



**Discipline Course-I**

**Semester-II**

**Paper No: Electricity and Magnetism**

**Lesson: Dielectric materials and Capacitors**

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## Learning Objective

This lesson aims at the following student learning objectives.

- About the dielectric material
- Two types of dielectrics
- Difference between the two types of dielectrics at the microscopic level.
- Properties of Dielectrics
  - a) Their characteristic parameters like real and imaginary dielectric permittivity
  - b) Effect of alternating electric field
- Understand the concept of 'capacitor'
- Know the reason for increase in capacitance of a capacitor as compared to an isolated conductor
- Know the general method of calculating capacitance of a capacitor.
- Calculate the capacitance of a
  - a) Parallel plate capacitor
  - b) Spherical capacitor
  - c) Cylindrical capacitor
- Understand the effect of dielectric on the capacitance of a capacitor; reason for the same
- Know the equivalent air-separation corresponding to a given thickness of a given thickness of a given dielectric.
- Know the formulae for the capacitance of different types of capacitors in the presence of dielectrics.
- Know about the different forms of practical capacitors.
- Energy stored in capacitors

### 13.1. Introduction

In our daily life, we are familiar with two types of materials, conductors and insulators. The conducting materials are used in the electrical circuits to allow the flow of electrons. On the contrary, the materials used to resist the flow of electrons are called insulators (or dielectrics). Some examples of dielectrics, the materials like glass, ceramics, polymers, and papers are good insulating materials. Dielectric materials can exist in all the three states of matter i.e. solids (glass, ceramics), liquids (silicon oil, vegetable oil, deionized water, liquid crystals) and gaseous (hydrogen, nitrogen). Liquid crystal is a particular type of material which possesses the properties of both liquid and solid. The dielectric materials are characterized by the following properties:

- i. *Polar and non-polar,*
- ii. *Dielectric constant,*
- iii. *Dielectric loss and phase difference,*
- iv. *Dielectric strength,*
- v. *Insulation resistance.*

We can discuss them in brief:

#### 13.1.1. Non-polar and polar dielectrics

We know that every material is composed of atom or molecules irrespective of its phase (solid, liquid or gas) and atoms or molecules are composed of negatively

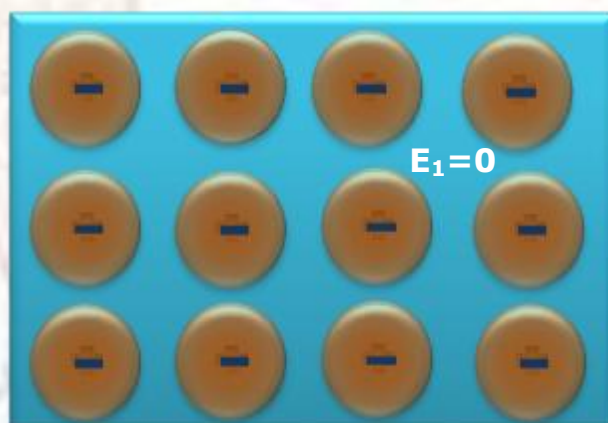
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charged electrons and positively charged nucleus. In some materials, the center of positive and negative charges coincides in the absence of external electric field; such materials are called non-polar dielectrics. On the application of external electric field, their center of mass gets separated up to some extent due to the force exerted by electric field on each charge. The negative charge is shifted opposite to the applied field direction whereas positive charge is shifted in the direction of field. This charge separation creates the dipoles in the material leading to induced dipole moment,  $\vec{p}$ , in it, which is proportional to the applied field strength  $\vec{p} = \alpha \vec{E}$ , where  $\alpha$  is the polarizability of atom or molecule and  $\vec{E}$  is the electric field. Examples of non-polar dielectrics are nitrogen  $N_2$ , hydrogen  $H_2$ , carbon dioxide  $CO_2$ , benzene etc.

### Value addition: Did you Know

#### Separation of positive and negative charges in non-polar dielectrics under external electric field

**Body text:** non-polar\_dielectric.swf



**Source:** Self

In case of polar dielectrics, the material possesses the permanent dipole moment even in the absence of external electric field i.e. the center of positive and negative charges are already separated even in the absence of electric field. For example HCl molecule, the Cl (chlorine) atom is highly electronegative in comparison to H (hydrogen) atom. The Cl atom tends to pull the electron cloud towards itself. Consequently, Cl gets excess of negative charge and becomes negatively charged

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whereas the H atom is deficient of it and becomes positively charged. This asymmetric distribution of electron cloud creates the permanent dipole moment of magnitude  $3.43 \times 10^{-30}$  C-m. In another example of CO (carbon mono oxide), the shifting of charges creates the dipole moment of lower magnitude  $0.4 \times 10^{-30}$  C-m than HCl because of lesser difference in the electronegativity between C and O atoms. Other examples are  $H_2O$ ,  $N_2O$ ,  $NH_3$  etc.

On the application of external electric field, the permanent dipole moment experiences the torque and tends to rotate along the direction of applied electric field. Mathematically,

$$\begin{aligned} \vec{F} &= q\vec{E}, \\ \vec{\tau} &= (\vec{r}_+ \times \vec{F}_+) + (\vec{r}_- \times \vec{F}_-) \\ \vec{\tau} &= \left[ \left( \frac{\vec{s}}{2} \right) \times (q\vec{E}) \right] + \left[ \left( -\frac{\vec{s}}{2} \right) \times (-q\vec{E}) \right] \\ \vec{\tau} &= q\vec{s} \times \vec{E} \\ \vec{\tau} &= \vec{p} \times \vec{E} \end{aligned} \quad (7)$$

Where  $\vec{p} = q\vec{s}$  is the dipole moment. Thus, a dipole moment,  $\vec{p}$ , experiences a torque,  $\vec{\tau}$ , in an external electric field  $\vec{E}$ .

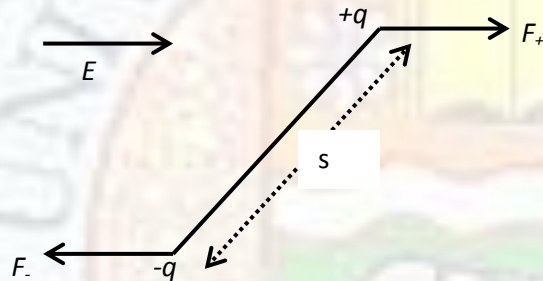


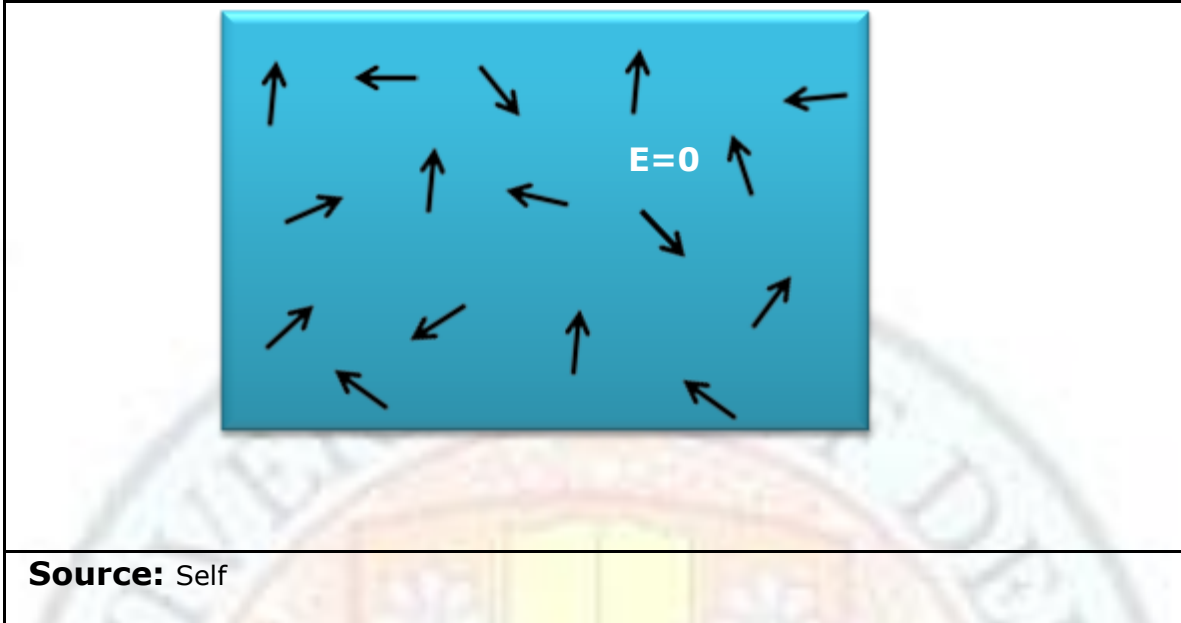
Fig. 2 Representation of electric forces on dipole in external electric field (Vectors are represented in bold alphabets).

### Value addition: Did you Know

**Orientation of permanent dipoles in polar dielectrics under external electric field**

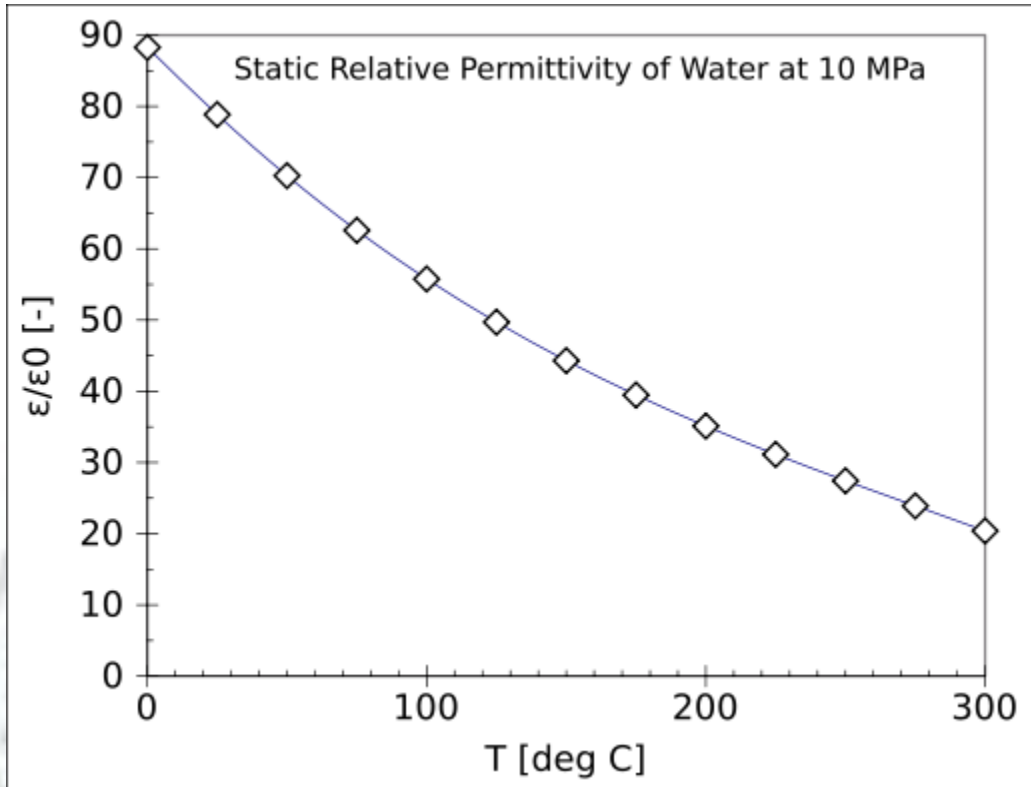
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### 13.1.2. Dielectric constant

Before going to dielectric constant let us discuss the permittivity. The permittivity of dielectric medium defines how finely it allows an electric field to pass through itself; We can determine it by how much the medium gets polarized in the presence of an electric field. It is denoted by ' $\epsilon$ ' and is measured in Farad per meter. The ratio of permittivity of dielectric material in the presence of an electric field strength to the permittivity of vacuum for the same electric field strength gives a property of material which is called as relative permittivity or dielectric constant i.e.  $\epsilon_r = \frac{\epsilon_m(\omega)}{\epsilon_0}$  where  $\epsilon_m(\omega)$  is the complex and frequency dependent permittivity of dielectric material whereas  $\epsilon_0$  is the permittivity of free space or vacuum. The determination of relative permittivity helps a lot in knowing the properties of a material such as electrical, optical etc. properties of material. The relative permittivity of material depends on certain parameters such as temperature, pressure, strength of magnetic field, electric field etc. Thus by knowing the behavior of such parameters we can decide the suitability of a particular material for particular application. For example, the static relative permittivity of water depends on the temperature as shown in Fig. 1.



**Fig. 1**

[http://en.wikipedia.org/wiki/File:Water\\_relative\\_static\\_permittivity.svg](http://en.wikipedia.org/wiki/File:Water_relative_static_permittivity.svg)

### 13.1.3. Dielectric loss and phase difference

Generally, the ideal dielectric material is one in which no electric current is allowed to pass through it but only electric field is permitted and the electric field can be generated within the material. But if the charges are allowed to pass through the dielectric material then the charges cannot be held on the opposite faces of the material in response to the electric field and hence electric field cannot be generated within the material. This phenomenon allows the loss in the dielectric permittivity or relative dielectric constant. Thus, the dielectric material which allow the electric current to pass through itself along with electric field are called lossy dielectric materials. Therefore, the dielectric constant is a complex quantity which is the function of both the properties.  $\epsilon^* = \epsilon' - j\epsilon''$ , where  $\epsilon'$  and  $\epsilon''$  are the real and imaginary part of dielectric permittivity of material respectively. The measurement of dielectric loss is very helpful in determining the various properties of material such as relaxation time of dielectric material.

In reality, there are two types of dielectric materials one is non-polar and another is polar. In non-polar, there is no permanent dipole moment (dipole moment is the product of magnitude of charge and the distance of their separation) associated with molecules but induced dipoles are generated on the application of external electric field whereas there is a permanent dipole moment in polar dielectrics. In ideal dielectric material, the dipoles are expected to follow the direction and magnitude of the applied electric field. In actual dielectrics, the dipoles respond to the **varying applied electric** field relative to another neighboring dipole which might not be responding freely and hence the dipole would be lagging the applied field. This lagging in response to the field is called phase difference. As a

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result, there is a loss in the dielectric properties which appear in the form of heat due to migration of charges. Thus, the overall behavior of dielectric permittivity can be written in frequency dependent complex form as  $\epsilon^*(\omega) = \epsilon'(\omega) - j\epsilon''(\omega)$ , where  $\epsilon'(\omega)$ ,  $\epsilon''(\omega)$ , and  $\omega$  are the real, imaginary part of dielectric material, and angular frequency of applied electric field respectively.

Now, if we consider the alternating electric field, then the total current density,  $J_{tot}$  can be considered as the combination of ohmic current density and capacitive current density. Then

$$\vec{J}_{tot} = \vec{J}_{conduction} + \vec{J}_{Displacement/capacitive}$$

$$\vec{J}_{tot} = \vec{J}_{conduction} + \frac{d\vec{D}}{dt}$$

Since applied electric field is varying  $\vec{E} = E_0 \exp(j\omega t)$ , so  $\vec{D} = D_0 \exp(j\omega t)$ , then

$$\vec{J}_{tot} = \vec{J}_{conduction} + j\omega \vec{D}$$

Now we know that  $\vec{J}_{conduction} = \sigma_{conduction}(\omega) \vec{E}$ ,  $\vec{D} = \epsilon_d(\omega) \vec{E}$ , and  $\sigma_{conduction}$  is electrical conductivity due to conduction therefore

$$\vec{J}_{tot} = \sigma_{conduction}(\omega) \vec{E} + j\omega \epsilon_d(\omega) \vec{E} \quad (1)$$

Where  $\vec{J}_{Displacement/capacitive} = \frac{d\vec{D}}{dt} = j\omega \epsilon_d(\omega) \vec{E}$ .

Using the relationship between conductivity and effective dielectric permittivity or complex permittivity  $\epsilon^*(\omega)$  as

$$\epsilon^*(\omega) = \epsilon_d(\omega) + \frac{\sigma_{conduction}(\omega)}{j\omega},$$

The total current density becomes

$$\vec{J}_{tot} = j\omega \epsilon^*(\omega) \vec{E} \quad (2)$$

Substituting the expression  $\epsilon^*(\omega) = \epsilon'(\omega) - j\epsilon''(\omega)$  in above Eq. (2).

$$\vec{J}_{tot} = j\omega \epsilon'(\omega) \vec{E} + \omega \epsilon''(\omega) \vec{E} \quad (3)$$

The real and imaginary parts are shown in Fig. 2(b).

The rate of work done by electric field i.e. power  $q\vec{v} \cdot \vec{E} = \vec{J}_{tot} \cdot \vec{E}$ . Then, the

The rate of loss of energy in unit volume of the material can be written as

$$W = \frac{dP_{loss}}{dV} \quad (4)$$

Where  $P_{loss}$  is the power loss in the dielectric under the application of alternating current field or electromagnetic radiations.  $dV$  is the volume element of the dielectric material.

$$W = \frac{dP_{loss}}{dV} = \frac{1}{2} \text{Re}[\vec{J}_{tot} \cdot \vec{E}^*] \quad (5)$$

Where  $\vec{E}$  is the applied alternating electric field.

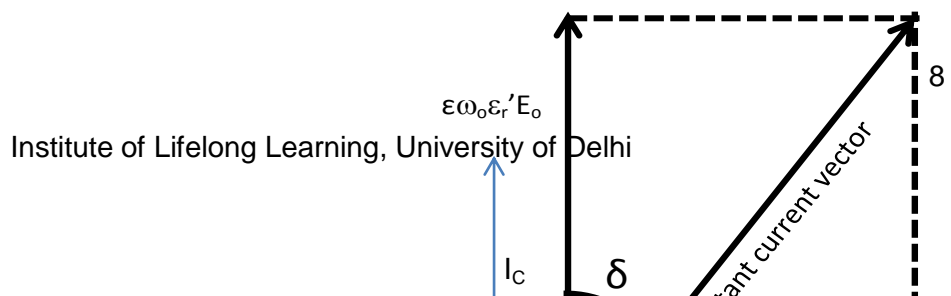
Substituting the expression of  $\vec{J}_{Tot}$  from Eq. (3) in Eq. (5), we get the real value as

$$W = \frac{dP_{loss}}{dV} = \frac{1}{2} [\omega \epsilon''(\omega) |\vec{E}|^2]$$

or

$$W = \frac{1}{2} \epsilon_0 \epsilon_r'' \omega E_0^2,$$

Where  $\epsilon_0$ ,  $\epsilon_r''$  and  $\vec{E}_0$  are the permittivity of free space, imaginary part of relative dielectric constant and applied electric field respectively. Since it is the combination of two properties one is capacitive (reactance) and other is due to current flow (resistive part), it can be represented as equivalent electrical circuit [Fig. 2(a)]





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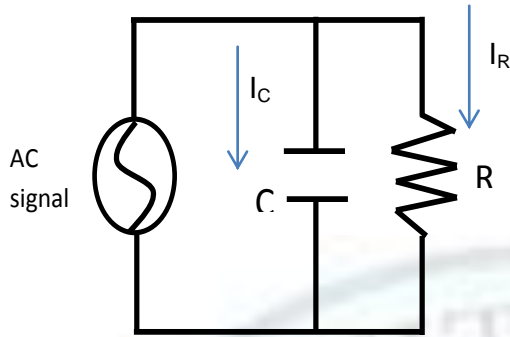


Fig. 2 (a) The parallel RC circuit, (b) phase diagram of real and imaginary contributions to dielectric constant in terms of vector components on the basis of Eq. (3).

Thus from Fig. 2 (b),

$$\tan\delta = \frac{\omega\epsilon_0\epsilon_r''\vec{E}_0}{\omega\epsilon_0\epsilon_r'\vec{E}_0} = \frac{\epsilon_r''}{\epsilon_r'}$$

Then the power loss can be written in terms of  $\tan\delta$  as

$$W = \frac{1}{2}\epsilon_0\epsilon_r'\omega\tan\delta\vec{E}_0^2 \quad (6)$$

The energy loss in dielectric materials is due to following reasons:

*i. ionization*, *ii. leakage current*, *iii. polarization*, and *iv. structural inhomogeneity*.

The dielectric loss due to ionization has been found to occur in solid and gaseous dielectrics. This can happen due to the ionization of gas molecules on the application of strong external electric field of strength greater than the threshold field. Threshold field is that electric field above which the bonds between polar charges (+ and -) breaks and positive and negative ions get separated. In solids it can appear due to the intrusion of ions or breaking of weak bonds.

The dielectric loss due to leakage current has been found in liquids and solids. The reason is that the ions and free charges are available in large quantity in solids and liquids and can respond to the applied electric field. On the other hand, dipolar losses have been observed in the radio frequency region which is due to the orientation of permanent dipoles. This is because the dipoles responds much faster than the ions in liquid and are observed at higher frequencies.

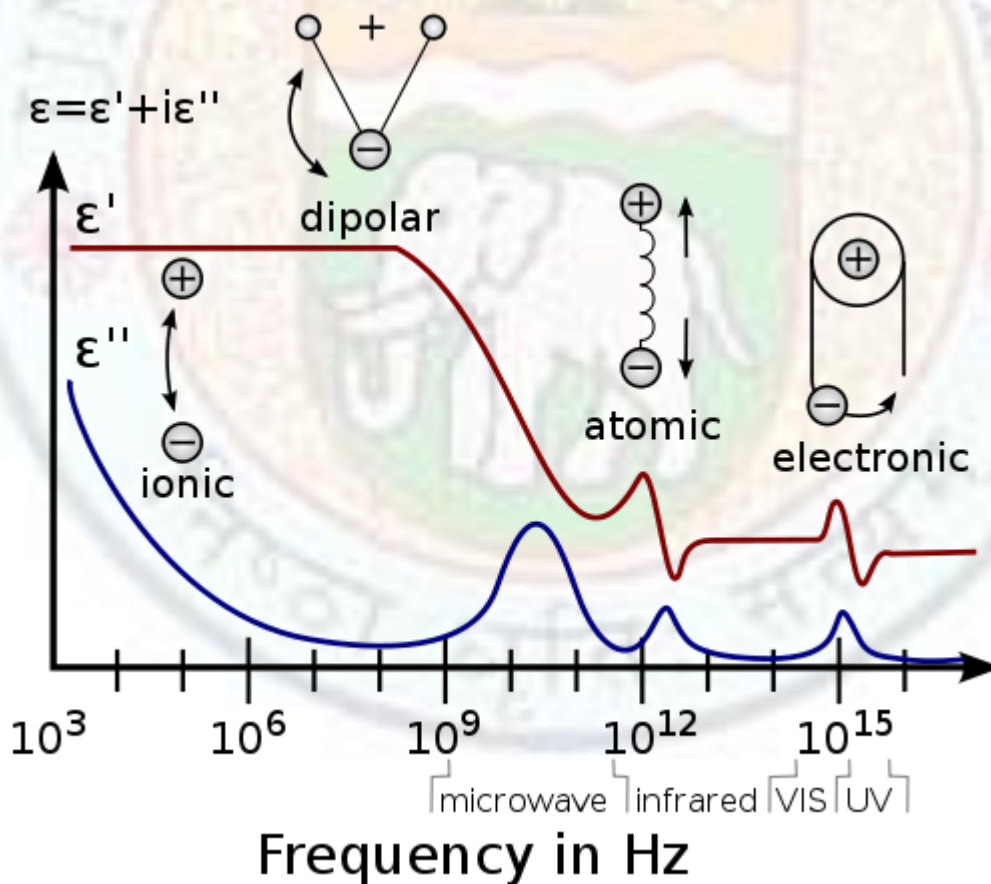
### Value addition: Did you Know

#### Dielectric constant as a function of frequency

**Body text:** The dielectric constant is a function of frequency i.e. it may have different values at various frequency ranges. The behavior of real part of dielectric

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constant ( $\epsilon_r'$ ) and  $\tan\delta$  or imaginary part ( $\epsilon_r''$ ) provide the important information about the ionic species, dipolar, electronic relaxation processes. It is helpful in determining the distribution of relaxation time for a particular relaxation process occurring inside the dielectric material at a particular range of frequency of applied ac field. The figure below shows the order of the relaxation processes frequency for a particular dielectric material like liquid crystals, polymers, rubbers and other soft matters. The relaxation time can be defined as the time taken by the dipole to reach the equilibrium state after the removal of applied field. The following figure represents the frequency ranges of various relaxation processes occurring in materials. The ionic relaxation process is a very slow process and takes longer time to relax than other processes so the frequency is large. Therefore, it appears in the range of low frequency. The dipolar process takes much lesser time than ionic process and hence the relaxation frequency range is higher than this. After this, the atomic polarization process takes lesser time than both the above process and appears at higher frequencies than these processes. The electronic relaxation process come at last and appears in the range of very high frequencies. The dielectric spectroscopy techniques are used to detect and analyze the low frequency processes such as ionic and dipolar. For the analysis of higher frequency processes, the separate techniques are used such as wave guide impedance spectroscopy techniques.



**Source: [http://en.wikipedia.org/wiki/Dielectric\\_spectroscopy](http://en.wikipedia.org/wiki/Dielectric_spectroscopy)**

**13.1.4. Dielectric strength**

Dielectric strength is the capability of a dielectric material to withstand without breaking down under the application of external electric field and describes its response to the field. It can be expressed as the voltage per unit thickness of the material and is denoted by

$$E_{br} = \frac{V_{br}}{d},$$

Where  $V_{br}$  and  $d$  are the breakdown voltage and thickness of the dielectric material. It is generally expressed in kV/mm or MV/m.

**13.1.5. Insulation resistance**

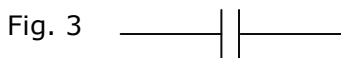
Insulating or dielectric materials are used to separate the two current carrying conductors in order to avoid the short circuit between them during conduction of electricity. The large potential difference between conductors may allow the leakage of current. Resistance is offered by the dielectric to the leakage current flow in two ways: (i) *over the surface* and (ii) *through bulk volume of insulators*. The resistance experienced by the electronic current between the two opposite edges of a square of unit area of insulating surface is called the surface resistivity whereas resistance offered by bulk material of unit cross section area per unit length of distance in the direction of current flow is called as the *volume resistivity*. The combine effect of surface and volume resistance of dielectric is called as insulation resistance.

Furthermore, the dielectric materials can also be classified in three ways:

S. No.	Name of dielectric	Polar/non-polar	Effect of electric field
i.	Simple dielectrics	Non-polar	Induces dipoles
ii.	Paraelectrics	Polar	Orients dipoles
iii.	Ferroelectrics	Polar	Orients domains of permanent dipoles

**13.2. Capacitors**

A capacitor is an electrical device used to store electrical energy by storing electric charges ( $\pm q$ ). It is an arrangement of two conductors separated by insulating material or dielectric materials. It is very important two terminal passive device used in electronic and telecommunication devices. Symbolically, it is represented in the electronic circuits as shown in Fig. 3



Capacitors have been classified in the following categories:

- 1) Parallel plate
- 2) Cylindrical

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### 3) Spherical

**Parallel plate:** Now suppose we have two parallel plates with area,  $A$ , of each plate arranged parallel to each other facing their planes and separated by a distance  $d$ , as shown in Fig. 4. If  $q$  and  $-q$  are the charges on the plates.

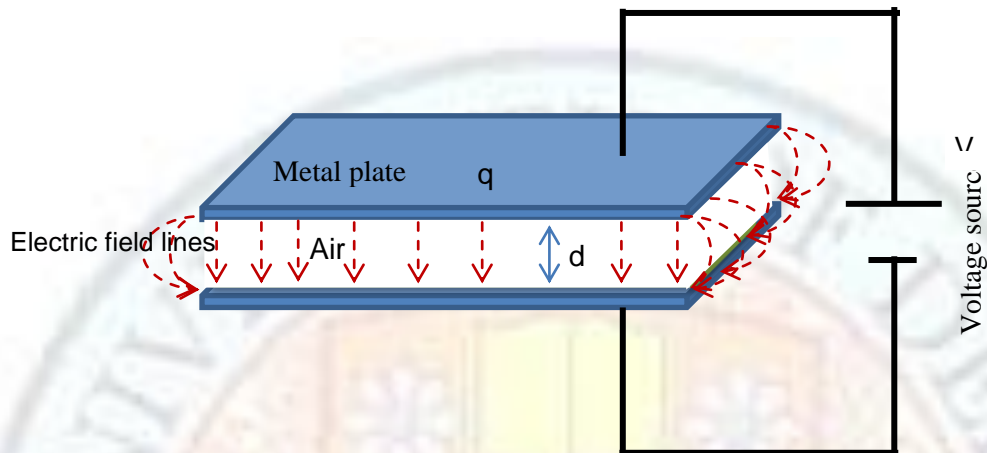


Fig. 4

Then, the electric field intensity of at a point inside the space between plates due to the plates is

$$\vec{E} = \frac{\sigma}{\epsilon_0} \quad (8)$$

Where  $\sigma$  is the density of surface charge on the inner surface of the plate. We know that  $\vec{E} = \frac{V}{d}$

Therefore, Eq.(8) can be written as

$$\frac{V}{d} = \frac{\sigma}{\epsilon_0} \quad (9)$$

We have surface charge density,  $\sigma = q/A$ , then

$$\frac{V}{d} = \frac{q}{\epsilon_0 A}$$
$$\frac{q}{V} = \frac{\epsilon_0 A}{d} = C \quad (10)$$

Eq. 10 represents the capacity of a parallel plate capacitor filled with air. It is represented by  $C$  and measured in Farad.

Now, these capacitors can be joined in two main combinations i.e. parallel and series as per the requirements of the circuit, Fig. 5.



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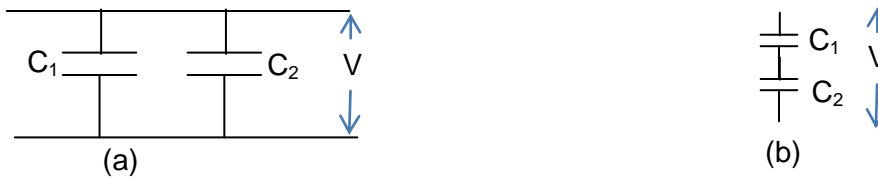


Fig. 5

In parallel combinations, Fig. 5 (a), both the capacitors give same potential across the combination,  $V$ . Then, the equivalent capacitance becomes

$$C = \frac{q}{V} = C_1 + C_2.$$

In series combination, each capacitor is required to acquire the charge due to the conservation of charge. Therefore, the equivalent capacitance of combination becomes

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

### 13.2.1. Capacitor filled with dielectric

If  $q$  be the charge given to the capacitor plates then the capacitance/capacity of the capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{Ed} \quad (11)$$

Where  $\sigma = \frac{q}{A}$  and is the free surface charge density of the plate of a parallel plate capacitor having surface area,  $A$  and separation between the plates,  $d$  whereas  $E$  is the electric field intensity.

$$C = \sigma A / (\sigma / \epsilon_m d) \quad (\text{since } E = \frac{\sigma}{\epsilon_m})$$

$$= \frac{\epsilon_m A}{d}$$

If the capacitance of a capacitor using air as a medium with permittivity  $\epsilon_0$  Eq. (10) is

$$C_0 = \frac{\epsilon_0 A}{d}$$

Where  $\epsilon_m$  is the permittivity of the dielectric material filled between metal plates of the capacitor.  $d$  is the separation between parallel plates.

$$\epsilon_r = \frac{C}{C_0} = \frac{\epsilon_m}{\epsilon_0} \quad (12)$$

Here  $\epsilon_r$  is the relative dielectric constant and is defined as the ratio of capacitance of the parallel plate capacitor in the presence of dielectric material and in its absence, then the capacity of a capacitor increases by a factor of  $\epsilon_r$ .

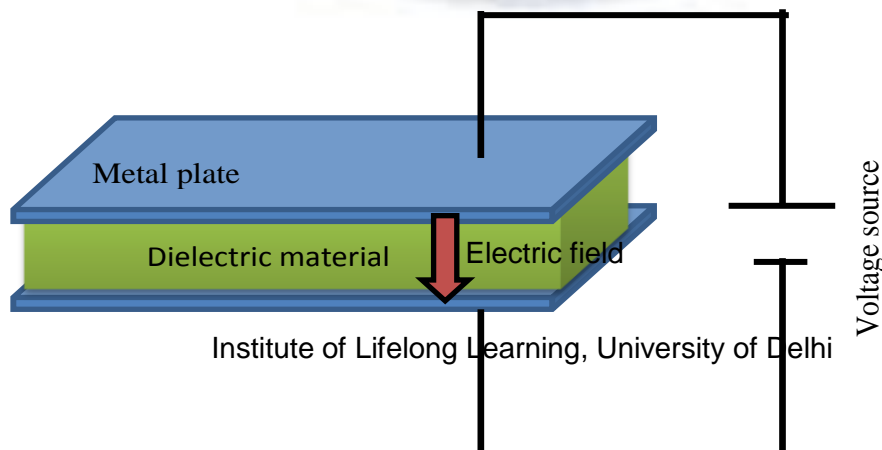


Fig. 6 Diagram of a capacitor filled with dielectric in closed circuit.

### 13.2.2. Energy stored in capacitor

Let us assume a capacitor of capacitance 'C' in farad. It is charged up to 'V' volts, then,

$$C = \frac{q}{V}$$

The work is stored in the form of energy. Let us calculate the equivalent work done  $Vdq$ , if the charge of the capacitor is increased by  $dq$ .

Let  $dw$  be the work done to establish a charge of  $dq$ .

Then,

$$V = \frac{dw}{dq}$$

$$dw = Vdq$$

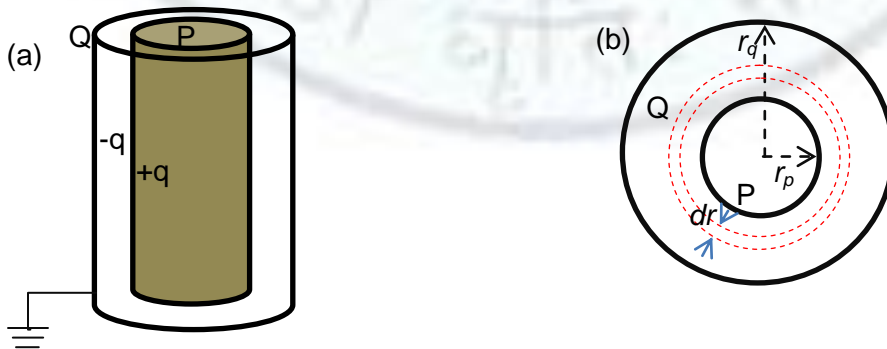
$$= \frac{q}{C} dq \quad (13)$$

When the capacitor is fully charged to 'V', the charge would be Q. Work done to charge a capacitor to total charge is

$$\begin{aligned} W_e &= \int_0^Q \frac{q}{C} dq = \frac{1}{2C} |q^2|_0^Q \\ &= \frac{1}{2} \frac{Q^2}{C} \\ &= \frac{1}{2} CV^2 \end{aligned} \quad (14)$$

' $W_e$ ' is the total electrical energy stored in the capacitor. The storage of energy depends on the capacity of capacitor which is proportional to the dielectric material filled in the capacitor. The energy stored in different types of capacitor can be obtained by knowing the expressions of capacitance of various capacitors such as cylindrical, spherical and parallel plate capacitors. The expression of parallel plate is shown in above discussion and rest of the cases are discussed as follows:

**Cylindrical capacitor:** Consider two concentric hollow cylinders P and Q (Fig. 7(a)) with their radii  $r_p$  and  $r_q$  as shown in Fig. 7(b).



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Fig. 7

The potential difference between two surfaces of the element of radial thickness,  $dr$ , considered in between two concentric cylinders P and Q.

$$dV = -E dr = -\frac{q}{2\pi\epsilon_0\epsilon_r r} dr \quad (15)$$

Exceeding the limits of element up to  $r_p$  and  $r_q$  by integration, we get

$$\begin{aligned} V &= \int_{r_q}^{r_p} -\frac{q}{2\pi\epsilon_0\epsilon_r r} dr \\ &= -\frac{q}{2\pi\epsilon_0\epsilon_r} \int_{r_q}^{r_p} \frac{1}{r} dr \\ &= -\frac{q}{2\pi\epsilon_0\epsilon_r} [\log r]_{r_q}^{r_p} \\ &= -\frac{q}{2\pi\epsilon_0\epsilon_r} [\log r_p - \log r_q] \\ &= -\frac{q}{2\pi\epsilon_0\epsilon_r} \left[ \log \frac{r_p}{r_q} \right] \\ &= \frac{q}{2\pi\epsilon_0\epsilon_r} \left[ \log \frac{r_q}{r_p} \right] \end{aligned} \quad (16)$$

If  $q = \lambda l$ , then

$$= \frac{\lambda l}{2\pi\epsilon_0\epsilon_r} \left[ \log \frac{r_q}{r_p} \right]$$

Then capacitance C can be written as

$$\begin{aligned} C &= \frac{q}{V} \\ &= \frac{q}{\frac{q}{2\pi\epsilon_0\epsilon_r} \left[ \log \frac{r_q}{r_p} \right]} \quad [\text{by using Eq. 16}] \\ &= \frac{2\pi\epsilon_0\epsilon_r}{\left[ \log \frac{r_q}{r_p} \right]} \end{aligned} \quad (17)$$

Eq. (17) represents the capacity of unit length cylinder.

The capacity of cylindrical capacitor of length  $l$  can be written as

$$C = \frac{2\pi\epsilon_0\epsilon_r l}{\left[ \log \frac{r_q}{r_p} \right]} \quad (18)$$

The capacity of cylindrical capacitor without dielectric i.e. air is considered between cylinders, can be written as

$$C = \frac{2\pi\epsilon_0 l}{\left[ \log \frac{r_q}{r_p} \right]}$$

The example of cylindrical capacitor is coaxial cable which is most frequently used in laboratories.

### **Spherical capacitor:**

The spherical capacitor consists of two concentric spheres with different radius. Let us consider two spheres with radii  $r_p$  and  $r_q$  with condition  $r_p < r_q$  as shown in Fig. 8.

The electric field  $\vec{E}$  is directed outward from the center O. Then, the potential difference between the two spheres can be written as

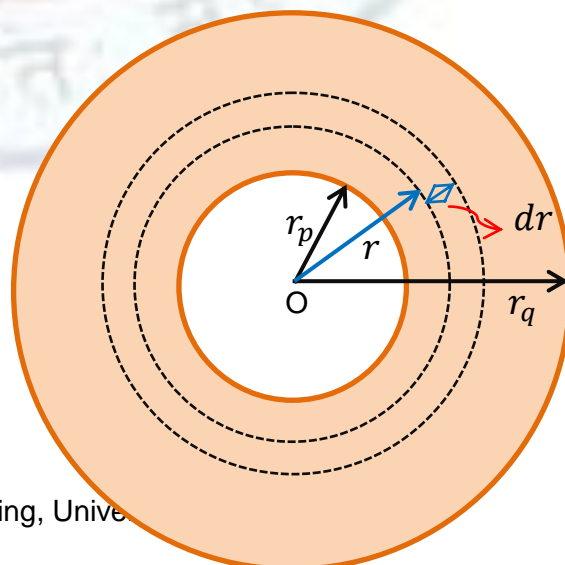


Fig. 8

## Dielectric materials and Capacitors

$$V = - \int_{r_q}^{r_p} \vec{E} \cdot d\vec{r} \quad (19)$$

The angle between  $\vec{E}$  and  $\vec{r}$  is zero due to the reason that both are directed outward.

Hence, the expression  $\vec{E} \cdot d\vec{r}$  becomes  $E dr$ . Then, Eq. (19) can be written as

$$V = - \int_{r_q}^{r_p} E dr \quad (20)$$

Now we know that the electric field at any point 'r' can be written as

$$E = \frac{q}{4\pi\epsilon_0 r^2}, \text{ then substituting this in Eq. (20), we get,}$$

$$V = - \int_{r_q}^{r_p} \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$= - \frac{q}{4\pi\epsilon_0} \int_{r_q}^{r_p} \frac{1}{r^2} dr$$

$$= - \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_q} - \frac{1}{r_p} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_p} - \frac{1}{r_q} \right]$$

Now, we know that capacitance  $C$  can be written as

$$C = \frac{q}{V} = 4\pi\epsilon_0 \frac{r_p r_q}{(r_q - r_p)} \quad (21)$$

This shows that the capacitance of a spherical capacitor depends on the their radii and permittivity. Here the permittivity of free space is used i.e  $\epsilon_0$ .

Now, if the capacitor is completely filled with dielectric material with permittivity  $\epsilon_m$ , then the Capacitance can be written as

$$C = 4\pi\epsilon_m \frac{r_p r_q}{(r_q - r_p)}$$

Using Eq. (12), we can get

$$C = 4\pi\epsilon_0 \epsilon_r \frac{r_p r_q}{(r_q - r_p)} \quad (22)$$

Thus if the high relative dielectric constant is filled in a spherical capacitor, then this increases its capacity linearly.

**Capacitance of isolated conductors:** In the above sections based on capacitance, the capacity of charge storage is based on two conductors placed adjacent to each other, such type of capacitance is called *mutual capacitance*. The isolated conductor also has the capacity to store charge and is termed as *self-capacitance*. This can be defined as the amount of electric charge required to raise its electrical potential by one unit i.e. one volt. Theoretically, the reference point for this system can be considered as hollow conducting sphere of infinite radius. The center of which is considered at another conductor at origin. Using Eq. (21), let  $r_q \rightarrow \infty$ , then the capacitance of conductor of radius  $r_p$  can be written as

$$C = 4\pi\epsilon_0 \frac{r_p r_q}{r_q \left(1 - \frac{r_p}{r_q}\right)}$$

Using limit  $r_q \rightarrow \infty$ , we get

$$C = 4\pi\epsilon_0 r_p \quad (23)$$

Eq. (23) shows that the capacitance of charged spherical conductor depends on the resistance of sphere i.e.  $r_p$ .

<http://en.wikipedia.org/wiki/Capacitance>



## Value addition: Did you Know

### Capacitors

**Body text:** In October 1745, [Ewald Georg von Kleist](#) of [Pomerania](#) in Germany found that charge could be stored by connecting a high-voltage [electrostatic generator](#) by a wire to a volume of water in a hand-held glass jar. Von Kleist's hand and the water acted as conductors, and the jar as a dielectric. Von Kleist found, after removing the generator that touching the wire resulted in a painful spark. In a letter describing the experiment, he said "I would not take a second shock for the kingdom of France. The term "[dielectric](#)" was coined by [William Whewell](#) (from "dia-electric") in response to a request from [Michael Faraday](#). After that the dielectrics are being used in the capacitor for large capacity of electrical storage. Following figure shows the first time fabricated device for the storage.

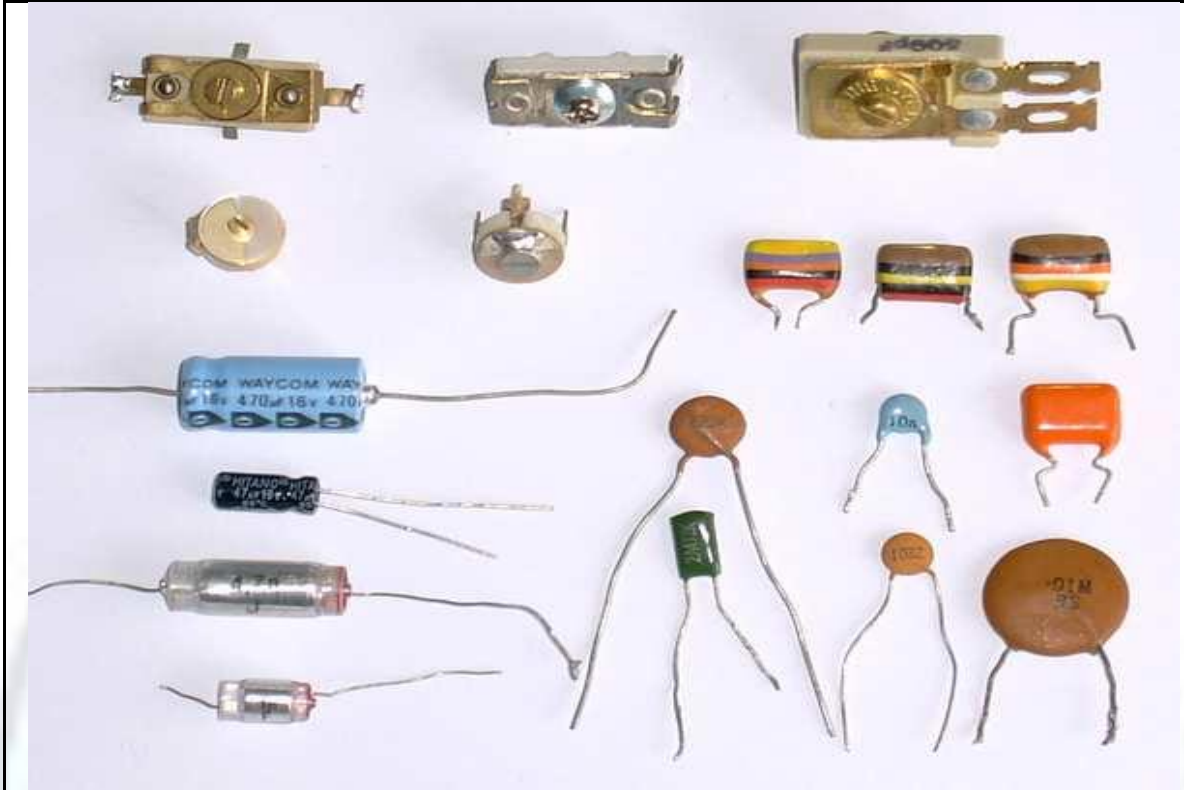


Battery of four [Leyden jars](#) in [Museum Boerhaave](#), [Leiden](#), the [Netherlands](#).

Source: [www.en.wikipedia.org/wiki/Capacitor](http://www.en.wikipedia.org/wiki/Capacitor)

Various forms of capacitors in the market

## Dielectric materials and Capacitors



Source: <http://www.google.co.in/imgres>.

Question Number	Type of question
1	Multiple questions

### Question

**13.3.** In dielectrics, the \_\_\_\_\_ are tightly bound to the nucleus of the atom

- 1) Proton
- 2) Neutron
- 3) Quarks
- 4) Electrons

**13.4.** Dielectrics are capable to maintain two large metal plates at a very \_\_\_\_\_ distance.

- 1) Infinite
- 2) Small
- 3) Zero
- 4) 1000m

## Dielectric materials and Capacitors

**13.5.** The \_\_\_\_\_ molecules have asymmetrical structure.

- 1) Polar
- 2) Non-polar
- 3) Monoatomic
- 4) None

**13.6.** The polar dielectrics have \_\_\_\_\_ dipole moment.

- 1) Zero
- 2) Unequal
- 3) Permanent
- 4) None

**13.7.** Hydrogen, nitrogen, and oxygen are \_\_\_\_\_ dielectrics.

- 1) Conducting
- 2) Non-polar
- 3) Polar
- 4) Asymmetric

**Answers** 1) 4 2) 2 3) 1 4) 3 5) 2

Question Number	Type of question
2	Subjective

### Question

1. Derive the expression for the combine capacity of a system of two series capacitors C1 and C2 filled with dielectric material  $\epsilon_1$  and  $\epsilon_2$ , respectively.
2. Derive the expression for the capacity of a parallel plate capacitor filled with two dielectrics.
3. A parallel plate capacitor consisting of two sheets of copper having area  $0.2 \text{ m}^2$ , separated by  $50 \mu\text{m}$ , and filled with a dielectric material of relative permittivity of 3. What is the capacity of this capacitor? Also calculate the energy storage in this system on the application of electric field 100 V.
4. Prove that the capacity a capacitor constituted by two concentric spheres having radius  $r$  and  $r'$  is  $4 \pi \epsilon_0 \epsilon_r (rr'/(r-r'))$ .

## References and further readings

1. **J. R. Reitz, F. J. Milford, and R. W. Cristy, "Foundations of electromagnetics theory", Narosa Publishing House, New Delhi.**
2. **David J. Griffiths, Introduction to Electrodynamics, 3rd edition.**
3. **K. B. M. Sahu, "Electricomagnetic fields", Second edition, Scitech publications (India) Pvt. Ltd. 2006.**
4. **S. L. Kakani and A. Kakani, "Material Science", New Age International Publishers, New Delhi.**
5. <http://en.wikipedia.org/wiki/Dielectric>

