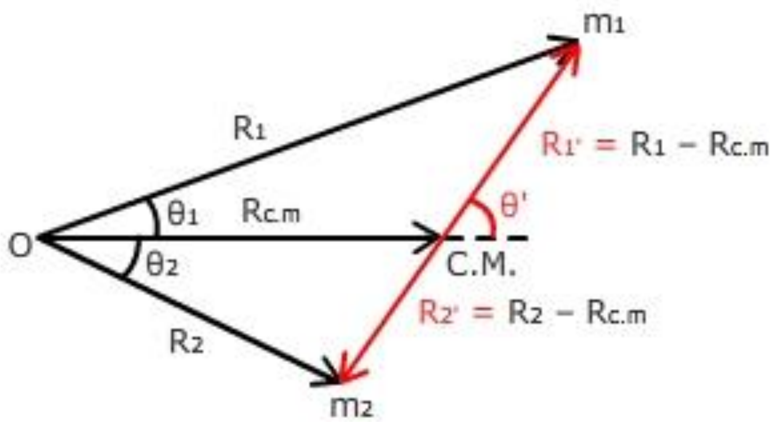

Fig 8.3

Let us denote the position vectors of first and second particles by $\mathbf{R_1}$ and $\mathbf{R_2}$ respectively and let $\mathbf{R_{C.M}}$ be the position vector of the Centre of mass in the laboratory frame.

Denoting the velocity of Centre of mass by $\mathbf{V_{C.M}}$, then the linear momentum of the Centre of mass is given by

$$\mathbf{P_{C.M}} = M\,\mathbf{V_{C.M}} = (m_1+m_2)\,\mathbf{V_{C.M}}$$

$$= (m_1+m_2)[(m_1\,\mathbf{v_1}+m_2\,\mathbf{v_2})/(m_1+m_2)]$$

$$= m_1\,\mathbf{v_1}+m_2\,\mathbf{v_2}$$

$$= \mathbf{p_1}+\mathbf{p_2}$$

$$= \mathbf{P}$$

So the linear momentum of the Centre of mass is equal to the total linear momentum of the two-particle system.

Now as shown in the figure, the position vectors of the particles in the C.M. frame is represented as $\mathbf{R_1'}$ and $\mathbf{R_2'}$ respectively. The separation between the two particles is given by in the two frames as

$$\mathbf{R} = \mathbf{R_1'} - \mathbf{R_2'}$$

$$= (\mathbf{R_1} - \mathbf{R_{C.M}}) - (\mathbf{R_2} - \mathbf{R_{C.M}})$$

$$= \mathbf{R_1} - \mathbf{R_2}.$$

Now in C.M. frame, the origin of the frame is itself at $\mathbf{R_{C.M}}$, hence we have

$$\mathbf{R_{C.M}} = [m_1\,\mathbf{R_1'}+m_2\,\mathbf{R_2'}]/(m_1+m_2)=0$$

So $\qquad m_1\,\mathbf{R_1'}+m_2\,\mathbf{R_2'}=0 \qquad \textbf{(1)}$

Now we also have $\qquad \mathbf{R_1'} - \mathbf{R_2'} = 0 \qquad \textbf{(2)}$

Adding and subtracting $m_2\,\mathbf{R_1'}$ in eq. (1) and using eq. (2), we have

$\mathbf{R_1'} = [m_2/(m_1+m_2)]\mathbf{R}$

$= (1/m_1)\,[m_1\, m_2\,/(m_1+m_2)]\mathbf{R}$

So $\mathbf{R_1'} = (m/m_1)\mathbf{R}$ **(3)**

Where $m=[m_1\, m_2/(m_1+m_2)]$ is the reduced mass.

Similarly we can have

$\mathbf{R_2'} = -\,(m/m_2)\mathbf{R}$ **(4)**

Now taking derivatives of eq. (3) and (4), we have velocities of the particles in the C.M. frame given by

$\mathbf{u_1'} = \dot{\mathbf{R}}_1' = (m/m_1)\,\dot{\mathbf{R}}$ (5)

$\mathbf{u_2'} = \dot{\mathbf{R}}_2' = -\,(m/m_2)\,\dot{\mathbf{R}}$ (6)

Also taking derivative of $\mathbf{R} = \mathbf{R_1'} - \mathbf{R_2'} = \mathbf{R_1} - \mathbf{R_2}$, we get

$$\dot{\mathbf{R}} = \dot{\mathbf{R}}_1' - \dot{\mathbf{R}}_2' = \dot{\mathbf{R}}_1 - \dot{\mathbf{R}}_2$$

$$= \mathbf{u_1'} - \mathbf{u_2'} = \mathbf{u_1} - \mathbf{u_2} = \mathbf{u} \qquad (7)$$

Where $\mathbf{u}$ is the relative velocity of the first particle w.r.t. second particle.

So using eq. (5) and (6), we have

$$\mathbf{k_1'} = m_1\, \mathbf{u_1'} = m\mathbf{u} = -m_2\, \mathbf{u_2'} = -\mathbf{k_2'} \qquad (8)$$

also $\mathbf{k_1'} = m\mathbf{u} = -\mathbf{k_2'}$ implies $k_1' = k_2'$ (9)

so *THE LINEAR MOMENTA OF THE TWO PARTICLES ARE EQUAL AND OPPOSITE IN THE C.M. FRAME OF REFERENCE.* This is the characteristic property of the C.M. frame.

Now applying the law of conservation of momentum in the Centre of mass frame, we have

Initial total momentum =final total momentum

Or $\mathbf{k_1'} + \mathbf{k_2'} = \mathbf{p_1'} + \mathbf{p_2'}$

Since $\mathbf{k_1'} = -\mathbf{k_2'}$,

So $\mathbf{p_1'} + \mathbf{p_2'} = 0$,

Implies $\mathbf{p_1'} = m\mathbf{u} = -\mathbf{p_2'}$ (10)

Also $p_1' = p_2'$ (11)

Centre of Mass Frame of Reference

Hence *BOTH INITIAL AND FINAL MOMENTA OF THE TWO PARTICLES ARE EQUAL AND OPPOSITE. ALSO THE TOTAL LINEAR MOMENTUM IN THE CENTRE OF MASS FRAME* ($\boldsymbol{P'=K_1'+K_2' = P_1'+P_2'=0}$) *IS ZERO*.

Now the conservation of kinetic energy demands

$$(k_1'^2/2m_1) + (k_2'^2/2m_2) = (p_1'^2/2m_1) + (p_2'^2/2m_2)$$

So using eq. (9) and (11), we have

$$k_1'^2/2m = p_1'^2/2m = (½)mu^2 \qquad (12)$$

so we have $u_1' = v_1'$ and $u_2' = v_2'$ (13)

SO THE MAGNITUDES OF INITIAL AND FINAL VELOCITIES OF THE PARTICLES REMAINS SAME IN C.M.FRAME.

Now we have relations connecting C.M.frame with the Laboratory frame, as

POSITION VECTORS $\mathbf{R_1=R_{C.M}+R_1'}$ (14)

$\mathbf{R_2= R_{C.M}+R_2'}$ (15)

INITIAL VELOCITIES $\mathbf{u_1= V_{C.M}+u_1'} = \mathbf{V_{C.M}}+(m/m_1)\mathbf{u}$ (16)

$\mathbf{u_2= V_{C.M}+u_2'} = \mathbf{V_{C.M}} - (m/m_2)\mathbf{u}$ (17)

FINAL VELOCITIES $\mathbf{v_1= V_{C.M}+v_1'} = \mathbf{V_{C.M}}+(m/m_1)\mathbf{u}$ (18)

$\mathbf{v_2= V_{C.M}+v_2'} = \mathbf{V_{C.M}} - (m/m_2)\mathbf{u}$ (19)

INITIAL MOMENTA $\mathbf{k_1}=m_1 \mathbf{u_1}=m_1 \mathbf{V_{C.M}}+m_1 \mathbf{u_1'}=m_1 \mathbf{V_{C.M}}+m\mathbf{u}$ (20)

$\mathbf{k_2}=m_2 \mathbf{u_2}=m_2 \mathbf{V_{C.M}}+m_2 \mathbf{u_2'}=m_2 \mathbf{V_{C.M}} - m\mathbf{u}$ (21)

FINAL MOMENTA $\mathbf{p_1}=m_1 \mathbf{v_1}=m_1 \mathbf{V_{C.M}}+m_1 \mathbf{v_1'}=m_1 \mathbf{V_{C.M}}+m\mathbf{u}$ (22)

$\mathbf{p_2}=m_2 \mathbf{v_2}=m_2 \mathbf{V_{C.M}}+m_2 \mathbf{v_2'}=m_2 \mathbf{V_{C.M}} - m\mathbf{u}$. (23)

Now we study the collision process with the help of geometrical diagrams. Since magnitudes of final momenta of the particles $\mathbf{p_1'}$ and $\mathbf{p_2'}$, in C.M. frame, are equal, we draw a circle at origin O and radius equal to $p_1'=p_2'=mu$, as shown below

Centre of Mass Frame of Reference

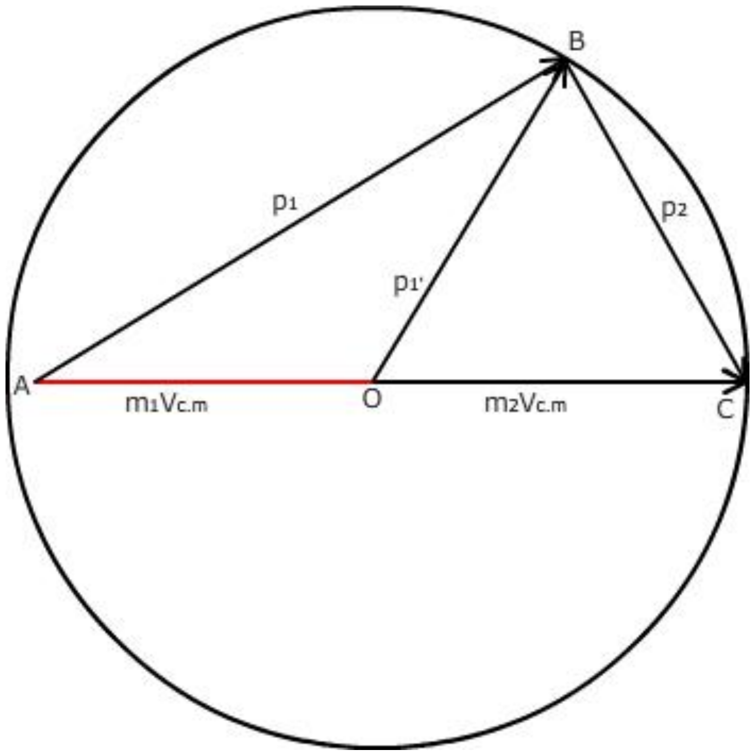

Fig 8.4 Vector Representation of Final Momenta

Let us draw vector **AC** such that vector **AO** represent m_1 $\mathbf{V_{C.M}}$ and vector **OC** represent $m_2\mathbf{V_{C.M}}$. Then by eq. (22) and (23), vector **AB** represent $\mathbf{p_1}$ and vector **BC** represent $\mathbf{p_2}$.

In laboratory system, we have chosen k_2=initial linear momentum of particle 2=0. Also the linear momentum of the Centre of mass in laboratory frame is given by

$$M\mathbf{V_{C.M}}= (m_1+m_2)\, \mathbf{V_{C.M}}=\mathbf{p_1}+\mathbf{p_2}=\mathbf{k_1}+\mathbf{k_2}=\mathbf{k_1} \qquad (\text{since } \mathbf{k_2}=\mathbf{0})$$

So $$\mathbf{V_{C.M}} = \mathbf{k_1} /(m_1+m_2) \qquad (24)$$

Now the vector **OC** is given by

$\mathbf{OC} = m_2\, \mathbf{V_{C.M}}$

$= m_2\, \mathbf{k_1} /(m_1+m_2)$

$=(m/m_1)\mathbf{k_1}$

$=(m/m_1)(m_1\, \mathbf{u_1})$

$= m\mathbf{u_1}$, and the magnitude of **OC** is mu_1. (25)

Now the vector **OB** is given by

$\mathbf{OB} = \mathbf{p_1}'$

$= m\mathbf{u}$

$= m(\mathbf{u_1}-\mathbf{u_2})$

$= m\mathbf{u_1}$, with magnitude given by mu_1. (26)

So $OC=OB=mu_1$ (27)

Hence we see from eq. (25) and (26), the magnitudes of the vectors **OC** and **OB** are equal and since point B lies on the circle, so the point C must lie on the circle, as shown below:

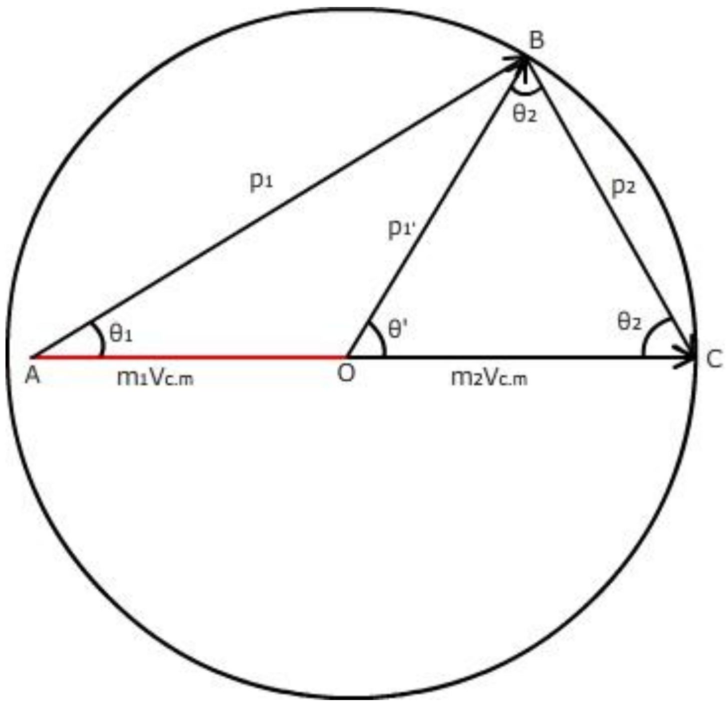

Fig 8.5 Vector Representation of Final Momenta

Now we think about the position of point A, if we take the ratio of length AO and OC, we have $OA=m_1\, V_{c.m}$ and $OC= m_2\, V_{C.M}$

So

$$OA/OC = m_1/m_2 \qquad (28)$$

Hence the position of point A is decided by the ratio of masses (m_1/m_2). We have three possibilities:

1. $m_1=m_2$ implies point A will lie on the circle.
2. $m_1>m_2$ implies point A will lie outside the circle.
3. $m_1<m_2$ implies point A will lie inside the circle.

Now we study these cases separately

Case1. $m_1=m_2$

In this case, the ratio OA/OC =1, so OA=OC. Hence point A lies on the circle as shown below:

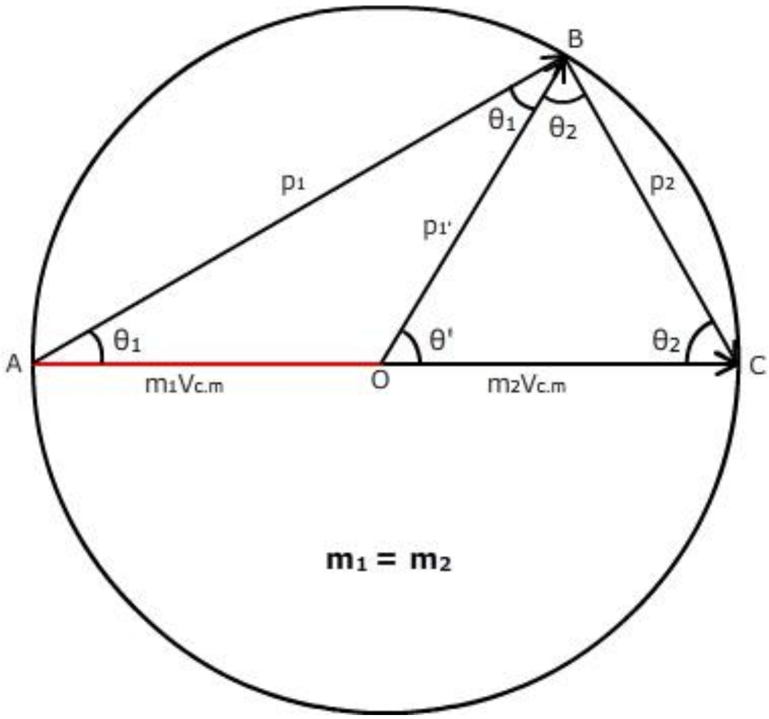

Fig 8.6 Vector Representation of Final Momenta

Here the angles of scattering in laboratory frame are related, as seen from the figure, as

$$\theta_1 = (1/2)\theta'$$

$$\theta_1 + \theta_2 = (1/2)\pi.$$

So after collision the two particles moves away at right angles to each other in the laboratory frame. Here θ_1 is the maximum angle of scattering.

Case II. $m_1 < m_2$

The ratio OA/OC is less than 1, so OA is less than OC. Hence point A will lie inside the circle as shown below:

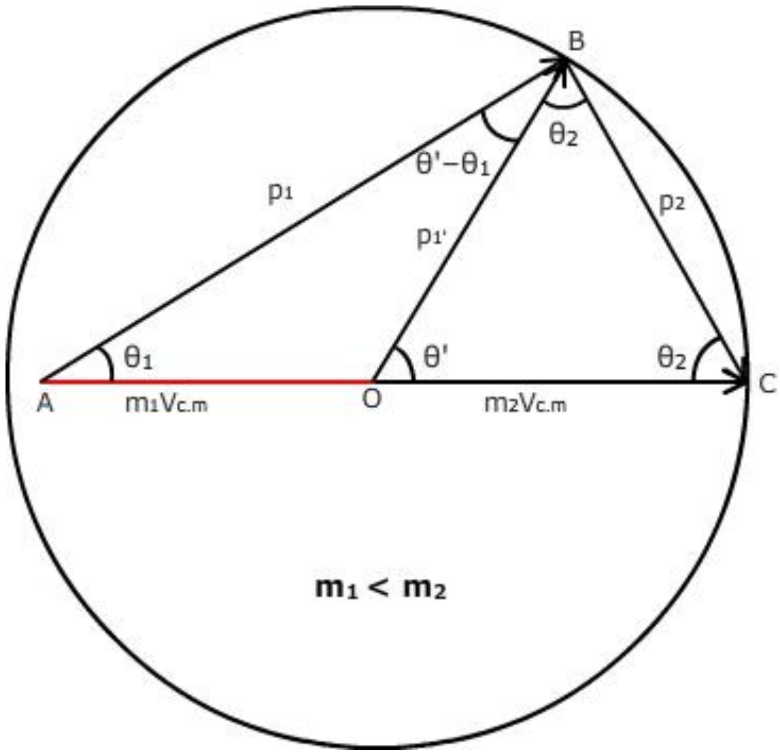

Fig 8.7 Vector Representation of Final Momenta

Here as we see that for a given ratio of masses and initial momenta, there exist only one value of final momenta **AB,** as given by the third side of the triangle formed.

Case III**.** $m_1>m_2$

The ratio m_1/m_2 is greater than 1, so OA is greater than OC. Hence the point A will lie outside the circle as shown:

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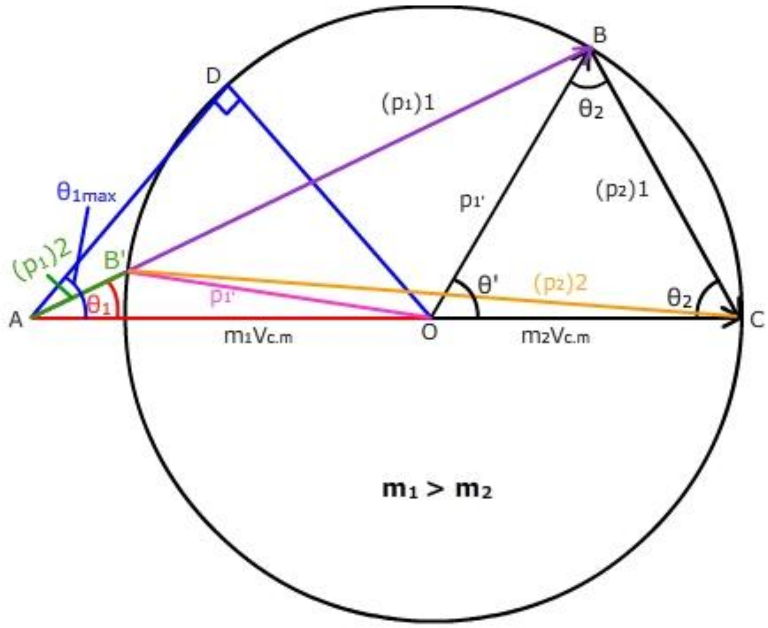

Vector Representation of final momenta

Fig 8.8

As shown in the figure, there exits two values of final momenta **(p_1)1, (p_2)1** and **(p_1)2, (p_2)2** of the two particles, respectively, for each value of initial momenta **(k_1)** and **(k_2)**. These are represented in the figure by the vectors **AB** and **AB'** for the first particle and by vectors **BC** and **B'C** for the second particle. Vectors **AB** and **BC** corresponds to forward scattering for which $\theta' < \pi/2$, whereas vectors **AB'** and **B'C** corresponds to the backward scattering for which $\theta' > \pi/2$. This was the case in C.M. frame.

In laboratory frame, scattering angle θ_1 is smaller than $\pi/2$, as shown in the figure, both for forward and backward scattering. The angle θ_1 varies from zero (when AB=AC, corresponding to no scattering) to maximum angle θ_{1max} (when AB=AD i.e. when AB is tangential to the circle).

So we have, from the fig,

$$\text{Sin } \theta_{1max} = OD/OA = OC/OA = m_2/m_1.$$

Now in the triangle Δ OBC, we have

$$2\theta_2 + \theta' = \pi$$

Or
$$\theta_2 = \frac{\pi - \theta'}{2}$$

Where θ_2 is the recoil angle for the second particle in lab frame.

Centre of Mass Frame of Reference

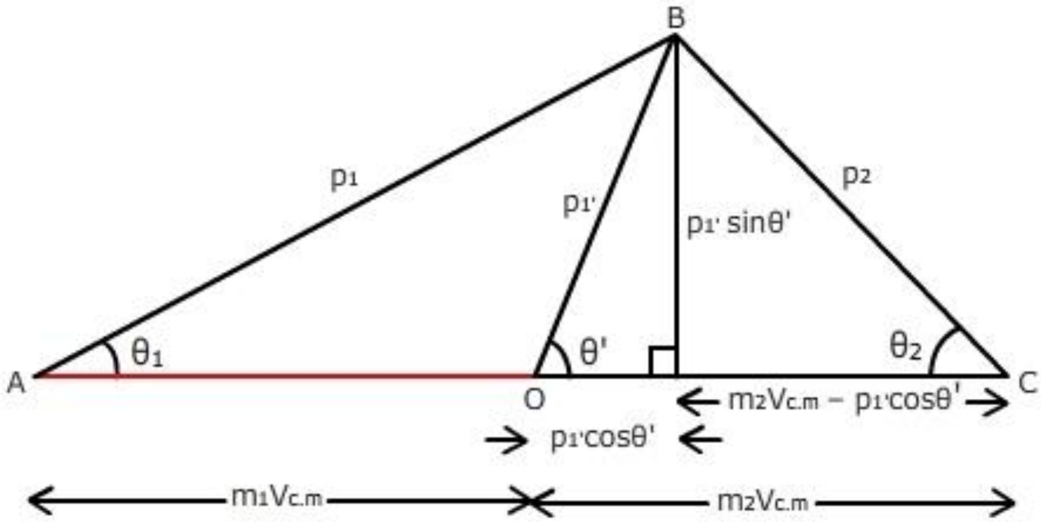

FIG 8.9

Now from figure. (9), we observe that

$$\tan\theta_1 = (p_1'\ \mathrm{Sin}\ \theta')/(m_1V_{C.M} + p_1'\ \mathrm{Cos}\ \theta')$$

$$= (\mathrm{Sin}\ \theta')/[(m_1V_{C.M}/p_1')+\mathrm{Cos}\ \theta']$$

But $p_1' = m_2V_{C.M}$,

So

$$\mathbf{\tan\theta_1 = (Sin\ \theta')/[(m_1/m_2)+Cos\ \theta']}$$

Now we consider the three case of ratio of masses,

(1) When $m_1=m_2$, we have

$$\tan\theta_1 = (\mathrm{Sin}\ \theta')/[(m_1/m_2)+\mathrm{Cos}\ \theta']$$

$$\tan\theta_1 = (\mathrm{Sin}\ \theta')/[1+\mathrm{Cos}\ \theta']$$

$$=(2\mathrm{Sin}\ (\theta'/2)\mathrm{Cos}(\theta'/2)\ /[2\mathrm{Sin}^2\ (\theta'/2)]$$

So

$$\tan \theta_1 = \tan(\theta'/2)$$

Hence

$$\theta_1 = \theta'/2$$

(2) When $m_1 > m_2$,

$$\tan \theta_1 = (\text{Sin } \theta')/[(m_1/m_2)+\text{Cos } \theta']$$

Or $$\tan \theta_1 = (m2/m1)(\text{Sin } \theta')/[1+(m2/m1)\text{Cos } \theta']$$

so as $m_1 >> m_2$, $m_2/m_1 \rightarrow 0$, so

$$\tan \theta_1 = 0 \qquad \text{and } \theta_1 = 0$$

This means there is no scattering, if a heavier particle strikes with a lighter particle at rest.

(3)When $m_1 < m_2$,

$$\tan \theta_1 = (\text{Sin } \theta')/[(m_1/m_2)+\text{Cos } \theta']$$

as $m_1 << m_2$, $m_1/m_2 \rightarrow 0$, so

$$\tan \theta_1 = (\text{Sin } \theta')/(\text{Cos } \theta')$$

$$\tan \theta_1 = \tan \theta'$$

or $$\theta_1 = \theta'$$

This result shows that the scattering angle for the lighter particle in the lab frame is equal to the scattering angle in the C.M. frame. We can also derive the expression for the scattering angle for the second particle in the laboratory Frame.

Again from the fig. (9), we have

$$\tan \theta_2 = (p_1' \text{ Sin } \theta')/(m_2 V_{C.M} - p_1' \text{ Cos } \theta')$$

$$= (\text{ Sin } \theta')/[(m_2 V_{C.M})/p_1' - \text{Cos } \theta']$$

Again $p_1' = m_2$ Vc.m, so

$$\tan \theta_2 = (\text{ Sin } \theta')/(1 - \text{Cos } \theta')$$

Hence

$$\tan\boldsymbol{\theta}_2 = \cot(\boldsymbol{\theta}'/2)$$

so $$\boldsymbol{\theta}_2 = (\boldsymbol{\pi} - \boldsymbol{\theta}')/2$$

4. Summary.

- In the C.M.frame of reference, the total linear momentum is zero, which is why this frame is also known as the zero-momentum frame of reference.
- Both initial and final momenta of the two particles are equal and opposite.
- The total linear momentum in the Centre of mass frame (**$P' = k_1' + k_2' = p_1' + p_2' = 0$**) is zero.
- The magnitudes of initial and final velocities of the particles remains same in C.M.frame
- We study the collision process with the help of geometrical diagrams. Since magnitudes of final momenta of the particles **p_1'** and **p_2',** in C.M. frame, are equal, we draw a circle at origin O and radius equal to $p_1' = p_2' = mu$, the magnitudes of the vectors **OC** and **OB** are equal and since point B lies on the circle, so the point C must lie on the circle,

- The position of point A is decided by the ratio of masses (m_1/m_2).
  - We have three possibilities:
    - (1) $m_1 = m_2$ implies point A will lie on the circle.
    - (2) $m_1 > m_2$ implies point A will lie outside the circle.
    - (3) $m_1 < m_2$ implies point A will lie inside the circle.

- The relation between scattering angles in lab frame and the C.M. frame is given by
    - $\tan\boldsymbol{\theta}_1 = (\text{Sin}\,\boldsymbol{\theta}')/[(m_1/m_2) + \text{Cos}\,\boldsymbol{\theta}']$
  - When $m_1 = m_2$, we have
    $$\tan\boldsymbol{\theta}_1 = \tan(\boldsymbol{\theta}'/2)$$
    and $$\boldsymbol{\theta}_1 = \boldsymbol{\theta}'/2$$
  - When $m_1 < m_2$
    $$\tan\boldsymbol{\theta}_1 = \tan\boldsymbol{\theta}'$$
    and $$\boldsymbol{\theta}_1 = \boldsymbol{\theta}'$$
  - We can also have relation for scattering angle $\boldsymbol{\theta}_2$ as
    $\tan\boldsymbol{\theta}_2 = \cot(\boldsymbol{\theta}'/2)$ and $\boldsymbol{\theta}_2 = (\boldsymbol{\pi} - \boldsymbol{\theta}')/2$
  - When $m_1 > m_2$
    $\tan\theta_1 = 0$ and $\theta_1 = 0$

5. Exercise.

Q1. Prove that in Centre of mass frame, the magnitude of velocities of particles remains unchanged in elastic collisions.

Q2. Two masses of 3kg and 6kg have their initial velocities $\mathbf{u_1}=2\mathbf{i}$ m/s and $\mathbf{u_2}=-3\mathbf{i}$ m/s. if the collision is perfectly inelastic, obtain the final velocity of the Centre of mass. Also find the final momentum of the system in (a) lab frame, (b) C.M. frame.

Q3. Two particles of mass 2kg and 4kg have their position vectors given by $\mathbf{R_1}=2t\mathbf{i}-3\mathbf{j}$ and $\mathbf{R_2}=3\mathbf{i}-2t\mathbf{j}$. Find (1) the position vectors at the time t=6s,(2) the position vector of Centre of mass at t=4s,(3) the velocity vectors at t=6s,(4) the velocity of Centre of mass at t=4s.

Q4. Two particles of equal mass are colliding head-on with initial speed of 10m/s and 5m/s towards each other. After collision if one of them move with 5m/s with an angle of 30 degree with respect to original direction. Find the speed and direction of the other particle in (1) lab frame,(2)C.M. frame.

Q5. Given that the ratios of two masses are 1/20, find the relation between the scattering angles in lab and C.M. frame of references. If one of the angles in lab frame is 30 degree, find the other angle.

Fill in the blanks:

Q6. In the C.M.frame of reference, the total linear momentum is ________________.

Q7. The Centre of mass frame of reference is also called____________________.

Q8. Both initial and final momenta of the two particles are _______________ in C.M.frame.

Q9. The magnitudes of initial and final velocities of the particles __________ in C.M.frame.

Q10. Since the Centre of mass moves with a constant velocity. So the frame associated with the C.M. also moves with the constant velocity . So C.M.frame of reference is an ________________.

Centre of Mass Frame of Reference

State whether following statements are true or false:

Q11. The centre of mass frame is a non-inertial frame of reference.

Q12. The total linear momentum of the C.M. of the system of is constant.

Q13. The linear momentum of the Centre of mass is equal to the total linear momentum of the system of particles.

Q14. The magnitudes of initial and final velocities of the particles remains same in C.M.frame.

Q15. In Centre of mass frame, the position vector of the Centre of mass, $\mathbf{R}_{\mathbf{C.M}}$, is taken to be at the origin of the axes.

Choose the appropriate option for the followin:

Q16. For two particles collision of equal masses ,after collision, the two particles moves away

(A) At right angles to each other in the lab frame.
(B) Moves in same direction in the lab frame.
(C) Moves in the oppsite direction in the lab frame.

Q17. The relation between scattering angles in lab frame and the C.M. frame is given by

(A) $\tan \boldsymbol{\theta}_1 = (\text{Sin } \boldsymbol{\theta}')/[(m_1/m_2)+\text{Cos } \boldsymbol{\theta}']$
(B) $\cot \boldsymbol{\theta}_1 = (\text{Sin } \boldsymbol{\theta}')/[(m_1/m_2)+\text{Cos } \boldsymbol{\theta}']$
(C) $\tan \boldsymbol{\theta}_1 = (\text{Sin } \boldsymbol{\theta}')/[(m_1/m_2) - \text{Cos } \boldsymbol{\theta}']$

Q18. The expression for the scattering angle for the second particle in the lab. Frame is

(A) $\tan \boldsymbol{\theta}_2 = \cot (\boldsymbol{\theta}'/2)$
(B) $\cot \boldsymbol{\theta}_2 = \cot (\boldsymbol{\theta}'/2)$
(C) $\tan \boldsymbol{\theta}_2 = \cos (\boldsymbol{\theta}'/2)$

Q19. For collision of equal masses , we have

(A) $\boldsymbol{\theta}_1 = \boldsymbol{\theta}'/2$
(B) $\boldsymbol{\theta}_1 < \boldsymbol{\theta}'/2$
(C) $\boldsymbol{\theta}_1 > \boldsymbol{\theta}'/2$

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Q20. For an elastic collision between two particles of masses m1 and m2 in the C.M. frame. Show that after collision m1 and m2 moves off in opposite direction with equal linear momentum and all values of scattering angle is permissible.

Q21. Consider two particles of masses m1 and m2 which collide and sick together on collision. Suppose m2 is at rest and m1 is moving with u1 velocity in +ve x direction before the collision. Discuss the motion of the system before and after the collision in the C.M.frame.

Q22. Two particles of equal masses moves with initial velocities u1 and u2 respectively, collide elastically. Discuss the motion before and after the collision in the C.M. frame.

Q23. When a very light particle collide elastically with a massive stationary particle. Find their finial velocities in lab and C.M. frame.

Q24 Show that the scattering angle and lab angle are related as

$$\tan \boldsymbol{\theta}_1 = (\mathrm{Sin}\ \boldsymbol{\theta}')/[(m_1/m_2)+\mathrm{Cos}\ \boldsymbol{\theta}']$$

Q25. Show that the linear momenta of the two particles are equal and opposite in the C.M. frame of reference

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