Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Magnetic Field Lesson 3.3: Magnetic Force

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Table of Contents

Chapter: Magnetic Force

Learning Objectives

After going through this lesson, the reader will be,

i. Able to know and appreciate the basic features of magnetic force.
ii. Know qualitatively and quantitatively the basic details of the magnetic force on a charged particle.
iii. Work out the trajectory of a charged particle in a magnetic field.
iv. Know the basic details of the magnetic force on a current element and a current carrying wire.
v. Workout the nature of the magnetic force between two current carrying wires.
vi. Able to obtain the quantitative relation for the magnetic force between two current carrying wires.
vii. Able to use the general relation, obtained above, for the special case of two long straight parallel current carrying wires.
viii. Understand why two 'like' currents attract and two 'unlike' currents repel each other
ix. Understand how one can define 'ampere' – the SI unit of current in terms of the magnetic force between two long straight parallel current carrying wires.

Introduction

We now know how the discovery of the magnetic force, on iron objects, led to the discovery of the phenomenon of magnetism. Later on, it was realized that unlike the case of two separate individual kinds of charges (positive and negative charges) in electricity, there are no free isolated magnetic poles in magnetism.

Oersted and Ampere's experiments with current carrying wires indicated that we can link the phenomena of magnetism with electric currents. Since currents are nothing but the effects of directed moving charges, it soon became clear that magnetism can be attributed to moving charges. It was but natural therefore to expect that moving charges would experience forces in a magnetic field.

We start this chapter by introducing the quantitative 'Lorentz Force' expression for the force on a moving charge in a magnetic field. We use this expression to know the details of the trajectory of a moving charged particle in a magnetic field. This expression is next used to find an expression for the magnetic force on a current elements and then on a current carrying wire.

The result, for the magnetic force on a current carrying wire, coupled with the expression for the magnetic field due to a current carrying wire is next used to obtain an expression for the magnetic force between two current carrying wires. This general result is next simplified for the special case of two long straight parallel current carrying wires. This special case is used to explain why two 'like' currents should attract each other while two 'unlike' currents should repel each other. The use of this special case result in defining the SI unit of current – the ampere- is brought out and a clear cut definition of this unit is given thereafter.

Magnetic Force on a Point Charge

The basic expression for the magnetic force, on a point charge, was provided by Lorentz. It is for this reason that this force on a point charge is often referred to as the magnetic part of the 'Lorentz Force' acting on it. According to Lorentz:

The magnetic force, **F**, on a charged particle, of charge q, moving with a velocity **v**, in a magnetic field, **B** is given by,

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

It follows that the magnitude F, of this force, is given by

$$F = qvB\sin\theta$$

where θ is the angle between the directions of **v** and **B**.

Special Features of the magnetic Force on a Charged Particle

The Lorentz force expression, given above, immediately brings out the following special features of the magnetic force on a charged particle.

(i) The magnetic force comes into play only for a moving charge. If a charge is at rest in a magnetic field, the magnetic force would be zero no matter how strong the charge on the particle is or how strong the magnetic field, in which it is present, is.

(ii) The magnetic force, on a moving charge, in a magnetic field becomes zero when the charged particle is moving parallel to, or along, the magnetic lines of the magnetic field.

(iii) The magnetic force, on a moving charged particle attains its maximum value (=qvB) when the charged particle is moving normal to the direction of the magnetic field.

(iv) The direction of the magnetic force, on a moving charged particle, is normal to the plane formed by **v** and **B**. This force, therefore, never acts along the direction of **v** and therefore, cannot increase the magnitude of **v**. We express this fact by saying that the magnetic force, on a charged particle due to a magnetic field, cannot bring about any change in the kinetic energy of the moving charged particle.

(v) The direction of the magnetic force is normal to the plane formed by **v** and **B** for all charged particles. It is in the sense of advance of a right handed screw, related from **v** to **B**, if the charged particle carries a positive charge. For a negatively charged particle the direction of the magnetic force on it is opposite to the sense of advance of a right handed screw rotated from **v** to **B**.

(vi) The magnetic force, on a particle, is zero when,

- any one of q, **v** or **B** has zero magnitude.
- **v** is parallel to **B** even though all three, i.e., q, **v** and **B**, have non-zero magnitude.

Trajectory of a Moving Charged Particle in a Magnetic Field

Consider a charged particle, or charge q and mass m, moving in a region in which a magnetic field **B** is present. The equation of motion of the charged particle would be,

$m \frac{d^2\mathbf{r}}{dt^2} = q\,(\mathbf{v} \times \mathbf{B})$

where, **r** is the instantaneous position vector of the moving charged particle.

Rewriting this equation in terms of velocity, we get,

$m \frac{d\mathbf{v}}{dt} = q\,(\mathbf{v} \times \mathbf{B})$

Therefore, $\mathbf{v}.\ m \frac{d\mathbf{v}}{dt} = \mathbf{v}.\ [q\,(\mathbf{v} \times \mathbf{B})]$

The right hand side, being equal to $q\,(\mathbf{v}.\ (\mathbf{v} \times \mathbf{B}))$, is zero.

Hence, $\mathbf{v}.\ m \frac{d\mathbf{v}}{dt} = m\,(\mathbf{v}.\ \frac{d\mathbf{v}}{dt}) = 0$

Or, $\mathbf{v}.\ \frac{d\mathbf{v}}{dt} = 0$

Or, $\frac{1}{2} \frac{d}{dt} (\mathbf{v}\ .\ \mathbf{v}) = \frac{1}{2} \frac{d}{dt} (v^2) = 0$

It follows that v^2 and therefore is a constant. Hence the speed of a charged particle moving in a magnetic field remains constant.

We usually express this result by saying:

'There is no change, in the kinetic energy of a charged particle, when it moves in a magnetic field.' Equivalently we can say: **'A magnetic field does not cause any change in the kinetic energy of a charged particle moving through it'**.

We can next rewrite the above equation of motion of the charged particle in terms of its three components. These would look like:

$m \frac{d^2x}{dt^2} = q\,(v_yB_z - v_zB_y)$

$m \frac{d^2y}{dt^2} = q\,(v_zB_x - v_xB_z)$

$m \frac{d^2z}{dt^2} = q\,(v_xB_y - v_yB_x)$

These equations get simplified by taking the direction of the magnetic field as one of the axis of co-ordinates. Doing that, and carrying out appropriate integrations (choosing appropriate 'boundary conditions' on the way), one can show that:

When a charged particle is moving with a velocity **v**, in a uniform magnetic field, the component of its velocity, along the co-ordinate axis, specified by the magnetic field, say v_B, does not change with time. The particle describes a circle of radius v_n/ω, (v_n=component of the velocity of the particle normal to the field direction and $\omega = qB/m$) in the plane normal to the field. The overall trajectory of the particle is, therefore, a helical one with the radius of the helix being equal to v_n/ω and the pitch of the helix being equal to v_B/ω.

When $v_B = 0$, i.e., the particle is moving in a plane normal to the field, the particle simply follows a circular path in its plane of motion. The centripetal force needed by the particle to keep moving along its circular path of radius R, say, is provided by the magnetic field. This force would now have a magnitude of qvB so that,

$$mv^2/R = qvB$$

or, $R = mv/qB$

The frequency of the charged particle, say υ, in this case, would be simply $v/2\pi r$, i.e.,

$$\upsilon = v/2\pi \, .qB/mv = qB/2\pi m$$

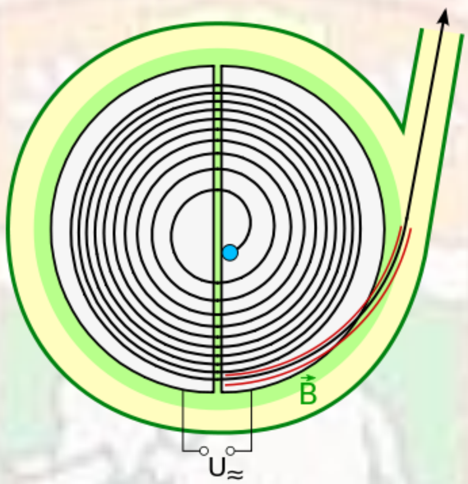
This frequency is therefore, dependent only on,

- the charge (q) of the particle
- the mass (m) of the particle
- the strength (B) of the magnetic field

It does not depend on,

- the speed of the charged particle
- the radius of its circular path

This fact is put to use in the cyclotron - a machine used for accelerating charged particles to high energies. The figure below shows a schematic sketch of a charged particle accelerated in a cyclotron and then ejected through a beamline.

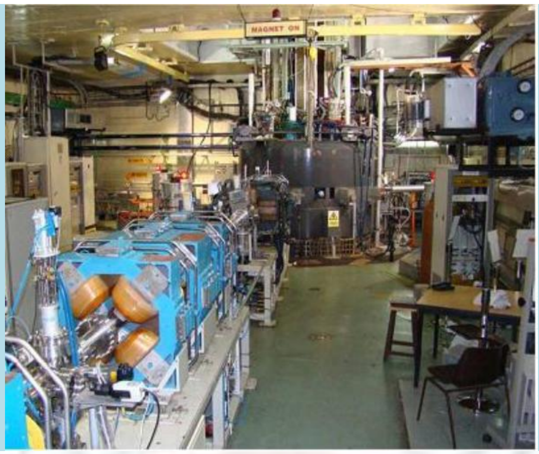


Did you know?

The cyclotron was designed by Lawrence and Livington in 1931. It was designed as an improvement on the 'Linear Particle Accelerator' which was a particle accelerator that had to be spread over distances of several kilometres. The cyclotron used the same idea – giving repeated accelerating pushes to charged particles – as was used in the linear accelerators. However, it made use of magnetic fields to bend the path of the accelerated particles into circles instead of letting them move along straight paths as was done in the case of linear accelerators. This made this machine much more compact and easier to manage.

Cyclotrons are now routinely used for accelerating protons for producing nuclear reactions that among other things can provide us with suitable radio isotopes of different elements. India also has a fair number of cyclotrons in its different laboratories.

For instance, the Variable Energy Cyclotron Centre (VECC), (a research unit of the Department of Atomic Energy, Government of India), is located in Kolkata and performs research in basic and applied nuclear sciences. The Centre houses a 224 cm cyclotron which has been operational since 1977.



Example 1:

A stream of deuterons and alpha particles moving in the x-y plane, are accelerated through the same potential. A uniform magnetic field **B**, directed along the z-axis, acts on both the streams. Find the ratio of the radii of the circular paths described by the two particles.

Solution: The radius, r, of the circular path, described by a charged particle, of charge q and mass m, in a normal magnetic field **B**, is given by

$$mv^2/R = qvB$$

$$\Rightarrow \quad r = mv/qB$$

Here, we have,

For deuterons, $\frac{1}{2}(2m)v_1^2 = eV$

For alpha particles, $\frac{1}{2}(4m)\, v_2^2 = 2eV$

where, m stands for the mass of the proton and V for the accelerating potential used.

Hence, $v_1^2/2v_2^2 = \frac{1}{2}$

Or $v_1^2/v_2^2 = 1$ or $v_1 = v_2$

Therefore $r_1/r_2 = 2mv_1/eB \times 2eB/4mv_2 = 1$

The two particles therefore describe circular paths of equal radii.

Magnetic Force on a current element

We can use the Lorentz Force expression, **F** = q (**v**X**B**), for the magnetic force on a charge q (moving with a velocity a **v**, in a magnetic field **B**) to obtain an expression for the magnetic force on a current element.

Consider a current element, of length dl, carrying a current I, to be present in a magnetic field **B**.

If these are n charge carriers per unit volume, each of charge q, moving with a drift velocity **v**, the current I is given by,

$$I = nqAv$$

where A is the area of cross section of the current element.

The force, on each charge carrier, say, d**F**, is given by

$$d\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

The total number, N, of charge carriers, in the current element of length dl, is given by,

$$N = n\,(Adl)$$

All these charge carriers move with the same drift velocity **v**. hence the total force; say d**F**, on the current element, is

$$d\mathbf{F} = N\,d\mathbf{F}' = n(Adl)\,q\,(\mathbf{v} \mathrm{X} \mathbf{B})$$

From the expression for the current I, we have,

$$nqA = I/v$$

hence, $d\mathbf{F} = Idl/v\,(\mathbf{v} \mathrm{X} \mathbf{B}) = I\,dl\,(\mathbf{v}/v \mathrm{~X~} \mathbf{B})$

therefore, $d\mathbf{F} = I\,dl\,(\check{v} \mathrm{~X~} \mathbf{B})$

It is usual to consider the current element dl itself as a vector quantity whose direction is the direction of flow of current. This direction is the direction of $\check{v}$, the unit vector directed along the direction of the drift velocity. We, therefore, put

$$(dl)\,\check{v} = d\mathbf{l}$$

Therefore, $d\mathbf{F} = I\,(d\mathbf{l} \mathrm{X} \mathbf{B})$

This, then, is the expression for the magnetic force on a current element d**l**, (carrying a current I) in a magnetic field **B**.

For a finite current carrying wire of length l, the total force, **F**, is given by

$$\mathbf{F} = \int_l I\,(d\mathbf{l} \mathrm{X} \mathbf{B})$$

When all the current elements, $d\mathbf{l}_i$, of a finite wire, are along the same direction, we can write

$$\mathbf{F} = I\,(\,\mathbf{I} \mathrm{~X~} \mathbf{B})$$

Where, $l = \sum d\mathbf{l}_i = \sum dli\,\check{\mathbf{v}} = \check{\mathbf{v}}\sum dl_i = \check{\mathbf{v}}l = \mathbf{l}$, in such a case.

Example 1:

A very small wire of length l and mass m is kept near the centre of a solenoid, normal to its axis. The solenoid has a length L and a diameter D and has p layers of insulated wire – each layer having N turns- wound around it. It is known that L >>D and D>>l and both the very small wire, and the axis of the solenoid are in the horizontal plane. If the wire were to carry a current i, find the value of the current, say I, flowing (in the correct sense) in the solenoidal wires, that can balance the weight of the small wire.

Solution:

The weight of the wire is mg and it acts along the vertically downward direction. The very small wire, when carrying a current i, would experience a force of magnitude (i l B) where B is the magnitude of the magnetic field along the axis produced by the current carrying solenoid. We write this because **l** is normal to the axis and **B** is along the axis of the solenoid.

Now, B = μ_0n I, where n=number of turns per unit length of solenoid = pN/L.

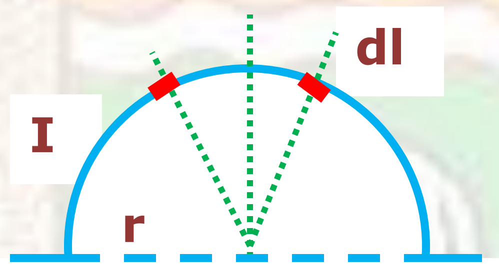
Therefore, B = μ_0p N I/L

We, therefore, need to have, i l (μ_0p N I/L) = mg

Therefore, I = mgL/ μ_0i p Nl

Example 2:

A wire, of length L, carrying a current I, is bent into a semicircular shape and is put in the x-y plane as shown in the figure. If a uniform magnetic field **B** (=B(-**k̂**)) were to act over the region in which the wire is located, find an expression for the magnitude of the total force F, experienced by the whole wire.



Solution

The force, dF, experienced by an element dl, of the wire is given by:

$$dF = I\,dl\,B \sin\frac{\pi}{2} = I\,dl\,B$$

This force has to be perpendicular to both d**l** and **B**. it is, therefore, directed along the raial direction. We break this force into a horizontal and a vertical component. These equal dFcosθ and dFsinθ, respectively.

Taking another symmetrically located current element dl, on the other side of the central vertical semi-diameter, we observe that the horizontal components cancel each other in pairs. The total force therefore is due to the vertical components only. We, therefore, have

$$F = 2\int_0^{\frac{\pi}{2}} I\, dl\, B\, cos\theta$$

But dl = rdθ

Hence, $F = 2\, I\, B\, r \int_0^{\frac{\pi}{2}} cos\theta d\theta = 2\, I\, B\, r\, |\sin\theta|_0^{\frac{\pi}{2}} = 2\, I\, B\, r$

But $2\pi r = L$ (given)

Therefore, $r = L/2\pi$

Thus the total force F, is given by

$$F = (I\, B\, L)/\pi$$

Magnetic force between two Current Elements

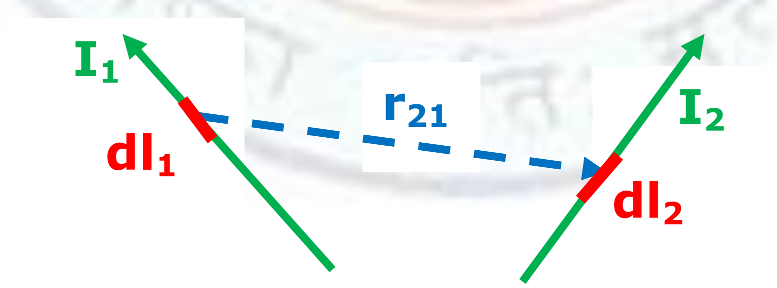
We have seen above that the force, d**F**, on a current element d**l**, in a magnetic field **B** is given by, d**F** = I (d**l** X **B**)

When a current element is present in the vicinity of another current element, we can think of the first current element to be present in the magnetic field produced by the second current element and vice-versa. Each current element would, therefore, experience a force due to the other current element. We can calculate this force by using the Biot-Savart law:

$$d\mathbf{B} = \mu_0\, I/4\pi\ .\ d\mathbf{l} \times \mathbf{r}/r^3$$

for the magnetic field produced by a current element. We need to remind ourselves here that the vector **r**, in the above expression, is the position vector of the field point with respect to the current element.

Consider two current carrying wires, carrying current I_1 and I_2, say, to be present in the vicinity of each other. Let $\mathbf{dl}_1$ and $\mathbf{dl}_2$ be two current elements in these two rays. Also let $\mathbf{r}_{21}$ be the position vector of the current element $\mathbf{dl}_2$ with respect to the current element $\mathbf{dl}_1$. The magnetic field, $\mathbf{dB}_1$ at the current element $\mathbf{dl}_2$, due to the current I_1 in the current element $\mathbf{dl}_1$ is given by:



$$\mathbf{dB}_1 = \mu_0\, I_1/4\pi\ \mathbf{dl}_1 \times \mathbf{r}_{21}/r_{21}^3$$

Therefore, the force, on the current element, $\mathbf{dl}_2$, is

$\mathbf{dF}_{21} = I_2\, \mathbf{dl}_2 \times \mathbf{dB}_1$

$= I_2\, \mathbf{dl}_2 \times [\mu_0 I_1/4\pi\, \mathbf{dl}_1 \times \mathbf{r}_{21}/r_{21}^3]$

$= \mu_0 I_1 I_2/(r_{21}^3 4\pi)\, [\mathbf{dl}_2 \times (\mathbf{dl}_1 \times \mathbf{r}_{21})]$

Similarly, the force $\mathbf{F}_{12}$, on the current element $\mathbf{dl}_1$, due to the current element $\mathbf{dl}_2$ is given by,

$\mathbf{dF}_{12} = \mu_0 I_1 I_2/(\mathbf{r}_{12}^3 4\pi)\, [\mathbf{dl}_1 \times (\mathbf{dl}_2 \times \mathbf{r}_{12})]$

here, $\mathbf{r}_{12}$ (=$-\mathbf{r}_{21}$) is the position vector of the current element $\mathbf{dl}_1$ w.r.t. to current element $\mathbf{dl}_2$.

The total force, on the current carrying wire 2, due to the current carrying wire 1, would be given by:

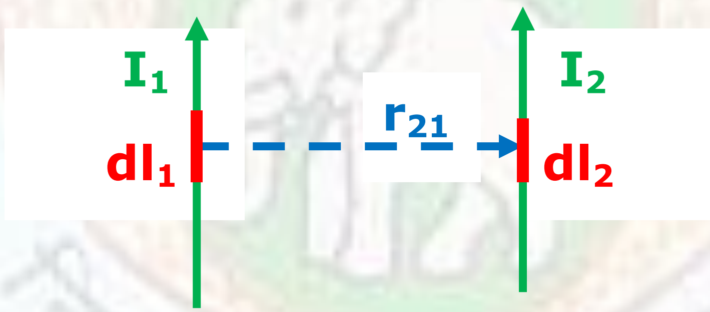
$\mathbf{F}_{21} = \mu_0 I_1 I_2/4\pi \int_1\int_2 \mathbf{dl}_2 \times (\mathbf{dl}_1 \times \mathbf{r}_{21})/\mathbf{r}_{21}^3$

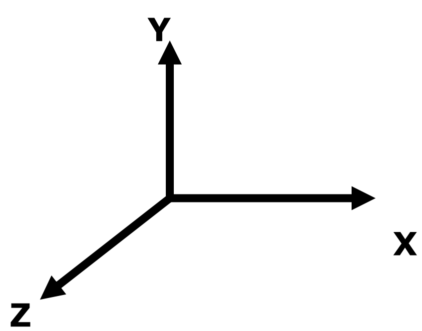
Similarly, the total force on the current carrying wire 1, due to the current carrying wire 2 would be given by:

$\mathbf{F}_{12} = \mu_0 I_1 I_2/4\pi \int_2\int_1 \mathbf{dl}_1 \times (\mathbf{dl}_2 \times \mathbf{r}_{12})/\mathbf{r}_{12}^3$

We then have, $\mathbf{F}_{12} = (-)\, \mathbf{F}_{21}$, i.e., the two forces are equal and opposite to each other.

The Special Case of two Straight Parallel Current Elements





For the special case of two long parallel straight current carrying wires, we can write (in terms of the co-ordinate axis shown),

$\mathbf{dl}_2 = (dl_2)\hat{\mathbf{j}}$

$\mathbf{dl}_1 = (dl_1)\hat{\mathbf{j}}$

and, $\mathbf{r}_{21} = r\hat{\mathbf{i}}$

therefore, $d\mathbf{F}_{21} = \mu_0 I_1 I_2 / 4\pi r^3 [dl_2 \mathbf{j} \times (dl_1 \mathbf{j} \times r\mathbf{i})]$

$= \mu_0 I_1 I_2 / 4\pi r^3 [dl_2 \mathbf{j} \times r dl_1 (-\mathbf{k})]$

$= \mu_0 I_1 I_2 / 4\pi r^3 . r(dl_2)(dl_1)[\mathbf{j} \times (-\mathbf{k})]$

$= \mu_0 I_1 I_2 / 4\pi r^2 (dl_2) \times (dl_1)(-\mathbf{i})$

Similarly, $d\mathbf{F}_{12} = \mu_0 I_1 I_2 / 4\pi r^3 (dl_1 \mathbf{j}) \times (dl_2 \mathbf{j} \times r(-\mathbf{i}))$

$= \mu_0 I_1 I_2 / 4\pi r^3 (dl_1 \mathbf{j}) \times r dl_2 (+\mathbf{k})]$

$= \mu_0 I_1 I_2 / 4\pi r^2 (dl_2)(dl_1)(\mathbf{j} \times \mathbf{k})$

$= \mu_0 I_1 I_2 / 4\pi r^2 (dl_2) \times (dl_1)(\mathbf{i})$

It is thus seen that $d\mathbf{F}_{12} = (-)d\mathbf{F}_{21}$

We also observe another interesting thing here. The two current elements carry currents in the same sense, i.e. they carry like currents. The force on current element 2 due to the current element 1 is along (-**i**), i.e. it is directed towards the current element 1. Similarly, the force on current element 1, due to the current element 2, is along (+**i**), i.e., it is directed towards current element 2. We can, therefore, say that the two current elements are attracting each other. We thus observe that:

'Like currents attract each other'.

Magnetic Force between two long (infinite) parallel current carrying straight wires

We now calculate the magnetic force between two long current carrying straight wires. Here we can make use of result for the magnetic field due to a straight, long, current carrying wire. The magnetic field due to wire 1, at any point of wire 2 is,

$\mathbf{B}_1 = (\mu_0 I_1 / 2\pi d)(-\mathbf{k})$

Hence, the force on length l, of wire 2, is:

$\mathbf{F}_{21} = I_2 \, l \times \mathbf{B}_1$

$= I_2 \, l \, \mathbf{j} \times (\mu_0 I_1 / 2\pi d)(-\mathbf{k})$

$= (\mu_0 I_1 I_2 / 2\pi d). \, l \, (-\mathbf{i})$

Similarly the force, on length l of wire 1 due to wire 2, is:

$\mathbf{F}_{12} = (\mu_0 I_1 I_2 / 2\pi d). \, l \, (+\mathbf{i})$

The two wires, carrying like currents are thus seen to attract each other. It is easy to realize that if the wire were to carry unlike currents, they would repel each other. We thus again confirm that:

'Like currents attract each other while unlike currents repel each other'.

The magnitude of the force on a unit length of either wire due to the other wire is seen to be:

$$F = \mu_0 I_1 I_2 / 2\pi d$$

The definition of Ampere-the SI unit of current

We use the above result for the force per unit length to define 'ampere', the SI unit of current. We observe that (since, $\mu 0 = 4\pi \times 10^{-7}$ SI units)

$$F = 2 \times 10^{-7} \text{ N/m}$$

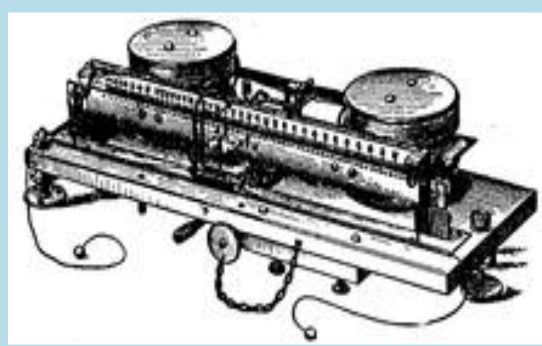
where, $I_1 = I_2 = 1$ ampere and $d = 1$m

Thus we can say that:

The 'ampere' is that amount of current which when flowing in each of two (infinitely) long parallel straight wires, kept one meter apart in vacuum, produces between them a force of 2×10^{-7} newton on each one meter length of either wire.

Did you know?

The nature of the force between two parallel current carrying wires was used by Ampere and by Kelvin to design instruments called a 'current balance'. Here the force between the two wires was balanced by the weight of a suitable mass. Using the value of the mass needed to attain 'balance', one could then calculate the value of the equal current flowing in each of the two wires.



Ampere's current balance

Torque on a Current Loop in a Uniform Magnetic Field

We know that:

(a) a current carrying loop is equivalent to a magnetic dipole.

(b) an electric dipole, when put in a uniform electric field, **E**, experiences a torque $\boldsymbol{\tau}$ where,

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

Here, **p** is the dipole moment of either electric dipole.

A combination of these two statements implies that a current carrying loop, when put in a uniform magnetic field, B, would experience a torque. This torque would be given by:

$$\boldsymbol{\tau} = \boldsymbol{\mu}_m \times \mathbf{B}$$

Here, $\boldsymbol{\mu}_m$ is the magnetic dipole moment associated with the current loop. As we have already seen,

$$\boldsymbol{\mu}_m = A\, I\, \hat{\mathbf{n}}$$

where, A is the area enclosed by the loop and I is the current through the loop. The unit vector $\hat{\mathbf{n}}$ is normal to the plane of the loop and is the sense of advance of a right handed screw rotated in the sense of flow of current through the loop.

Hence, $\boldsymbol{\tau} = A\, I\, \hat{\mathbf{n}} \times \mathbf{B}$

It follows that, $|\boldsymbol{\tau}|=0$ when the plane of the coil is normal to the direction of the magnetic field. Also, $|\boldsymbol{\tau}|$would be maximum when the plane of the coil is parallel to the direction of the magnetic field.

For a given length of a wire, drawn into a loop, the torque $\boldsymbol{\tau}$, would be maximum (for given values of I and **B**) when the wire is shaped into a circular loop. This is because for a given perimeter, the circle encloses the maximum area.

The torque, experienced by a current loop, in a magnetic field, is put to practical uses. The moving coil galvanometer, a common instrument used in the laboratory, is a well known example of a device making use of this torque.

Did you know?

The moving coil galvanometer generally uses a rectangular coil. This coil has two of its sides parallel to the field lines of the magnet between which the coil moves. To ensure that this condition remains satisfied, the magnet used in a moving coil galvanometer, is designed to produce a radial magnetic field.

Summary

- A particle of charge q, moving with a velocity **v**, in a magnetic field **B**, experiences a force **F** where,

  $$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

  This force is usually referred to as the magnetic Lorentz Force acting on a charged particle.
- We can associate several special features with the magnetic force acting on a charged particle.
- A magnetic field, acting on a moving charged particle, causes the particle to move along a helical trajectory. If the magnetic field is directed normal to the plane of motion of the particle, the particle would describe a circular path in its plane of motion.
- The circular trajectory described by a charged particle in a normal magnetic field, is put to use in the cyclotron – a device for accelerating charged particles.
- A current element, d**l**, carrying a current I, experiences a force d**F**, in a magnetic field **B**. We have,

  $$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$$
- Two current carrying elements $d\mathbf{l}_1$ and $d\mathbf{l}_2$, carrying currents I_1 and I_2, exert forces on each other. This force is given by

  $$d\mathbf{F} = \mu_0\, I_1 I_2 / 4\pi\, r^3 .(d\mathbf{l}_2) \times (d\mathbf{l}_1 \times (-\,\mathbf{r})$$

  or $$d\mathbf{F} = \mu_0\, I_1 I_2 / 4\pi\, r^3 .(d\mathbf{l}_2) \times (d\mathbf{l}_1 \times (-\,\mathbf{r})$$

  depending on which current element is considered as experiencing the force due to the other current element.
- For two straight parallel current elements, separated by a distance r, in vacuum, the force d**F** has a magnitude dF given by:

  $$dF = (\mu_0\, I_1 I_2 / 4\pi\, r^2)(dl_2)(dl_1)$$
- Two long straight current carrying wires, kept parallel to each other separated by a distance r in vacuum, exert a force on each other. This force has a magnitude ($\mu_0\, I_1 I_2 / 2\pi d$) per unit length of either wire.
- We can use the above expression to define 'ampere', the SI unit of current.
- It turns that like currents attract each other while unlike currents repel each other.
- A current carrying coil experiences a torque in a magnetic field.
- The torque experienced by a current carrying coil, in a magnetic field, **B**, equals ($\boldsymbol{\tau} = \boldsymbol{\mu}_m \times \mathbf{B}$). Here $\boldsymbol{\mu}_m$ is the magnetic moment associated with the coil and has a magnitude equal to the product of the current I through the coil and the area of the coil.
- The direction of $\hat{n}$, of the magnetic moment vector associated with a current carrying coil, is normal to the plane of the coil. Its sense is that of advance of a right handed screw rotated along in the sense of flow of current. The torque $\boldsymbol{\tau}$ experienced by a current loop, in a magnetic field is given by

  $$\boldsymbol{\tau} = I\, A\, \hat{n}\, \mathbf{B}$$

Exercise

Fill in the Blanks

Fill in the blanks in the following statements with appropriate words/expressions:

(i) The Lorentz magnetic force, on a charge q, moving with a velocity of **v**, in a magnetic field, **B**, is given by **F**=_________________

(ii) A moving charged particle experience maximum force in a magnetic field when it is moving ___________ to the magnetic field lines.

(iii) The magnetic force, on a current element, is ______________proportional to the length of the current element.

(iv) The general expression for the force between two current elements, is obtained by using the ___________law and the expression for the ______________(magnetic) force.

(v) A current carrying coil, present in a magnetic field, experiences ____________torque when the magnetic field lines are directed normal to the plane of the coil.

Answers:

(i) q (**v** X **B**)

(ii) perpendicular

(iii) directly

(iv) Biot-Savart; Lorentz

(v) zero

True or False

State whether the following statements are true or false:

(i) A charged particle, moving in a magnetic field, can still experience zero force.

(ii) A charged particle, moving in a magnetic field, always follows a circular trajectory.

(iii) The force between two current elements is inversely proportional to the product of the currents through them.

(iv) We use the expression, for the force between any two straight parallel current carrying wires for defining the SI unit of current.

(v) The magnitude of the torque, experienced by a current carrying coil in a magnetic field, depnds on the area enclosed by the coil.

Answers:

(i) true

(ii) false: the general trajectory is a helical one

(iii) false (the force is directly proportional to the product of the two currents)

(iv) false (we use the expression for the force per unit length between two long (infinite) straight parallel current carrying wires for defining the SI unit of current.

(v) true

Multiple Choice Questions

In the following questions, a statement is followed by four choices only one of which is a logically valid choice. You have to select that correct choice in each case.

(i) A particle of charge q moves with a velocity of **v**, in a magnetic field, **B.** The magnitude of the magnetic force, experienced by the particle,

(a) would be always zero

(b) would be always equal to $1/\sqrt{2}$ qvB

(c) would be always equal to q v B

(d) can be anywhere between zero to qvB

Answer: (d)

Justification for the answer: The magnetic force **F** is given by

F = q (**v** X **B**)

Hence the magnitude is qvBsinθ. This can vary from zero when θ=0 to qvB when $\theta = \frac{\pi}{2}$. Hence choice (d) is correct choice.

(ii) An electron beam and a proton beam are both accelerated from rest through a potential V. The beams, while moving in a horizontal plane, are subjected to a uniform magnetic field having its field lines in the vertical direction. The ratio of the radii of the circular paths, described by the electron and the proton would be nearly

(a) 1850

(b) 43

(c) 1/43

(d) 1/1850

Answer: (c)

Justification for the answer: The radius of the particle is given by,

$$mv^2/r = qvB$$

$$\text{or } r = mv/qB$$

Here the two particles have equal kinetic energies (=eV). Hence their momenta (momentum = $\sqrt{2mK} = \sqrt{2meV}$) would be in the ratio of the square roots of their masses. The other quantities (charge and B) being equal, the ratio of the radii of their circular paths is given by:

Radius of electron path/radius of proton path = (mass of electron/mass of proton)$^{1/2}$ ≈$(1/1850)^{1/2}$ ≈1/43.

Hence choice (c) is the correct choice.

(iii) Two long straight parallel wires, carrying antiparallel currents of 2.0A and 4.0A respectively are kept 50 cm apart in vacuum. The magnitude of the force experience by a 5cm long segment of either wire would then be

(a) 3.2×10^{-9} N

(b) 1.6×10^{-9} N

(c) 3.2×10^{-9} N

(d) 1.6×10^{-7} N

Answer: (d)

Justification for the answer: The force, per unit length, on either wire equals $\mu_0 I_1 I_2 / 2\pi d$. Hence the force, on a length l equal $(\mu_0 I_1 I_2 / 2\pi d) \times l$. Substituting the values, we get

$F = 4\pi \times 10^{-7} \times 2 \times 4 / 2\pi(1/2) \times 5/100$ N

$F = 160 \times 10^{-9}$ N $= 1.6 \times 10^{-7}$ N

Hence choice (d) is the correct choice.

(iv) A square current carrying coil is kept in the region of a uniform magnetic field whose magnetic field lines are parallel to two of the sides of the square coil. We can then say that:

(a) all four sides of the square would experience equal forces, all in the same direction.

(b) only two sides would experience equal forces which would act along the antiparallel directions

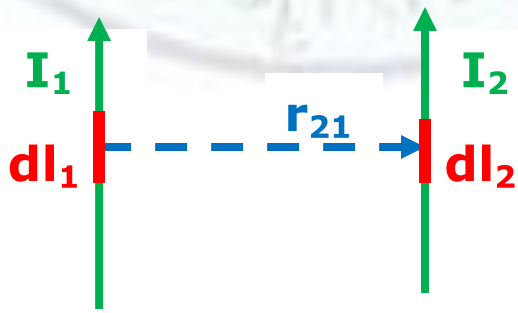
(c) all four sides of the square would experience equal forces with the two pairs of parallel sides experiencing forces in antiparallel directions.

(d) only two sides would experience equal forces which would act along parallel directions.

Answer: (b)

Justification for the answer: The force is zero on the two sides of the square that are parallel to the magnetic field lines. The other two sided (normal to the magnetic field lines) experience equal forces but these forces have anti parallel directions. This is because these two sides carry currents along anti parallel directions.

(v) For the two current elements shown here, the magnitude of the force, experienced by either element would be



(a) $(\mu_0 I_1 I_2/ 4\pi r^2)(dl_2)(dl_1)$

(b) $(\mu_0 I_1 I_2/ 2\pi r^3)(dl_2)(dl_1)$

(c) $(\mu_0 I_1/ 4\pi I_2 r^2)(dl_2)(dl_1)$

(d) $(\mu_0 I_1/ 4\pi I_2 r^3)(dl_2)(dl_1)$

Answer: (a)

Justification for the answer: The magnetic field, due to the first current element, at the location of the second element, is

$d\mathbf{B} = \mu_0 I_1/4\pi \, . \, d\mathbf{l_1} \times r\mathbf{i}/r^3$

$= \mu_0 I_1/4\pi \, . \, d\mathbf{l_1} \times (-\mathbf{k})/r^2$

Therefore the force on the second current element is

$d\mathbf{F} = I_2 (d\mathbf{l}_2 \times d\mathbf{B}) = (\mu_0 I_1 I_2/ 4\pi r^2)(dl_2)(dl_1)(-\mathbf{i})$

therefore, the magnitude of the force is $(\mu_0 I_1 I_2/ 4\pi r^2)(dl_2)(dl_1)$

hence choice (a) is correct.

Short Notes

Write short notes on:

(i) Lorentz magnetic force

(ii) Circular trajectory of a charged particle in a magnetic field.

(iii) Magnetic force on a current element

(iv) Define the SI unit of current.

(v) Torque on a current loop in a magnetic field.

Essay Type Questions

(i) State and discuss the special characteristics of the magnetic force on a charged particle.

(ii) Discuss with appropriate reasoning the condition under which a charged particle moving in a magnetic field, describes:

(a) circular (b) helical trajectory

(iii) Starting from the Lorentz magnetic force expression, obtain an expression for the magnetic force experienced by a current element.

(iv) Obtain expressions for the magnetic force between

(a) two current elements (b) two current carrying wires

(v) Obtain an expression for the magnetic force between two long straight current carrying wires, kept parallel to each other, in vacuum and separated by a distance r.