Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Electric dipole and its behavior under electric field

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Table of Contents

Learning Objectives

After going through this chapter, the student would be able to

- Define the term 'Electric dipole'.
- Appreciate the role of 'electric dipoles' at the microscopic level.
- Calculate the electric field due to dipole at an axial and at an equatorial point.
- Appreciate that a dipole would experience only a torque in a uniform electric field but both a torque and a force in a (highly) non-uniform field.
- Calculate the torque on a dipole in a uniform electric field.
- Calculate the force on a dipole in non-uniform field.

4.1 Electric Field due to a dipole

Electric dipole

Consider a system of two opposite point charges having same magnitude, separated by a small distance (2a) as shown in figure 3.1.

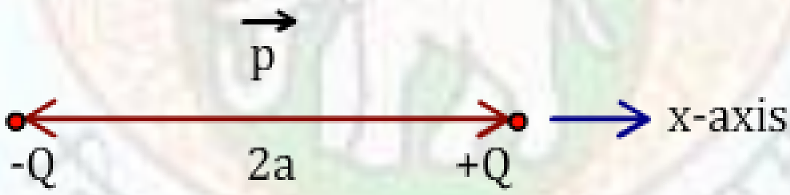


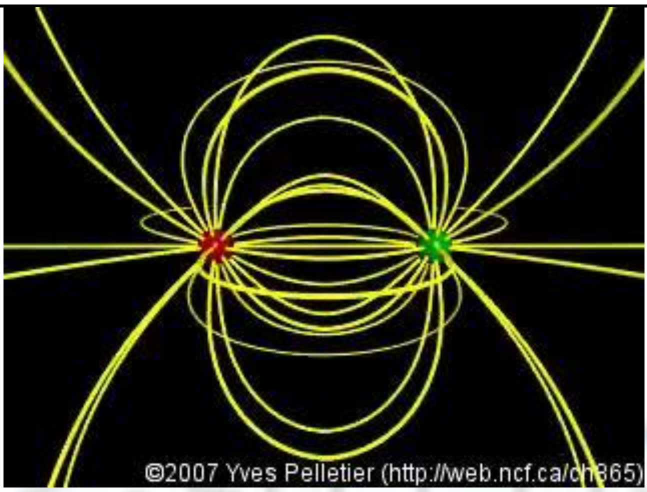
Figure 3.1 Dipole moment $(\vec{p})$ of an electric dipole

It is the product of magnitude of either one of the charges and the distance between them. It is a vector quantity. It points from –Q to +Q.

$$\vec{p} = 2aQ\,\hat{x}$$

Its S.I. units is '(coulomb).(meter)' or C-m.

Value addition: Did you Know
Electric field of a dipole



Both the balls are representing point charges opposite in nature.

Source: http://www.youtube.com/watch?v=bG9XSY8i_q8&feature=fvwrel

(a) Electric field at a point P, present on the axial line of the electric dipole, at a distance r from the center of the dipole (fig. 3.2).

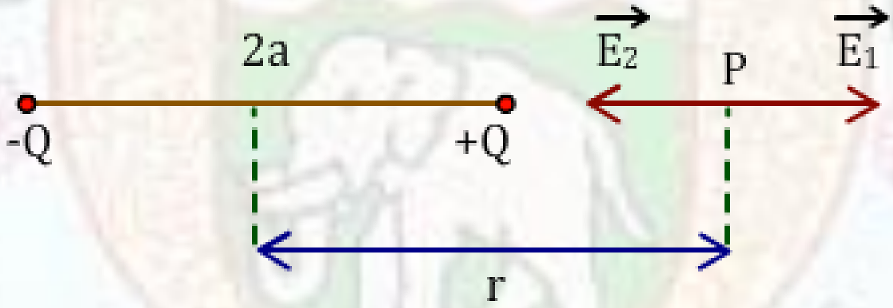


Figure 3.2 Electric field at a point on the axial line of an electric dipole.

$E_1 > E_2$

$$E = \frac{1}{4\pi \in_0}\left[\frac{Q}{(r\text{-}a)^2} - \frac{Q}{(r+a)^2}\right]$$

$$= \frac{1}{4\pi \in_0}\frac{2p}{r^3} \qquad \text{(Eq. 4.1)}$$

where p is the dipole moment of electric dipole and it is equal to

$p = 2aq$

$\vec{E}$ in the same direction as that of $\vec{p}$.

(b) Electric field at a point P, present on the equatorial line of the electric dipole, at a distance r from the center of the dipole (fig. 3.3).

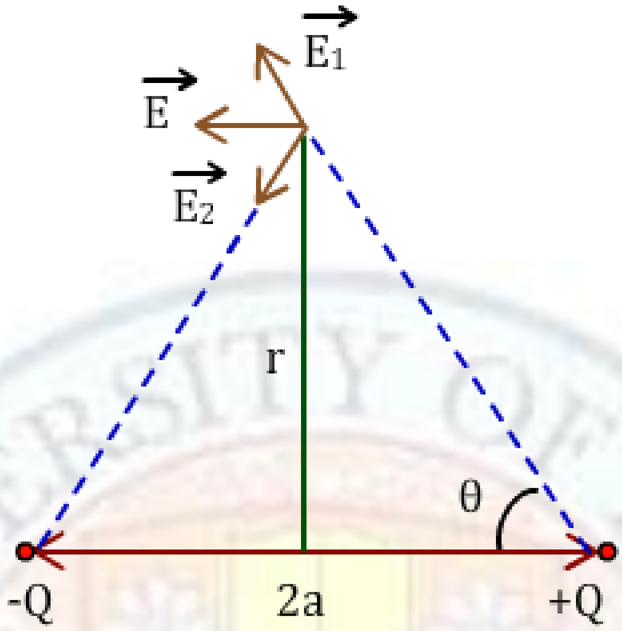


Figure 3.3 Electric field at a point on the equatorial line of an electric dipole.

$E_1 = E_2 = E$

$E = E_1\cos\theta + E_2\cos\theta = 2.E\cos\theta$

$$= \frac{2}{4\pi\in_0}\frac{Q}{r^2+a^2}\frac{a}{\sqrt{r^2+a^2}}$$

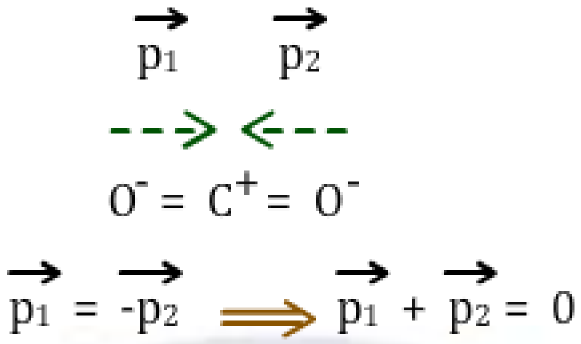
$r \gg a$

$$E = \frac{1}{4\pi\in_0}\frac{p}{r^3} \qquad \text{(Eq. 4.2)}$$

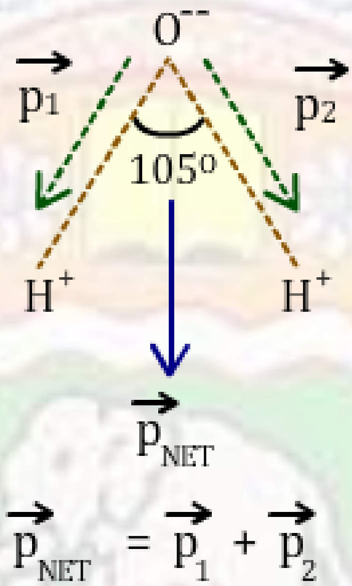
$\vec{E}$ in the opposite direction of $\vec{p}$ and, for the same magnitude of the distance of the field point,

E(axial) = 2×E(equitorial).

Value addition: Did you Know
Polar and non-polar molecules.
Body text: Non-Polar molecules are those molecules having zero dipole moment in absence of external field. Eg.: CO_2



Polar molecules are those molecules having non-zero dipole moment in absence of external field. Eg. H_2O



Reference: Electricity and magnetism. By D C Tayal (Himalaya Publishing House, 1988).

4.2 Torque on an Electric Dipole

Let an electric dipole be placed in a uniform (external) electric field ($\vec{E}$) with its axis making an angle θ with the direction of field as shown in figure 3.4:

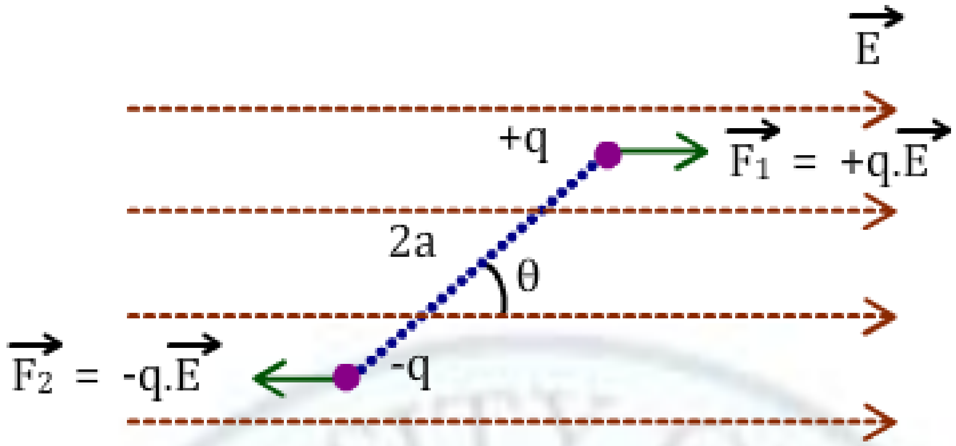


Figure 3.4 Torque acting on an electric dipole placed in uniform electric field.

Here, due to presence of the field, each charge will experience a force. But the net force on dipole is zero because

$$\vec{F}_1 = -\vec{F}_2$$

$$\vec{F}_{NET} = \vec{F}_1 + \vec{F}_2 = 0$$

$\vec{F}_{NET} = 0$. However, the dipole does experience a torque. The two equal and opposite forces, acting on the dipole, are not acting at the same point. This means that their lines of action are different. They, therefore, constitute a couple and the moment of this couple gives the torque experienced by the dipole.

Now, by definition the torque, associated with a couple is the product of either force and the perpendicular distance between them.

∴ Torque = (Force)×(perpendicular distance)

$$\tau = F\ (2a \sin\theta)$$
$$= qE\ (2a \sin\theta)$$
$$= pE \sin\theta \quad \text{where } p=2aq \text{ (dipole moment)}$$

In vector notation,

$$\vec{t} = \vec{p}' \;\; \vec{E} \qquad \text{(Eq. 4.3)}$$

Direction of the torque will be perpendicular to the plane containing the dipole moment and the field. This torque tends to align the dipole in such a way that its axis gets aligned along the field.

4.3 Potential Energy of an Electric Dipole

When the dipole axis is making an angle θ with the field direction ($\vec{E}$), the torque, acting on it, is

$$\vec{t} = \vec{p}\,\acute{}\,\vec{E}$$

$\therefore \quad \tau = p.E.\sin\theta$

The work done, by the torque, in turning the dipole through a further angle dθ,

dW= $\tau.d\theta = p.E.\sin\theta\, d\theta$

The total work done in moving the dipole, from its initial position θ_1 to final position θ_2, can be viewed as the change in potential energy of the dipole.

$$\int dW = W = pE\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

$$\Delta U = W = pE(\cos\theta_1 - \cos\theta_2)$$

We adopt the convention that the P.E. equals zero when $\theta_1=\pi/2$. Then the potential energy for any orientation $\theta_2 = \theta$, is given by

U = -p.E.cos θ = $-\vec{p}.\vec{E}$ (Eq. 4.4)

$\therefore$ When $\theta_1=0$ and $\theta_2= \theta$, the difference of potential energy at two positions, is given by

W=ΔU=pE(1-cos θ) (Eq. 4.5)

Value addition: Did you Know

Gradient, Divergence and Curl

Body text:

1. Del operator or Differential operator $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

2. Gradient of a scalar field Φ is a vector field. It is denoted by $\vec{\nabla}F$ and is defined as

$$\vec{\nabla}F = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)F = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k}$$

The gradient, $\vec{\nabla}F$, points in the direction of maximum increase of the function Φ. Its magnitude equals the most rapid rate of change of the scalar field.

For example, temperature is a scalar field. The gradient of temperature, known as temperature gradient, is a vector quantity. Its direction is that in which the temperature changes most rapidly.

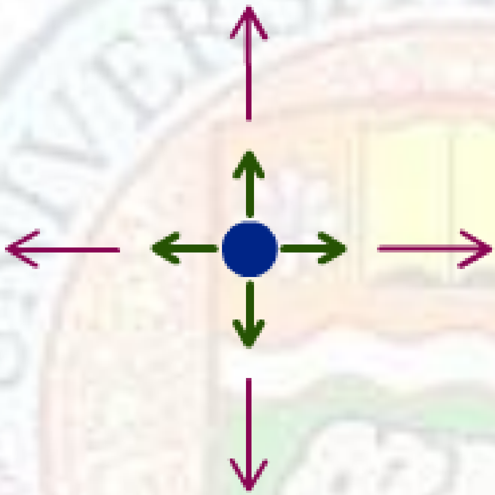
3. Divergence of a vector field $\vec{A}$ is a scalar field. It is denoted by $\vec{\nabla}.\vec{A}$ and is defined as:

$$\vec{\nabla}.\vec{A}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right).(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

The divergence $\vec{\nabla}.\vec{A}$ is a measure of how much the vector $\vec{A}$ diverges or what is its 'rate of flow'.

By convention, a point, of positive divergence, is viewed as a 'source'.

A point of negative divergence is viewed as a 'sink'.



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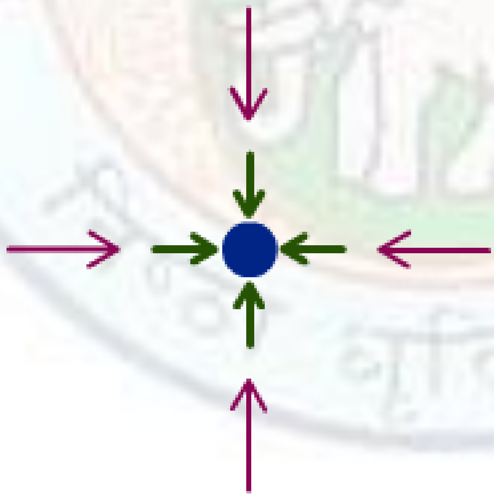
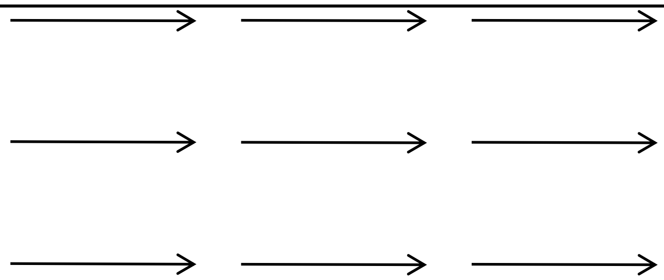


Sink

For a vector field, if the divergence equals zero, i.e., $\vec{\nabla}.\vec{A}=0$, the field $\vec{A}$ is referred to as a solenoidal field.



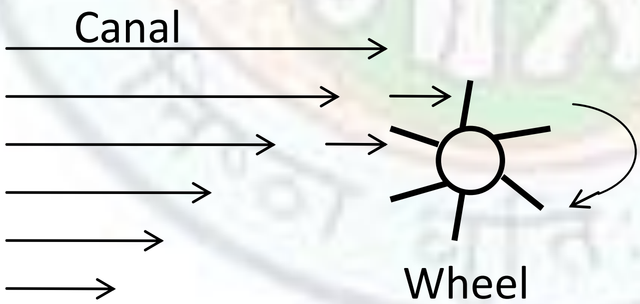
4. Curl of a vector field, $\vec{F}$ is a vector field. It is denoted by $\vec{\nabla}\times\vec{F}$ and is defind as

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ F_x & F_y & F_z\end{vmatrix}$$

$$=\hat{i}\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)+\hat{j}\left(\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\right)+\hat{k}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)$$

The curl of a vector field, $\vec{F}$, i.e., $\vec{\nabla}\times\vec{F}$ is a measure of how much the vector $\vec{F}$ curls around.

To understand clearly the meaning of the curl of a vector and its direction, let us consider a small paddle wheel in the path of the flow of water in a canal. The velocity of the water layer decreases downwards, it is nearly zero at the bottom and is maximum at the top. The wheel will turn in the clockwise direction, the figure showing that there is a small circulation of water round about the wheel. The axis about which the wheel rotates gives the direction of the curl.



<u>NOTE</u>: A vector field, for which the curl is zero at all points of space, is known as an irrotational field. Such a field is also conservative in nature, i.e. its line integral, around a closed path, equals zero.

Reference:

1. David J. Griffiths, Introduction to

Electrodynamics, 3rd edition.

2. **Electricity and magnetism. By D C Tayal (Himalaya Publishing House,1988).**

4.4 Force on an Electric Dipole in a non-uniform Electric Field

Let an electric dipole be placed in a non-uniform electric field. In this case (fig. 3.5) the net force is not zero, because the field, at the location of the positive and the negative charges, would not be the same. Hence

$\vec{F}_{+} \neq \vec{F}_{-}$

Since $\vec{F}_{+}$ now does not exactly balance $\vec{F}_{-}$, there will be a net force on the dipole. This would be in addition to the torque experienced by it.

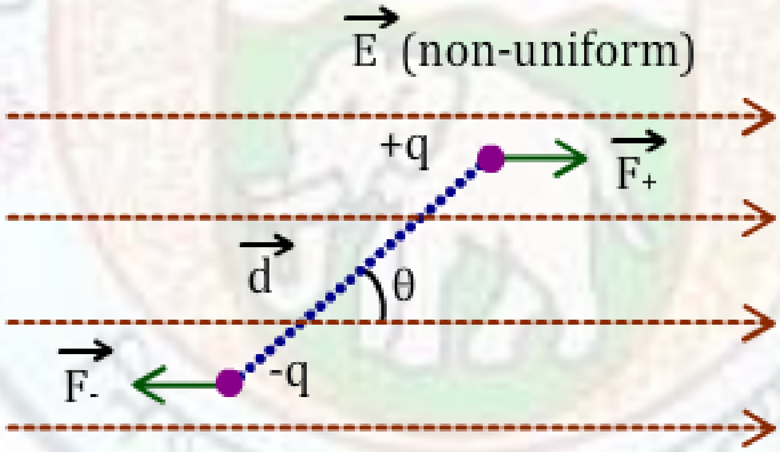


Figure 3.5 Torque acting on an electric dipole placed in a non-uniform electric field.

$$\vec{F}_{NET} = \vec{F}_{+} + \vec{F}_{-} = q(\vec{E}_{+} - \vec{E}_{-}) = qDE$$

Assuming the dipole to be very short,

the small change in E_x would be given by $\Delta E_x = dE_x$

A theorem on partial derivatives states that

$$dE_x = dx\left(\frac{\partial E_x}{\partial x}\right) + dy\left(\frac{\partial E_x}{\partial y}\right) + dz\left(\frac{\partial E_x}{\partial z}\right)$$

We can rewrite it as

$$dE_x = (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}).\left(\frac{\partial E_x}{\partial x}\hat{i} + \frac{\partial E_y}{\partial y}\hat{j} + \frac{\partial E_z}{\partial z}\hat{k}\right)$$

$$dE_x = (\vec{d}.\vec{\nabla})E_x$$

Extending this result to all components, we can write

$$\Delta\vec{E} = (\vec{d}.\vec{\nabla})\vec{E}$$

$$\therefore\ \vec{F} = q(\vec{d}.\vec{\nabla})\vec{E} = (\vec{p}.\vec{\nabla})\vec{E} \qquad \text{(Eq. 4.6)}$$

For a perfect dipole of infinitesimal length, the torque, about the center of dipole, even in a non-uniform field is

$$\vec{\tau} = \vec{p} \times \vec{E}$$

About any other point

$$\vec{\tau} = \vec{p} \times \vec{E} + \vec{r} \times \vec{F} \qquad \text{(Eq. 4.7)}$$

where force $\vec{F}\ (= (\vec{p}.\vec{\nabla})\vec{E})$ and field $\vec{E}$ are both non-uniform.

Value Addition: Electric Dipole

For more interesting information the students are requested to go to the following weblink:

http://en.wikipedia.org/wiki/Electric_dipole_moment

For an illustrative and interesting description of the electric dipole the students are advised to go to the following YouTube videos:

1. https://www.youtube.com/watch?v=513aIefo6QA

<iframe width="420" height="315" src="//www.youtube.com/embed/513aIefo6QA" frameborder="0" allowfullscreen></iframe>

2. https://www.youtube.com/watch?v=kdUKuBu8UH0

<iframe width="420" height="315" src="//www.youtube.com/embed/kdUKuBu8UH0" frameborder="0" allowfullscreen></iframe>

Question Number	Type of question
1	MCQ

Question

(1) Curl of electric field is __________.

Option(s)	(a) Zero (b) 1 (c) 10 (d) None

Question

(2) Electric field is ___________.

Option(s)	(a) Rotational (b) Irrotational (c) Radial (d) None.

Question

(3) $\int curlEdr$ is equal to

Option(s)	(a) $\oint E.dr$ (b) ∇E dv (c) ∇.E (d) None

Correct Answers	1) a 2) b 3) a.

Question Number	Type of question
2	Subjective

Question

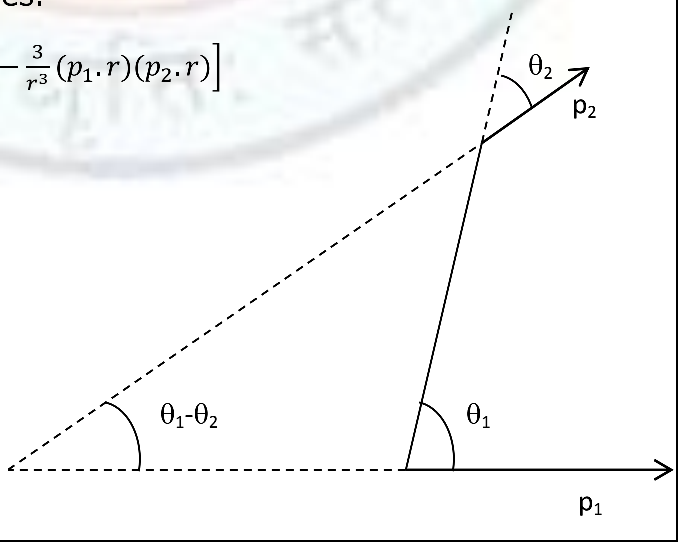
Derive an expression for the interaction potential energy of two short electric dipoles separated by a distance. If one of the dipoles in include at an angle θ_1 to the radius vector joining them, show that in the state of equilibrium, the other dipole would make an angle θ_2 with it given by

$$tan\theta_2 = -\frac{1}{2} tan\theta_1$$

Correct Answers

We are left with two dipoles so first of all we should recall the formula for interaction energy of two short electric dipoles.

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{p_1.p_2}{r^3} - \frac{3}{r^3}(p_1.r)(p_2.r)\right]$$



$$p_1.r = |p_1||r|cos\theta_1$$

$$p_2.r = |p_2||r|cos\theta_2$$

$$p_1.p_2 = |p_1||p_2|cos(\theta_1 - \theta_2)$$

$$U = \frac{1}{4\pi\varepsilon_o r^3}[|p_1||p_2|cos(\theta_1 - \theta_2) - 3|p_1||p_2|cos(\theta_1)cos(\theta_2)]$$

As the dipoles are in equilibrium, the

$$F = -grad\, U = 0$$

$$F = -\frac{dU}{d\theta_2} = 0$$

$$\frac{1}{4\pi\varepsilon_o r^3}[|p_1||p_2|sin(\theta_1 - \theta_2) + 3|p_1||p_2|cos(\theta_1)sin(\theta_2)] = 0$$

$$\frac{|p_1||p_2|}{4\pi\varepsilon_o r^3}[sin(\theta_1 - \theta_2) + 3cos(\theta_1)sin(\theta_2)] = 0$$

$$[sin(\theta_1)cos(\theta_2) - \cos(\theta_1)\sin(\theta_{12}) + 3cos(\theta_1)sin(\theta_2)] = 0$$

$$[sin(\theta_1)cos(\theta_2) + 2cos(\theta_1)sin(\theta_2)] = 0$$

$$tan\theta_2 = -\frac{1}{2}tan\theta_1$$

Hence proved.

References:

1. **David J. Griffiths, Introduction to Electrodynamics, 3rd edition.**
2. **Electricity and magnetism. By D C Tayal (Himalaya Publishing House,1988).**