Discipline Course-I

Semester -I

Paper: Mechanics IB

Lesson: Collision and Centre of Mass System

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Ch.7. Collision and Centre of Mass System

1. Collisions.
2. Inelastic collisions.
3. Elastic collisions of particle.
4. Motion of Centre of mass.
5. Total linear momentum about the Centre of mass.
6. Two body Problem-equivalent one body problem.
7. Summary.
8. Exercise.

Objective

After studying this chapter you will understand:

- The phenomena of collision and types of collision
- How the law of conservation of linear momentum is applied for the collision problem
- The Inelastic collision and its application and calculation of the loss of kinetic energy
- The one –dimensional elastic collision in lab frame
- The two- dimensional elastic collision in the lab frame and special cases of collision of two particles
- The motion of the Centre of mass
- The calculation of linear momentum of Centre of mass
- The two body problem and how can the two body problem is reducible to an equivalent one body problem

Collision and Centre of Mass System

1. Collision

Collision is the phenomena in which two particles exchange their momentum and kinetic energy. When there is the loss of kinetic energy during the collision, we call this type of collision as inelastic collision. When the kinetic energy remains conserved in the collision, we call this type of collision as elastic collision. The linear momentum always remains conserved in inelastic as well as elastic collisions.

Let us study collision by some example

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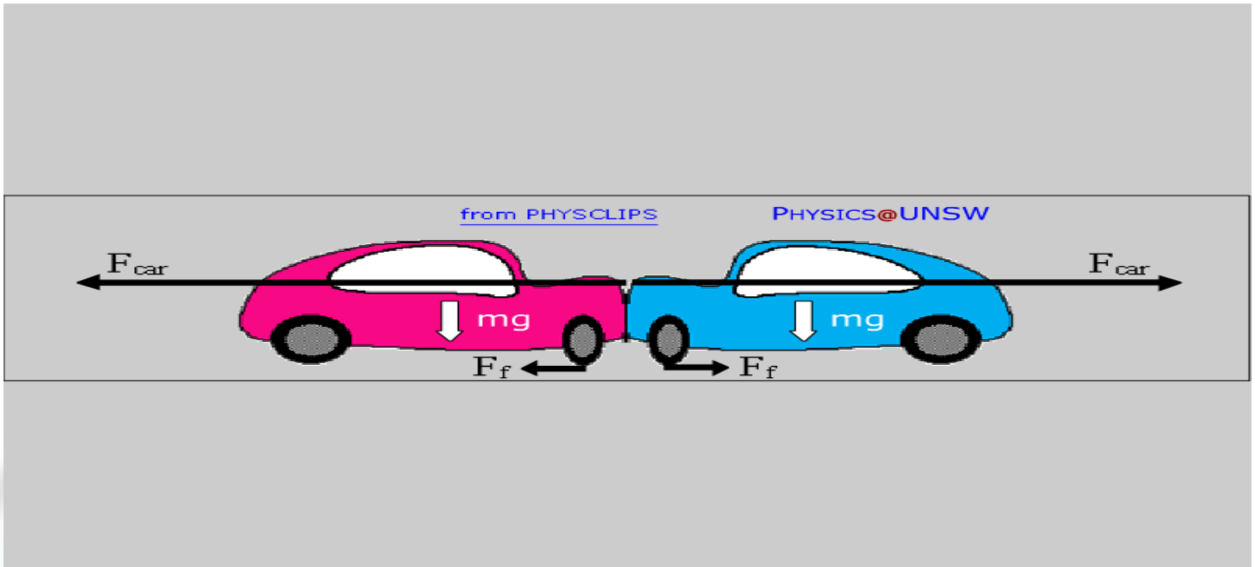

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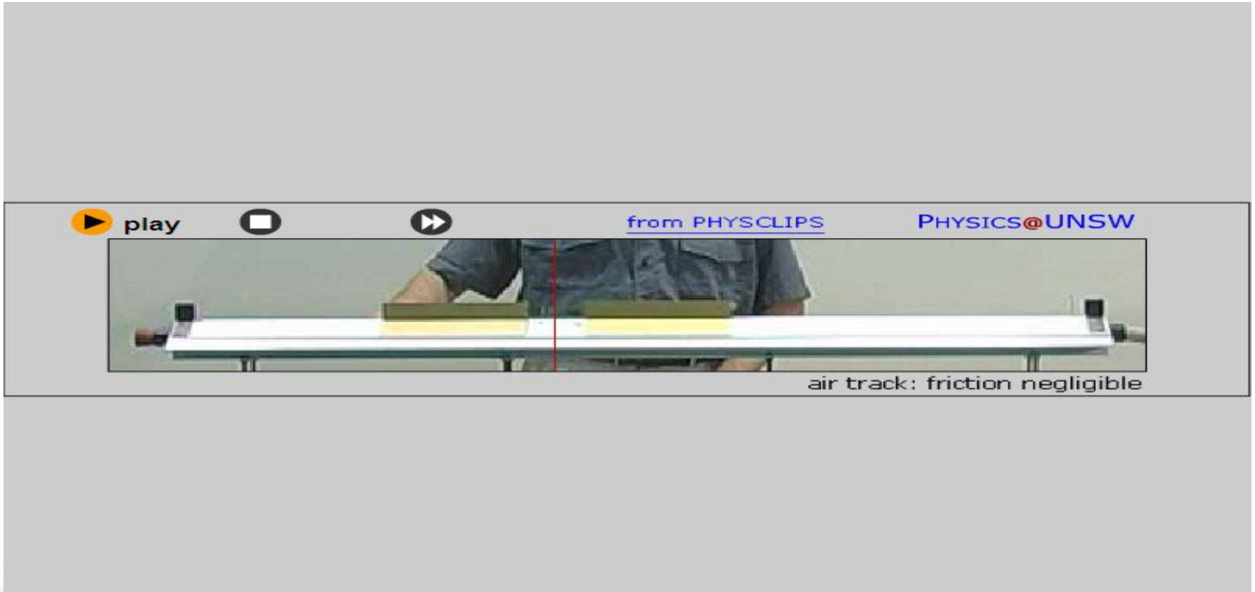

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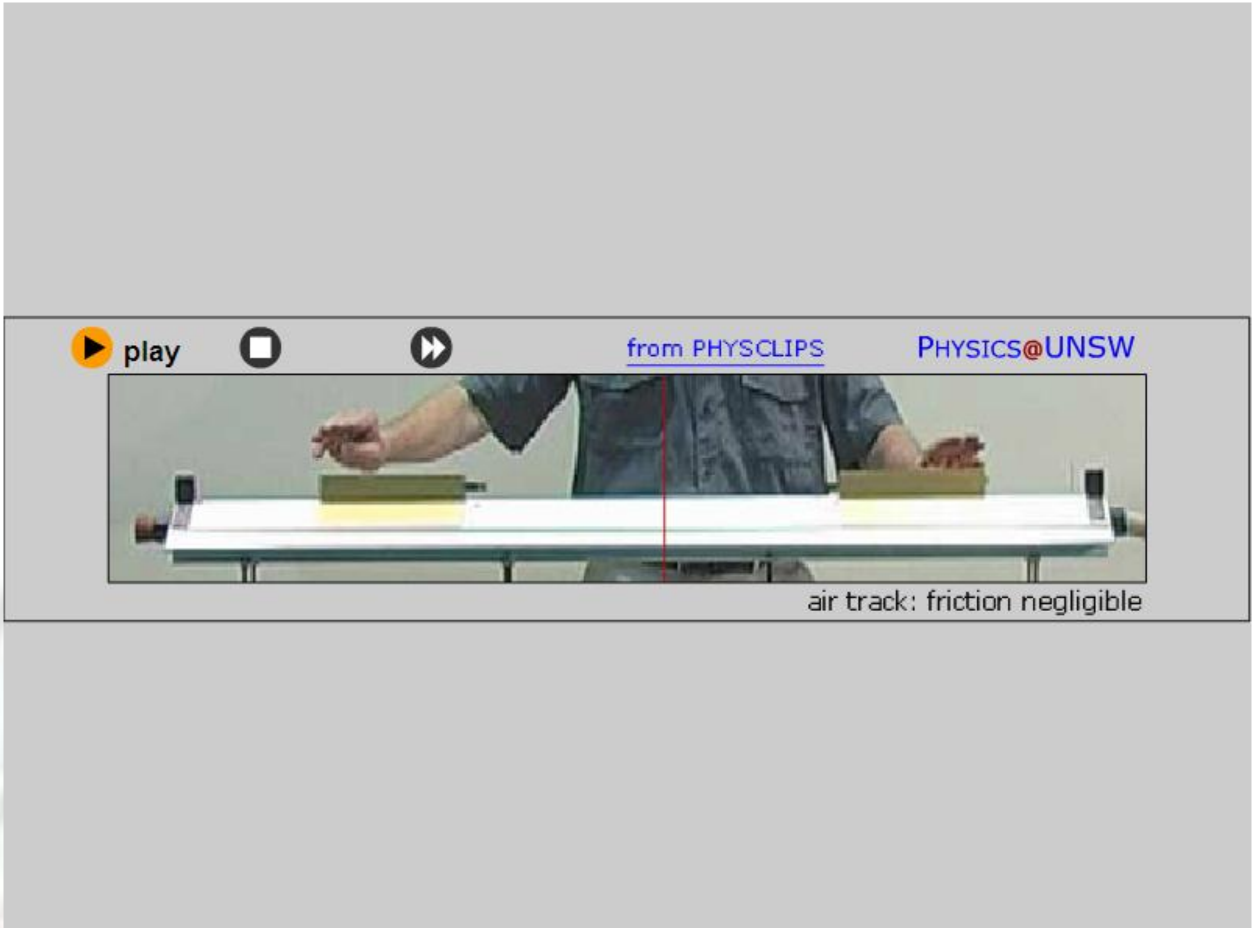

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2. Inelastic Collisions

Now we study inelastic collision in detail. As we have stated earlier in this type of collision only linear momentum is conserved and the kinetic energy is not conserved.

The example of such type of collision is when a bullet is fired on a wall or wooden block and it gets embedded in the block. Suppose the mass of bullet is m and mass of wooden block is M. Let us suppose that initially wooden block is at rest and the bullet is fired with a muzzle velocity **v**. After piercing and embedding in the block the bullet-wooden block system move with a common velocity **V**. Let as assume also that motion here is one-dimensional.

Then applying principle of conservation of linear momentum

Total initial momentum =Total finial momentum

So $m\mathbf{v} + M(0) = (m + M)\mathbf{V}$

So the common velocity is $\mathbf{V} = (m/(m+M))\mathbf{v}$

We can also calculate the loss of kinetic energy as follows:

The initial kinetic energy $K_i = (½)\, m\, v^2$

The finial kinetic energy $K_f = (½)(m+M)V^2$

So the loss of kinetic energy $\Delta K = K_f - K_i = (½)[(m+M)V^2 - mv^2]$

Also $\mathbf{V} = (m/(m+M))\mathbf{v}$

So

$$\Delta K = K_f - K_i = (½)[(m+M)(\, m/(m+M))^2\, v^2 - mv^2\,]$$

$$= (½)[(m^2/(m+M))\, v^2 - mv^2]$$

$$= (1/2)\, [(m^2\, v^2 - m^2\, v^2 - mMv^2)/(m+M)]$$

$$= -(½)(mM/(m+M))v^2\,.$$

Here negative sign shows that change is negative, so there is the loss of kinetic energy during inelastic collision.

We can also calculate fraction change in kinetic energy as

We know fractional change in kinetic energy = change in kinetic energy/initial kinetic energy

So fractional loss $= \Delta K/\, K_i = (\, K_f - K_i\,)/\, K_i$

$$= (½)[(mM/(m+M))v^2]\, /\, (½)\, m\, v^2$$

So

$$\text{Fractional loss in K.E} = \left[\frac{M}{m+M}\right]$$

Hence we see that fractional loss of kinetic energy in inelastic collision of two masses is the ratio of the mass of the stationary body to their combined mass.

For example in our bullet wooden block system if

M=100g   m=10g   v=200m/s

Then

$\mathbf{V}$ = (10/110)X 200 m/s = 18.18 m/s

Initial kinetic energy = (1/2) X10X10^{-3} X200^2 =200J

Finial kinetic energy =(1/2) X 110 X 10^{-3} X 18.18^2 =18.178 J

And

Loss in kinetic energy   =-(½)(mM/(m+M))v^2 =-(1/2) (1000X10^{-6}/110X10^{-3})X40000

=-181818.18X10^{-3} J =-181.82 J

And the fractional loss in K.E = loss / initial energy = 181.82/ 200=0.9091.

Also from the formula of the fraction loss, we have

Fractional loss $=\left[\frac{M}{m+M}\right]$ = 100/110 =0.90909.

The two answer match as they should be.

3. Elastic collision

Now we study the elastic collision of two particles. As we know in this type of collision both, the linear momentum and total kinetic energy remains conserved.

Let us first study one-dimensional elastic collision and later we move on to two-dimensional case.

Let us consider collision of two particles of mass m and M let us also assume that the heavy mass is initially at rest for simplicity.

Now **u** be the initial velocity of mass m and **v** and **V** be the finial velocities of mass m and M after the collision respectively, then the conservation of linear momentum demands

$m\mathbf{u} + M(0) = m\mathbf{v} + M\mathbf{V}$

or $m\mathbf{u} = m\mathbf{v} + M\mathbf{V}$ (1)

so

$\mathbf{V} = m(\mathbf{u}-\mathbf{v})/ M$

Also conservation of kinetic energy demands

$(½)mu^2 = (½)[mv^2 + MV^2]$

So

$mu^2 = [mv^2 + MV^2]$ (2)

Now multiplying eq (2) by m, we have

$m^2u^2 = m^2v^2 + mMV^2$ (3)

And taking square of eq (1), we have

$m^2u^2 = m^2v^2 + M^2V^2 + 2mM\mathbf{v}.\mathbf{V}$ (4)

Now comparing eq. (3) and eq. (4), we have

$mMV^2 = M^2V^2 + 2mM\mathbf{v}.\mathbf{V}$

or

$V^2 (mM - M^2) = 2mM\mathbf{v}.\mathbf{V}$

Or

$\mathbf{V} = [2m/(m-M)]\ \mathbf{v}$ (5)

Now we write expression for **v** and **V** in terms of **u**

From eq. (1), we have

$\mathbf{v} = \mathbf{u} - (M/m)\mathbf{V}$

so from eq. (5)

$\mathbf{v} = \mathbf{u} - (M/m)[2m/(m-M)]\ \mathbf{v}$

$\mathbf{v}[1 + (2M)/(m-M)] = \mathbf{u}$

or

$\mathbf{v} = [(m-M)/(m+M)]\ \mathbf{u}$ (6)

also from eq. (5) and (6), we have

$\mathbf{V} = [2m/(m-M)]\ \mathbf{v} = [2m/(m-M)][(m-M)/(m+M)]\ \mathbf{u}$

So

$\mathbf{V} = [2m/(m+M)]\mathbf{u}$ (7)

So we have finial velocities of the two particles given by

Now we study some special cases:

(1) When m=M, we have **v**= 0 and **V**=**u**, so the particle exchange their velocities with each other after the collision.

(2) When $m \ll M$, we have **v** ≅ - **u** and **V**= 0, so the lighter particle just bounce back and the heavier particle remains stationary.

(3) When $m \gg M$, the moving particle is much heavier then **v** ≅ **u** and **V** ≅ 2**u**, so the lighter particle moves twice the speed of heavier particle and the heavier particle continues its motion with the same speed.

Now we consider the motion of both particles before the collision in one-dimension (hence the vector sign is being omitted on the velocity). So we have the initial velocities as u and U of masses m and M respectively. Also let v and V are their finial velocities after the collision. Then conservation of linear momentum equation:

$$mu + MU = mv + MV \qquad (1)$$

or

$$m(u - v) = M(V-U) \qquad (2)$$

and conservation of kinetic energy demands

$$(½)\, mu^2 + (½)\, MU^2 = (½)mv^2 + (½)MV^2 \qquad (3)$$

Or

$$mu^2 + MU^2 = mv^2 + MV^2 \qquad (4)$$

or

$$m(u^2 - v^2) = M(V^2 - U^2) \qquad (5)$$

now dividing eq. (5) by eq. (2), we get

$$u+v = U+V$$

or

u-U = V-v (6)

so the eq.(6) shows that in one dimensional elastic collision, the relative velocity with which the two particles approach each other before the collision is same as the relative velocity with which the two particles move away from each other after the collision.

Now we wish to express final velocities of the two particles in terms of their initial velocities. For this we write eq. (6) as

V= v+u-U (7)

And

v= V+U – u (8)

And using (7) and (8) in (2), so we have

v=[(m - M)/(m+M)]u + [2M/(m+M)]U

V= [2m/(m+M)]u +[(M - m)/(m+M)]U

Now we have special cases:

(1) When m=M, v= U and V=u, as expected the particle exchanges their speeds.
(2) When $m \ll M$, so v= -u + 2U and V= U.
(3) When fast moving particle is heavier or $m \gg M$ so v= u and V=2u- U.

We will study 2 and 3 dim collision in the next chapter when we study them in centre of mass frame of reference, since in the lab frame these problems are hard to solve.

4.Motion of Centre of mass.

Now having understood the concept of Centre of mass, we study the motion of the Centre of mass, for this we assume a system of N–particles so that their inter-particle distance remains constant as the system moves. Also let M be the total mass of the system and it remains constant during the motion. So according to the definition of the Centre of mass, we have

$$\mathbf{R}_{C.M} = \frac{m_1\mathbf{R}_1+m_2\mathbf{R}_2+m_3\mathbf{R}_3+\cdots+m_n\mathbf{R}_n}{m_1+m_2+m_3+\cdots+m_N} = \sum_{i=1}^{N} m_i\mathbf{R}_i/M$$

Or $M\mathbf{R}_{C.M} = m_1\mathbf{R_1} + m_2\mathbf{R_2} + m_3\mathbf{R_3} + \ldots + m_i\mathbf{R_i}$

Let us take the differentiation of above equation

$M\,(d\mathbf{R}_{C.M}/dt) = m_1(d\mathbf{R_1}/dt) + m_2(d\mathbf{R_2}/dt) + \ldots + m_N(d\mathbf{R_N}/dt)$

Now denoting the velocity of Centre of mass by $\mathbf{V}_{C.M}$, and velocities of individual particles by $\mathbf{V}_1, \mathbf{V_2}, \mathbf{V}_3, \ldots \mathbf{V_N}$, respectively. We have

$M\mathbf{V}_{C.M} = m_1\mathbf{V}_1 + m_2\mathbf{V}_2 + m_3\mathbf{V}_3 + \ldots + m_n\mathbf{V}_n = \sum_i^N m_i\mathbf{V}_i$

So the velocity of centre of mass , $\mathbf{V}_{C.M}$, is given by

$$\mathbf{V}_{C.M} = (1/M)\sum_i^N m_i\mathbf{V_i}$$

Also we have

$$M\mathbf{V}_{C.M} = m_1\mathbf{V}_1 + m_2\mathbf{V}_2 + m_3\mathbf{V}_3 + \ldots + m_n\mathbf{V}_n = \mathbf{P}_{c.m} = \mathbf{P}$$

Or $M\mathbf{V}_{C.M}$ = **TOTAL LINEAR MOMENTUM OF THE SYSTEM**

So

$$\mathbf{V}_{C.M} = \mathbf{P}/M$$

So the velocity of Centre of mass is the total linear momentum of the system of particles divided by the total mass of the system of particles. Or in other words, the total linear momentum of the system is the product of the total mass of the system and the velocity of the Centre of mass.

Now for an isolated system of particles, when no external force is acting on the system, we know that the total linear momentum **P** remains constant. Hence

$\mathbf{P} = \mathbf{P}_{C.M} = M\,\mathbf{V}_{C.M} = \mathbf{constant}$, since total mass is also constant,

So

$$\mathbf{V}_{C.M} = \text{constant}$$

So, if **no external force is acting on the system, the velocity of centre of mass is constant.**

We can similarly find the expression for acceleration of Centre of mass, when we take the derivative of the velocity of Centre of mass equation, we have

$M\,d\mathbf{V}_{C.M}/dt = m_1 d\mathbf{V_1}/dt + m_2 d\mathbf{V_2}/dt + m_3 d\mathbf{V_3}/dt + \ldots + m_N d\mathbf{V_N}/dt$

Or $M\,\mathbf{a}_{c.m} = m_1\mathbf{a_1} + m_2\mathbf{a_2} + m_3\mathbf{a_3} + \ldots + m_N\mathbf{a_N} = (\sum_i^N m_i\mathbf{R_i})$

Or the acceleration of the Centre of mass is given by

$\mathbf{a}_{C.M} = (\sum_i^N m_i \mathbf{a}_i)/M$

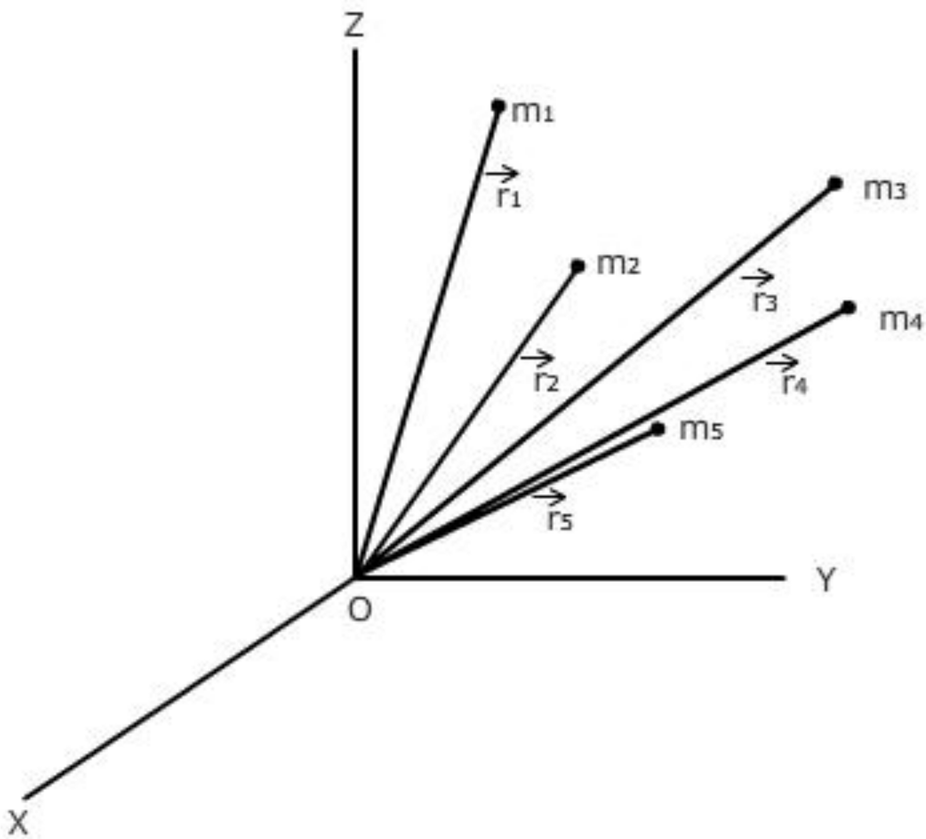
also from the Newton's second law, we have

$M\,\mathbf{a}_{C.M} = m_1\,\mathbf{a}_1 + m_2\,\mathbf{a}_2 + m_3\,\mathbf{a}_3 + \ldots + m_N\,\mathbf{a}_N = (\sum_i^N m_i\,\mathbf{a}_i) = \sum_i^N \mathbf{F}_1 = \mathbf{F}_{external}$

So, we have the product of total mass of the system of particles and the acceleration of Centre of mass is equal to the total external force acting on the system. When the external force is zero, acceleration is zero. Hence the velocity of Centre of mass is constant as expected.

5.Total linear momentum about the Centre of mass.

Let us now consider system N particles, assuming its Centre of mass C at the position vector $\mathbf{R}_{C.M}$ with respect to some inertial frame of reference as shown in the figure below.



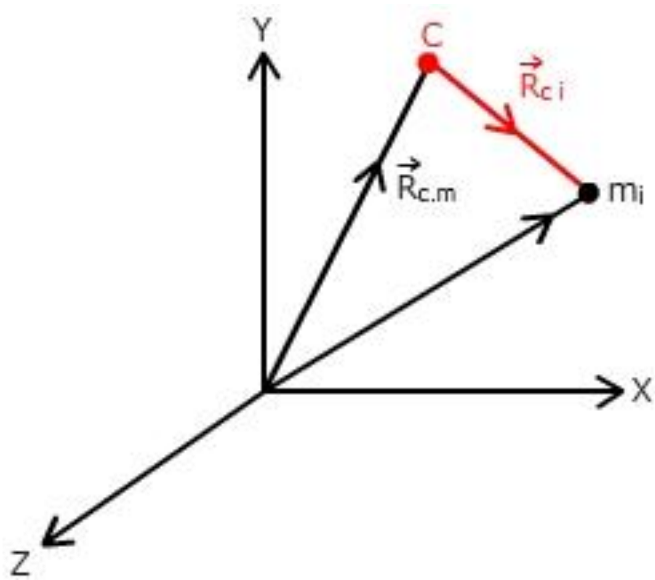
Now consider there are N particles in a system having masses m_1, m_2, m_3,…m_N, respectively. Now if the position vectors of each particle is given by $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \ldots, \mathbf{R}_N$ respectively.

The position vector of Centre of mass is

$$\mathbf{R}_{C.M} = \frac{m_1\boldsymbol{R}_1 + m_2\boldsymbol{R}_2 + m_3\boldsymbol{R}_3 + \cdots + m_N\boldsymbol{R}_N}{m_1 + m_2 + m_3 + \cdots + m_N} = \sum_{i=1}^{N} m_i \mathbf{R}_i / M$$

Collision and Centre of Mass System

Let us consider a specific mass particle m_i at the position $\mathbf{R_i}$, as shown in figure below:



Now the position vector of this mass m_i with respect to the Centre of mass C is $\mathbf{R_{Ci}}$ and is written as

$$\mathbf{R_{Ci} = R_i - R_{C.M}}$$

Or $$\mathbf{R_i = R_{C.M} + R_{Ci}}$$

For all other masses we can have similar expressions for the position vectors with respect to the Center of mass C.

Now we have $$M\,\mathbf{R_{C.M}} = \sum_{i=1}^{N} m_i\,\mathbf{R_i}$$

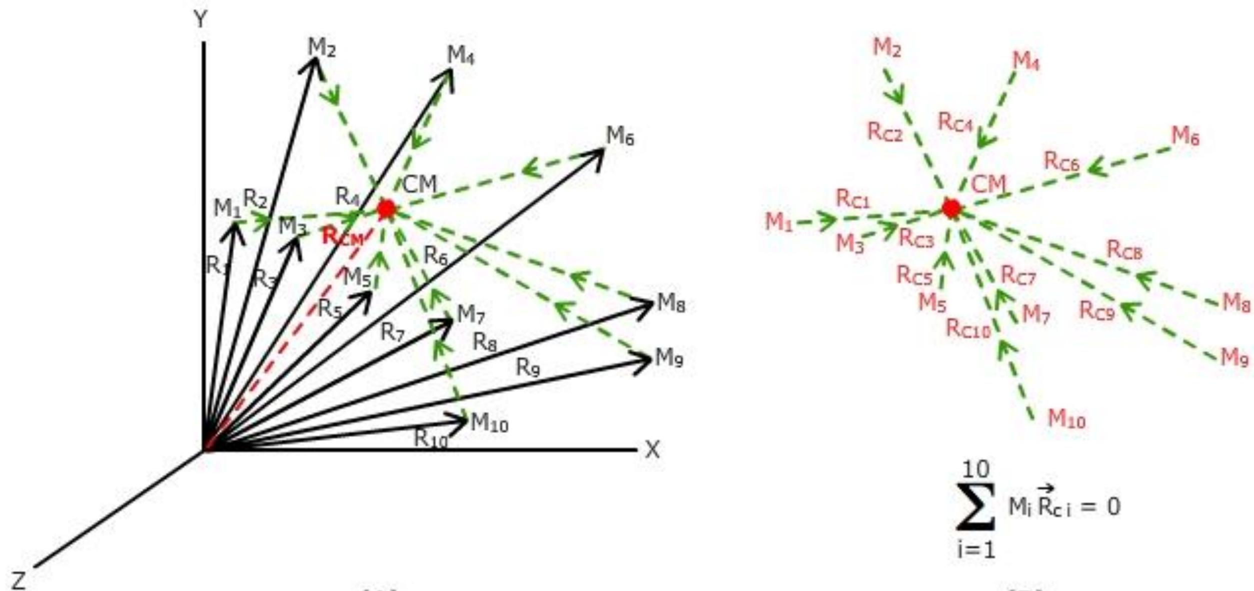
Or $$M\,\mathbf{R_{C.M}} = \sum_{i=1}^{N} m_i\,(\mathbf{R_{C.M} + R_{Ci}})$$

Or $$M\,\mathbf{R_{C.M}} = \sum_{i=1}^{N} m_i\,\mathbf{R_{C.M}} + \sum_{i=1}^{N} m_i\mathbf{R}c_i$$

Now $$\sum_{i=1}^{N} m_i\,\mathbf{R_{C.M}} = M\,\mathbf{R_{C.M}} \quad \text{since } \sum_{i=1}^{N} m_i = M.$$

So $$M\,\mathbf{R_{C.M}} = M\,\mathbf{R_{C.M}} + \sum_{i=1}^{N} m_i\,\mathbf{R_{Ci}}$$

Hence $$\sum_{i=1}^{N} m_i\,\mathbf{R_{Ci}} = 0$$



(A) (B)

Or, we can write it explicitly:

"The sum of the product of the position vector with respect to the Centre of mass of all the particles of a system of N particles and their respective masses is zero."

This is a very important result, since if we take the differentiation of this equation, we have

$\sum_{\mathbf{i=1}}^{\mathbf{N}} \mathrm{m_i}$ $\mathbf{dR}c_{\mathrm{i}}$ **/dt** $=\sum_{\mathbf{i=1}}^{\mathbf{N}} \mathrm{m_i}$ $\mathbf{V}c_{\mathrm{i}}$ = 0.

Here $\mathbf{V}c_{\mathrm{i}}$ **=** $\mathbf{dR}c_{\mathrm{i}}$ **/dt** is the velocity of ith particle with respect to the Centre of mass.

Now the summation of the product of mass mi and its velocity $\mathbf{V}c_{\mathrm{i}}$, $\sum_{\mathbf{i=1}}^{\mathbf{N}} m_{\mathrm{i}} \mathbf{V}c_{\mathrm{i}}$,

Which give the total linear momentum $\mathbf{P_C} = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{P}_{c_{\mathrm{i}}} = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathrm{mi}\, \mathbf{V}c_{\mathrm{i}}$ of all the particles about the Centre of mass. Hence

$$\mathbf{P_C = 0}$$

"The total linear momentum of the system of N particles with respect to the Centre of mass frame of reference is zero."

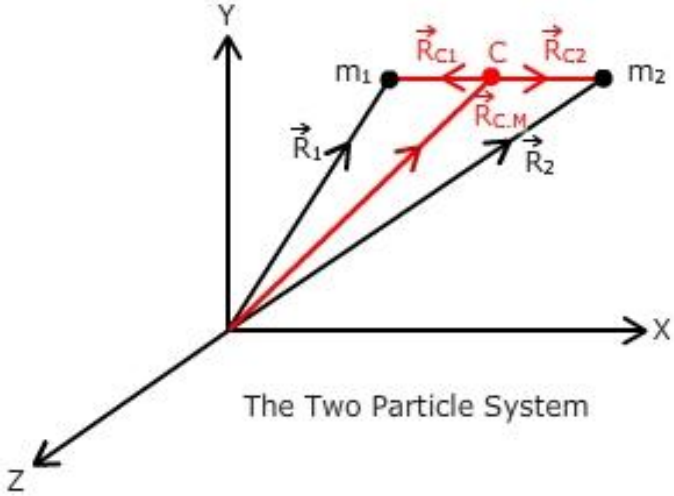
So due to this result we can say that the Centre of mass frame of reference is a zero-momentum frame of reference. In next chapter we will use this result to solve the collision problem in the Centre of mass frame of reference.

6. Two body Problem-equivalent one body problem

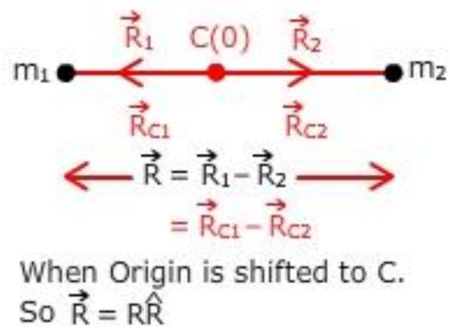
Collision and Centre of Mass System

Now we study motion of two particles explicitly and find out some important relations, we will then reduce the two-particle problem into equivalent one body problem.

Consider two particles of mass m_1 and m_2, having position vectors $\mathbf{R_1}$ and $\mathbf{R_2}$, respectively with respect to an inertial frame of reference as shown in the figure.



The Two Particle System



Now the position of Centre of mass is given by

$\mathbf{R_{C.M}} = (m_1\mathbf{R_1}+m_2\mathbf{R_2})/(m_1+m_2)$

We have the position vectors of the two masses with respect to the Centre of mass frame of reference is given by

$$\mathbf{R_{C1}} = \mathbf{R_1}-\mathbf{R_{C.M}} \quad \text{and} \quad \mathbf{R_{C2}} = \mathbf{R_2}-\mathbf{R_{C.M}}$$

Now if we shift our origin of Co-ordinate axes at the Centre of mass, then

We have $\mathbf{R_{C.M}} = 0$,

So $\mathbf{R_{C1}} = \mathbf{R_1}$ $\quad$ $\mathbf{R_{C2}} = \mathbf{R_2}$

And $m_1\,\mathbf{R}_1 + m_2\,\mathbf{R}_2 = \mathbf{0}$

Or $m_1/m_2 = -\,\mathbf{R_2}/\mathbf{R_1}$

Hence, the Centre of mass of the two-particle system divides the straight line joining the Centre of two masses in the inverse ratio of the two masses m_1 and m_2. So the heavy mass m2 lies nearer to the Centre of mass position $\mathbf{R_{C.M}}$.

Now expression for the velocity of the Centre of mass is given by

$\mathbf{V_{C.M}} = (m_1\mathbf{V_1}+m_2\mathbf{V_2})/(m_1+m_2)$

Similarly the expression for the acceleration of the centre of mass is given by

$$\mathbf{a_{C.M}} = (m_1\mathbf{a_1}+m_2\mathbf{a_2})/(m_1+m_2)$$

Now the total linear momentum of the two particles is given by

$$\mathbf{P} = m_1 \mathbf{V_1} + m_2 \mathbf{V_2}$$

$$= (m_1+m_2)\mathbf{V_{C.M}}$$

$$= M\, \mathbf{V_{C.M}}$$

$$= \mathbf{P_{C.M}}.$$

Hence,

$\mathbf{P} = \mathbf{P_{C.M}}$

So the total linear momentum of the two-particle system is equal to the linear momentum of the Centre of mass. So this result suggests that the two particles can be replaced by a single particle situated at the Centre of mass position.

Now we wish to reduce this two particle system into an equivalent one body system.

Suppose there is no external force acting on the system of two particles under consideration, but only the internal forces. Then from previous discussions the velocity of the Centre of mass is constant. So

$\mathbf{V_{C.M}} = (m_1\mathbf{V_1}+m_2\mathbf{V_2})/(m_1+m_2)$ = constant

Also, as the Centre of mass lies on the line joining the centers of the two masses, the force acting on the first particle due to the second particle, $\mathbf{F_{12}}$ and the force acting on the second particle due to the first, $\mathbf{F_{21}}$ is directed towards the Centre of mass. Hence the two internal forces are the central forces.

Now we have the position vectors of the two masses with respect to the Centre of mass frame of reference is given by

$\mathbf{R_{C1}} = \mathbf{R_1} - \mathbf{R_{C.M}}$ and $\mathbf{R_{C2}} = \mathbf{R_2} - \mathbf{R_{C.M}}$

Or

$\mathbf{R}_{c_1} - \mathbf{R}_{c_2} = \mathbf{R_1} - \mathbf{R_2} = \mathbf{R}.$

Now the force on mass m_1 is

$$\mathbf{F_{12}} = \mathbf{F(R)} = f(R)\,\hat{\mathbf{R}}$$

And the force on mass m_2 is

$$\mathbf{F_{21}} = -\mathbf{F_{12}} = -\mathbf{F(R)} = -f(R)\,\hat{\mathbf{R}}.$$

From Newton's second law

$$m_1\ddot{\mathbf{R}}_1 = \mathbf{F_{12}} = f(R)\,\hat{\mathbf{R}} \quad \text{or} \quad \ddot{\mathbf{R}}_1 = (1/m_1)\, f(R)\,\hat{\mathbf{R}}$$

And

$m_2\ddot{\mathbf{R}}_2 = \mathbf{F}_{21} = -f(R)\,\hat{\mathbf{R}}$ or $\ddot{\mathbf{R}}_2 = -(1/m_2)\, f(R)\,\hat{\mathbf{R}}$

so from above two equations

$\ddot{\mathbf{R}}_1 - \ddot{\mathbf{R}}_2 = [\,(1/m_1) + (1/m_2)\,]\, f(R)\,\hat{\mathbf{R}}$

Now $\ddot{\mathbf{R}}_1 - \ddot{\mathbf{R}}_2 = \ddot{\mathbf{R}}$ (since $\mathbf{R}_{C1} - \mathbf{R}_{C2} = \mathbf{R}_1 - \mathbf{R}_2 = \mathbf{R}$)

Also $\frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1+m_2}{m_1 m_2} = \frac{1}{m}$

So defining a single mass particle of mass m (also called as the reduced mass m) of the two masses m_1 and m_2 as m, where m is given by

$m = \frac{m_1 m_2}{m_1+m_2}$

So we can have our equation of motion of two-particle system in terms of the reduced mass m as

$$m\,\ddot{\mathbf{R}} = f(R)\,\hat{\mathbf{R}}$$

so the above equation looks like as that of a single particle of mass m having position vector $\mathbf{R}_{C.M}$ under the action of a force $f(R)\,\hat{\mathbf{R}}$

so we have reduced a two particle problem in to **an equivalent one particle problem with reduced mass m .**

7. **Summary**

- The collision phenomena involve exchange of linear momenta of the particles.
- The total linear momentum remains conserved in elastic as well as inelastic collisions.
- The total kinetic energy remains conserved in elastic collisions only.
- The finial velocity in completely inelastic collision is given by is

$$\mathbf{V} = (m/(m+M))\mathbf{v}$$

- The finial velocity of two particles in terms of their initial velocities is given by

$$v=[(m - M)/(m+M)]u + [2M/(m+M)]U$$

$$V= [2m/(m+M)]u +[(M - m)/(m+M)]U.$$

- The Centre of mass of a discrete system of particles is defined as

$$\mathbf{R}_{C.M} = \frac{m_1\boldsymbol{R_1}+m_2\boldsymbol{R_2}+m_3\boldsymbol{R_3}+\cdots+m_N\boldsymbol{R_N}}{m_1+m_2+m_3+\cdots+m_N}$$

$=\sum_{i=1}^{N} M_i\mathbf{R_i}/M$, Where M is the total mass of the system of particles

- For a continuous distribution of mass, we have

$$\mathbf{R}_{C.M} = (1/M)\int \mathbf{R}\,dm$$
$$= (1/M)\int \mathbf{R}\rho dV.$$

- The velocity of Centre of mass when the system of particles is moving is given by

$$\mathbf{V}_{C.M} = (1/M)\sum_{i}^{N} m_i\mathbf{v_i}$$

- If no external force is acting on the system, the velocity of Centre of mass is constant.

- The sum of the product of the position vector with respect to the Centre of mass of all the particles of a system of N particles and their respective masses is zero.

- The total linear momentum of the system of N particles with respect to the Centre of mass frame of reference is zero.

- The equation of motion of two particle system in terms of the reduced mass m as

$$m\ddot{\mathbf{R}} = f(R)\,\hat{\mathbf{R}}, \quad \text{where } \ddot{\mathbf{R}}_1 - \ddot{\mathbf{R}}_2 = \ddot{\mathbf{R}} \quad \text{and} \quad m = \frac{m_1m_2}{m_1+m_2}$$

8. Exercise.

Collision and Centre of Mass System

Q1. The two particles have same mass. If one of them collide the other which is at rest, with a speed of 10m/s. find the speeds of them after the collision.

Q2. A wooden block of 50g is suspended by a rope 10m long from the ceiling. A bullet is fired toward the block with a muzzle speed of 200m/s. the bullet get embedded in the block and the embedded system is displaced from it mean position. If the mass of bullet is 10g, find the amplitude of displacement.

Q3. Two balls of 30 and 50 g are moving with speeds of 10m/s and 5m/s respectively. Find their speeds after the collision, assume the collision is elastic.

Q4. Find the ratio of finial and initial kinetic energy of the inelastic collision of two cars of masses 1000kg and 2000kg moving towards each other with the speeds 40m/s and 50 m/s respectively.

Q5. A particle with mass m moving with initial velocity u collide another particle of mass M, at rest, elastically. If the finial speed after the collision of M is V, then show that V is given by

$V=[2mu/(m+M)]$.

Q6. The two particles of masses 10kg and 20 kg are separated by a distance on 1m. find the Centre of mass of the system.

Q7. The electron revolves around a nucleus containing one proton. Find the Centre of mass of the electron-proton system.

Q8. Three equal masses are situated at the vertices of an equilateral triangle of side 2m. Find the position of the Centre of mass.

Q9. Find the Centre of mass of a uniform rod of mass m and length L.

Q5. Centre of mass is at point P(1,2,3) when system consist of masses 3,4 and 5kg. if the Centre of mass shifts to Q(2,4,6) on removing 5kg mass, what was its position?

Q6. The position vectors of two masses of 3kg and 5kg are -2**i-j+k** and **2i+j-k** respectively. Find the position vector of the Centre of mass and its distance from the origin.

Q7. The position vectors of two masses are given by **R1=** t^2 **i+**2**j+**3t **k** and **R2=** $2t^2$ **i+j+**3 **k**, find the position vector of the Centre of mass and the Velocity of the Centre of mass at t=5s. Given masses are m1=2kg and m2=3kg.

Q8. A bomb in flight explodes into two fragments when its velocity is 5**i+j-k.** if the smaller mass m flies with velocity 10**i+**20**j-k** ,find the velocity of larger mass 4m.

Q9. Find the Centre of mass of (1)A solid hemisphere (2) A thin hemispherical shell, of radius r.

Q10. Show that a system of two planets revolving around the sun can be reduced to a one-particle system.

Fill in the blanks:

Q11. The Collision of two particles can be ___________ or in-elastic.

Q12. If no _________________ is acting on the system, the velocity of Centre of mass is constant.

Q13. The sum of the product of the position vector with respect to the Centre of mass of all the particles of a system of N particles and their respective masses is ______________.

Q14. The total linear momentum of the two particle system is equal to_______________.

Q15. The total linear momentum of the system of N particles with respect to the Centre of mass frame of reference is ______________.

State whether following statements are true or false:

Q16. The kinetic energy is conserved in all types of collision.

Q17. The Centre of mass of two particles always remains fixed in their collision.

Q18. The linear momentum in collision remains conserved.

Q19. We can reduce a two body problem in one body problem.

Q20. The total energy remains conserved in all types of collisions.

Choose the most appropriate option for the following:

Q21. The kinetic energy and total linear momentum is conserved in
(A) Eleatic collisions
(B) Inelastic collisions
(C) Both types of collisions.

Q22. The motion a system of particle can be easily described with the help of
(A) The Centre of mass of system
(B) The individual velocity of the system of particles
(C) The individual momentum of system of particles.

Q23. The velocity of Centre of mass is given by

(A) $\mathbf{V_{C.M}} = (1/M) \sum_i^N mi\mathbf{Vi}$

(B) $\mathbf{V_{C.M}} = (1/M)(\sum_i^N mi\mathbf{Vi})^{1/2}$

(C) $\mathbf{V_{C.M}} = [(1/M) \sum_i^N mi\mathbf{Vi}^2]^{1/2}$

Q24. When a particle moving with a velocity u collide with another similar particle at rest, then after the collision

(A) The second particle remains at rest and first particle bounce back with its earlier speed.

(B) They just exchange their velocity as before the collision.

(C) The first particle continues to move with its original velocity and the second particle starts to move with the velocity of first particle in the same direction.

Q25. If a very massive particle collide with a lighter stationary particle, so after the collision

(A) Both the particles move with equal velocity in same direction.

(B) The massive particle bounce back with double speed and lighter particle moves with original speed of massive particle in forward direction.

(C) The massive particle continue to be in same direction with unchanged speed while the lighter particle moves in same direction with double the speed of the massive particle.