Subject :Physics

Lesson Name: Centrifugal force

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Centrifugal Force

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Learning Objectives

After reading this lesson, you should be able to

a) Understand the concept of circular motion

b) Define centripetal force

c) Develop the concept of centrifugal force

d) Generalize the expression of D' Alembert principle

e) Learn about centripetal acceleration

Centrifugal Force

Chapter: Title   Centrifugal Force

2.1 Introduction

Here we will discuss the motion of rotation about a fixed axis (as shown in (Fig. 2.1)). All points on the axis remain stationary during the motion, while other points in the body move in circles concentric with the axis and in planes perpendicular to it. Such a motion is known as circular motion.

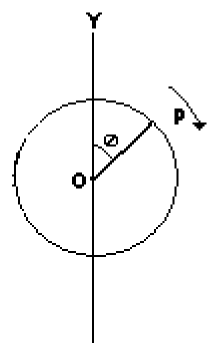

Fig. 2.1

The position of every point in a body rotating in this way is completely described if the angular position of any radius of the body is known, relative to some fixed direction i.e. if the rotating body is represented by the circle of Fig. 2.1. OP is radius fixed in the body, angle θ which OP makes with y-axis is sufficient to determine the position of every point in the body.

2.2 Centripetal and centrifugal force

Consider the experiment of tying a stone or weight to a cord, and whirling the stone in a circle. While the stone is revolving it can be felt to pull outward on one's hand, and equally an inward pull on the stone must exist.

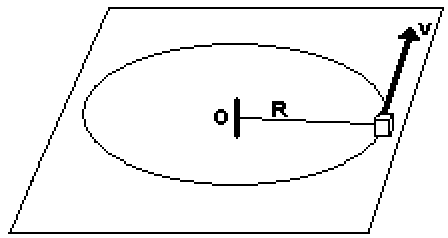

Fig. 2.2

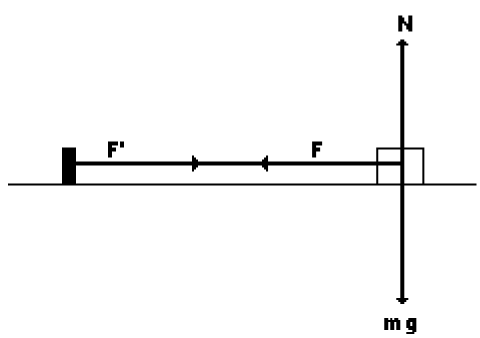

Fig. 2.3

Suppose a pin *O* is set into a horizontal frictionless table top as in Fig. 2.2. A small body of mass *m* is attached to the pin by a cord of length *R*, and set revolving about it with an angular velocity ω, a tangential velocity v_T, and a radial acceleration $a_R = \frac{v_T^2}{R} = \omega^2 R$.

According to Newton's second law, a force must be exerted on the body to produce this radial acceleration, and the direction of this force must be the same as the direction of the acceleration or towards the centre of the circle. It is therefore known as a central or centripetal force. (The term "centripetal" means literally "seeking a center") Since

$$\vec{F} = m\vec{a} \qquad \vec{a} = \frac{v_T^2}{R} = \omega^2 R \qquad (a)$$

And the magnitude of the centripetal force is

$$F = \frac{mv_T^2}{R} = m\omega^2 R \qquad (b)$$

Here the cord is providing the inward force, which is evidently in tension and which therefore exert an outward force, equal and opposite to the centripetal force, on the pin at the center. This outward force is called a centrifugal force. (The term "centrifugal" means literally "fleeing a center". The force diagram of the system is shown in Fig. 2.3 where *F* and *F′* are the equal and opposite forces exerted by the cord on the bodies to which its ends are attached. Force *F* is centripetal, force *F′* the centrifugal force. Both centripetal and centrifugal forces always constitute an action-and-reaction pair, the former being the resultant inward force on the revolving body and the latter the reaction to this force. Centripetal forces, like other forces, are pushes or pulls exerted on some material body by some other material body, and their designation as "centripetal" refers only to the effect they produce (a change in direction) and not to something inherently different in their nature.

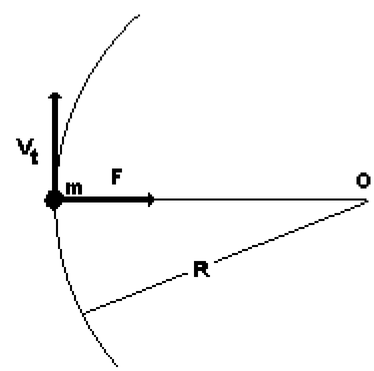

Fig. 2.4(a)

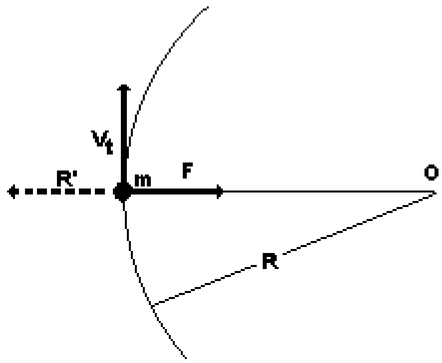

Fig. 2.4(b)

The D' Alembert principle may be applied to circular motion as well as to motion in a straight line. Fig. 2.4(a) and Fig. 2.4(b) represents a body of mass *m* moving with a tangential velocity v_T in a circular path of radius *R* about a center at *O*. In Fig. 2.4(a), a resultant inward force *F* is exerted by connecting cord on the body and this resultant force equals the product of mass and radial acceleration, $\frac{mv_T^2}{R}$. The D'Alembert viewpoint, in Fig. 2.4(b) is that the body is in equilibrium under the combined action of the force *F* and the fictitious outward force $\frac{mv_T^2}{R}$. When D'Alembert principle is used, the fictitious outward force $\frac{mv_T^2}{R}$ is called a "centrifugal force".

2.3 D' Alembert's principle

Consider a body of mass *m*. If $\vec{a}$ is the acceleration of a body in any given direction, and $\vec{F}_1, \vec{F}_2, \ldots\ldots$ are the components of the applied forces in that direction, Newton's second law may be expressed as

$$\vec{F}_1 + \vec{F}_2 + \ldots\ldots = \sum \vec{F} \qquad (a)$$

in which $\sum \vec{F}$ represents the sum of the components of the applied forces in the direction of acceleration. This is named as effective force.

Equation (*a*) may be written

$$\sum \vec{F} - m\vec{a} = 0 \qquad (b)$$

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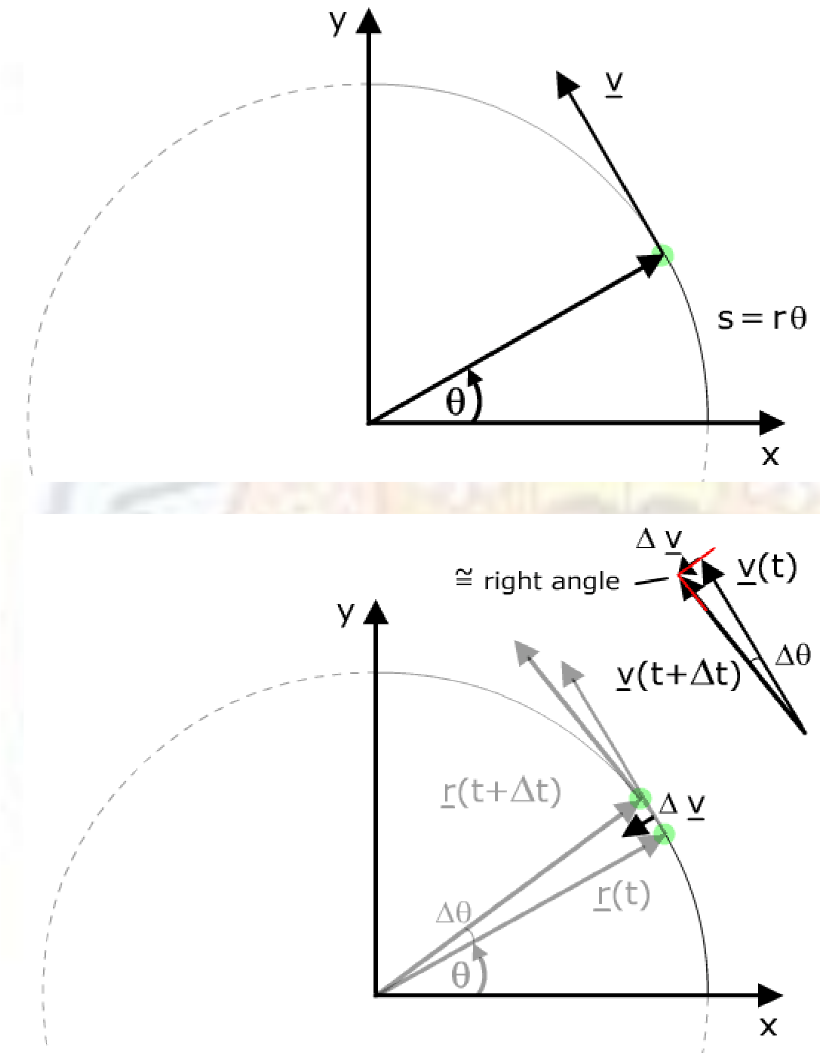
In this form the equation represents D' Alembert's principle. The quantity represented by $m\vec{a}$ is now called the reversed effective force. If a body is accelerated by a system of forces, a fictitious force, which is equal to the product of the mass multiplied by the acceleration and in a direction opposite to the acceleration, may be added to the applied forces and the resulting equation will be an equation of equilibrium. When the motion of the body is translational one, the reversed effective force is applied at the center of mass of the body. When the body has an angular as well as linear acceleration, a reversed effective force must be applied to each particle of the body. The resultant of the reversed effective forces on the whole body is the equivalent of the actual external force system acting on the body and in general does not act along a line passing through the center of mass. In rotating bodies, the radial component of the reversed effective force is often called "centrifugal force".

Centripetal Acceleration and Centripetal Force

Circular motion always involves an acceleration, which is directed towards the center of the circular motion called Centripetal acceleration.



For animations and details kindly click on the link given below:

http://www.animations.physics.unsw.edu.au/jw/circular.htm#calculating

Credits: Authored and Presented by Joe Wolfe

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Centrifugal Force

Summary

1. A body is having rotation about a fixed axis in which all points on the axis remain stationary during the motion, while other points in the body move in circles concentric with the axis and in planes perpendicular to it. Such a motion is known as circular motion.

2. If a stone is tied to a cord, and whirl the stone in a circle. While the stone is revolving it can be felt to pull outward on one's hand, and equally an inward pull on the stone must exist.

3. A small body of mass *m* is attached to the pin by a cord of length *R*, and set revolving about it with an angular velocity ω, a tangential velocity v_T, then radial acceleration will be $a_R = \frac{v_T^2}{R} = \omega^2 R$.

4. According to Newton's second law, a force must be exerted on the body to produce the radial acceleration, and it will act towards the centre of the circle. It is named as a central or centripetal force. It is given by

$$F = \frac{mv_T^2}{R} = m\omega^2 R$$

5. An inward force will come into account due to the cord, which is mainly in tension and therefore exert an outward force, equal and opposite to the centripetal force, on the pin at the center. This outward force is called a centrifugal force. (The term "centrifugal" means literally "fleeing a center").

6. Both centripetal and centrifugal forces always constitute an action-and-reaction pair, the former being the resultant inward force on the revolving body and the latter the reaction to this force.

7. Centripetal forces, like other forces, are pushes or pulls exerted on some material body by some other material body, and their designation as "centripetal" refers only to the effect they produce (a change in direction) and not to something inherently different in their nature.

8. If a body is accelerated by a system of forces, a fictitious force which is equal to the product of the mass multiplied by the acceleration and in a direction opposite to the acceleration may be added to the applied forces and the resulting equation will be an equation of equilibrium.

9. When the motion of the body is translational one, the reversed effective force is applied at the mass center of the body. When the body has an angular as well as linear acceleration, a reversed effective force must be applied to each particle of the body.

10. The resultant of the reversed effective forces on the whole body is the equilibrant of the actual external force system acting on the body and in general does not act along a line passing through the mass center. In rotating bodies, the radial component of the reversed effective force is often called "centrifugal force".

Centrifugal Force

Some illustrative examples

Ex1 Obtain the centrifugal force exerted by 4000lbs. van while rounding a curve of 100 ft. radius at 40 miles an hour.

Sol Velocity=40 m.p.h=176/3 ft./sec

Centrifugal force= $\frac{mv^2}{r} = \frac{4000 \times 176 \times 176}{9 \times 100 \times 32} = 4302 \;\; lbs\, wt.$

Ex2 An automobile moving with a constant speed of 15 miles an hour takes a turn around a circular path of radius 10 ft. Find the inclination to the vertical?

Sol. Say the automobile leans at an angle θ. to the vertical. The reaction of the ground also acts along the same line inclined θ to the vertical. This reaction is split into two components, the vertical component which is balancing its weight but the horizontal component provides the centripetal force required to keep the automobile moving along in a circle. Suppose *T* poundals is the reaction of the ground on the body of mass *m* moving in a circle of radius *r*. Then

$$T\, Cos\theta = mg$$

$$T\, Sin\theta = \frac{mv^2}{r}$$

Dividing $\tan\theta = \frac{v^2}{rg} = \frac{22^2}{10 \times 32} = 1.51$ [15 miles/hr.=22ft/sec.] $\theta = \tan^{-1}(1.51)$

Ex3 The roadway bridge over a canal is in the form of an arc of radius 200ft. Calculate the maximum speed with which a car can cross the bridge without leaving the ground at the highest point?

Sol. The car while crossing the curved bridge exerts a centrifugal force $\frac{mv^2}{r}$ due to which it tends to leave the ground at the highest point. But if the centrifugal force just becomes equal to the weight of the car acting vertically below towards the centre of curvature of the curved canal bridge, the car will stick to the ground, so

Centrifugal force = weight of the car

$\frac{mv^2}{r} = mg$ i.e. $\frac{mv^2}{200} = mg$ (In F.P.S. g=32ft/s^2)

Or $v^2 = 200 \times 32$

$$v^2 = 6400$$

$v = 80$ ft/sec

Ex4 The following Figure 2.5 shows a smooth straight tube moving around horizontal plane about a vertical axis through *C*. A mass coming out through the tube has a velocity v_p at a distance *p* from *C*. Find the velocity v_q at the corner of the tube at a distance *q* from *C* if the tube rotates uniformly at $\omega rad/\sec$. Calculate the pressure between the mass and the tube just before the mass emerges.

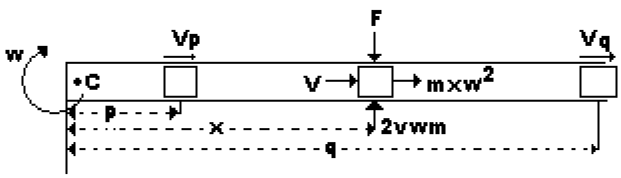

Fig. 2.5

Sol. Let the reference frame revolves with the tube about the center *C*. the tube is then a straight radial line in the frame. A free-body diagram of the body at *x* distance from *C* shows a centrifugal force $mx\omega^2$ acting outwards along the tube, a Coriolis force $2mv\omega$ acting laterally on the body. The weight of the body downward is balanced by an upward pressure from the tube wall. The body moves in a straight line and has no acceleration normal to the tube. Therefore, $F = 2mv\omega$. The force $mx\omega^2$ accelerates the mass along the tube. The acceleration is not constant,

and the following energy equation can be used

$$\frac{mv_p^2}{2} + m\omega^2 \int_p^q x dx = \frac{mv_q^2}{2} \qquad (a)$$

The integral is easily evaluated as $\frac{m\omega^2(q^2 - p^2)}{2}$, or the same result can be obtained by noting that the centrifugal force varies uniformly from *p* to *q* like the force of a linear spring so that the mean value of the initial and the final centrifugal forces is the average value.

Evaluating the integral and solving for v_q,

$$v_q = \sqrt{v_p^2 + \omega^2 (q^2 - p^2)} \qquad (b)$$

The force *F* equals the coriolis force, and at *q*, it is given by

$$F_q = 2m\omega\sqrt{v_p^2 + \omega^2 (q^2 - p^2)} \qquad (c)$$

Questions For Practice

1. Explain the centrifugal forces.

2. Calculate the values of centrifugal force on a mass of 45g placed at a distance of 5cm from the axis of a rotating frame of reference, if the angular speed of rotation of the frame be 25rad/sec.

3. Obtain the centrifugal force exerted by 2000lbs. car while rounding a curve of 300 ft. radius at 20 miles an hour.

References/ Bibliography/ Further Reading References

1. “Principles of mechanics” John L.Synge and Byron A. Griffith

2. “Mechanics” Boyd and Ott Third Edition

3. “University Physics” Sears F.W., Zemansky

4. “Numerical examples and Problems” Chatterjee

5. “Mechanics” D.S.Mathur, revised by P.S. Hemne

6. “Physics” Resnick, Halliday and Walker