Chapter 6.1: Analysis of AC circuits using Kirchhoff’s laws

Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Chapter 6.1: Analysis of AC circuits using Kirchhoff’s laws

Lesson Developer: Dr. Narmata Soni
College/ Department: Hans Raj College, University of Delhi

LEARNING OBJECTIIVES

Going through this chapter, the reader would know

- the basic difference between A.C and D.C. Circuits.
- the significance of the phase difference between the current and the voltage in A.C circuits.
- the need for using complex variables for currents, voltages and impedances in A.C circuits
- the basic ideas of the algebra of complex numbers.
- the modified form of Kirchhoff's laws for A.C circuits.
- the details of using Kirchhoff's laws for analyzing different types of A.C circuits.

INTRODUCTION

We start this chapter by pointing out the basic differences between a.c and d.c circuits. The importance and significance of the phase difference, between the current flowing and the voltage applied, in an a.c circuit is highlighted. The basic ideas, on the algebra of complex numbers, are recapitulated and the need for using the complex equivalents of the currents, voltages and impedances in a.c circuits is clearly explained.

The modified form of Kirchhoff's laws for a.c circuits is stated and explained. The details of using these laws, for analyzing different types of circuits are discussed and explained.

<u>AC Circuits</u>

An AC circuit, as we know, is a circuit that is 'powered', or 'driven', by an alternating voltage source. The simplest type of an alternating voltage is the sinusoidal source, i.e., a 'source' for which the (instantaneous) voltage is a sine, or cosine, function of time. We represent the time dependence, of the voltage of such a source, by

$$V = V_o \, Sin \, \omega t$$

$$V = V_o \, Cos \, \omega t$$

The current, flowing in different branches of such a 'source', would also be a sinusoidal function of time.

<u>The (usual) components of an AC circuit:</u>

Unlike the simple DC circuits, an AC circuit usually has, in addition to resistors, inductances and capacitance also as its essential components. These can be connected to form simple series and parallel circuits as well as relatively complicated circuits involving 'cross connections'.

A significant difference between an AC circuit, and a DC circuit, as we know, is that the current flowing in an AC circuit is usually not in phase with the voltage applied. The current flowing in pure inductor lags behind the applied voltage, in phase, by $\frac{\pi}{2}$. Similarly the current flowing, in a pure capacitor, leads the applied voltage, in phase, by $\frac{\pi}{2}$. The current flowing, in a pure resistor, however, remains in phase with the applied voltage.

The well-known '<u>Phasor Diagram'</u> approach incorporates these 'phase relationships', between the 'current' and 'voltage', for each of the three components – the resistor, the capacitor and the inductor. This approach provides us with a convenient method for analyzing AC circuits made from a series combination of these three components. The more general approach, for analyzing all types of AC circuits, is however, the one based on the use of 'complex numbers'.

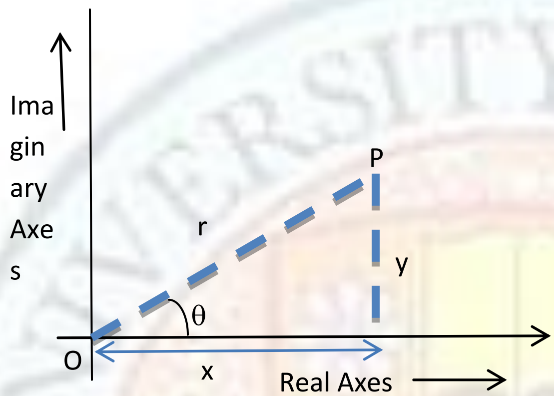
<u>A Quick Recapitulation of the basic ideas of Complex numbers</u>

A complex number, Z, is represented in term s of its real part (x), and its imaginary part (y), as

$$Z = x + j\,y \quad (j = \sqrt{-1})$$

This complex number can be represented by a point P on the 'Argand diagram'. In this diagram, the two axes correspond to the real and imaginary parts of the complex number and the point P has these two parts, (i.e. x and y), as its coordinates.

The point P on the Argand diagram, can also be assigned the (polar) coordinates r and θ, as shown here.



We clearly have

$$x = r\cos\theta \text{ } and \text{ } y = r\sin\theta$$

$$Z = x + j\,y = r\,(\cos\theta + j\sin\theta) = r\,e^{j\,\theta}$$

'r' and 'θ' are referred to as the 'modulus' and 'argument' of the complex number, Z. We can, therefore, represent a complex number, Z,

(i) Either through its real (=x) and the imaginary (=y) parts
(ii) Or through its modulus (= r) and argument ($= \theta$).

It is the latter representation that is almost invariably used while analyzing AC circuits using the 'complex number based approach'

The interrelations between (x, y) and (r, θ) are:

$$x = r\cos\theta \text{ } and \text{ } y = r\sin\theta$$

and $r = \sqrt{x^2 + y^2}$; $\theta = tan^{-1}({}^{y}/_{x})$

We can use these relations to convert the (real, imaginary part) representation of a given complex number, into its equivalent (modulus, argument representation) and vice versa.

<u>Two interesting Results:</u>

Let there be a complex number, Z, where

$$Z = x + j\,y$$

We then have

$$Z' = j\,Z = j\,x + j^2\,y \;=\; -y + j\,x$$

In the modulus, argument form, the two numbers Z and Z', would be represented as

$Z = r\,e^{j\,\theta}\ where\ r = \sqrt{x^2 + y^2}\ ;\ \theta = tan^{-1}\left({}^{y}/_{x}\right)$ and

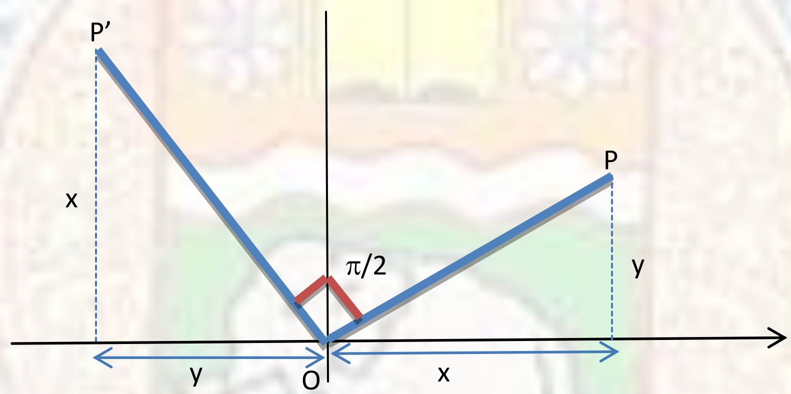
$Z' = r'\,e^{j\,\theta'}\ where\ r' = \sqrt{(-y)^2 + x^2}\ ;\ \theta' = tan^{-1}\left({}^{x}/_{-y}\right)$

We observe that r' = r

And $\tan\theta' = -\left({}^{x}/_{y}\right) = -\cot\theta$

$$\therefore\ \theta' = \left({}^{\pi}/_{2} + \theta\right)$$

On the Argand diagram, the numbers, Z and Z', would correspond to points P and P' such that OP' and OP and the angle between OP'and OP equals ${}^{\pi}/_{2}$.



We can express these results by saying that:

Multiplication of a complex number, by j, can be viewed as equivalent to a simple rotation, of its representative (position vector) line, on the Argand diagram, through${}^{\pi}/_{2}$.

In a similar way, we can see that the division of a complex n umber by j (which is equivalent to its multiplication by –j) would result in a simple rotation of its representative (position vector) line, on the Argand Diagram, through$(-{}^{\pi}/_{2})$.

These interesting results are put to use while analyzing AC circuits through the complex number based approach.

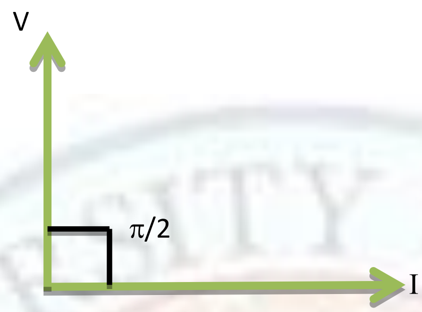
<u>Assigning 'Complex Impedances' to the Inductor and the Capacitor</u>

$$V = V_o\ Sin\ \omega t$$

Is applied across a pure inductor, the current flowing in the inductor is given by

$$I = I_o\ sin\left(\omega t - {}^{\pi}/_{2}\right)\ where\ I_o = \frac{V_o}{\omega L}$$

The impedance (X_L) of a pure inductor (L), therefore, equals ωL and the current, in it, lags in phase, with respect to the applied voltage by $\pi/_2$. The 'phasors', corresponding to the current and voltage, are, therefore, represented as shown.



Now voltage = current x impedance

This result, and the phase relation between the current and voltage, can be linked with the effect, of multiplication by j, on a given complex number. This can be done by assigning a 'complex Impedance', Z_L, to an inductor, where

$$Z_L = j\,\omega L$$

We now imagine the 'current flowing' and the 'voltage applied' (across a pure inductor) to be replaced y their 'complex equivalents', given by

$$V = V_o\, Sin\, \omega t = Im(V_o\, e^{j\,\omega t})$$

And

$$I = I_o\, sin\left(\omega t - \pi/_2\right) = Im\left(I_o\, e^{j\,(\omega t - \pi/_2)}\right)$$

here I_m implies that we need to take the imaginary part, of the complex numbers within the brackets, to get the actual 'voltage' and 'current' values.

The multiplication of this complex current, by$j\,\omega L$, thus results in a (complex) voltage value that not only has the correct magnitude but also has the correct phase relationship with the (complex) current. Assignment of the equivalent complex impedance $(=j\,\omega L)$to a pure inductor (and imagining the current and voltage to be replaced by their complex representations) thus helps us to incorporate not only the effect associated with its magnitude $(=\omega L)$but also gives us the correct phase relationship between the 'current' and 'voltage'.

A similar analysis when done for a pure capacitor, which shows that we need to assign it a complex impedance, Z_C, where

$$Z_C = \frac{1}{j\,\omega C} = \frac{-j}{\omega C}$$

For a pure resistor, its equivalent complex impedance, Z_R, would still be equal to R only. This is because the 'applied voltage' and the 'current flowing' are in phase with each other for a pure resistor.

<u>The 'Complex Number Based Approach' for Analyzing AC Circuits:</u>

The above results suggest that we can analyze AC circuits, using complex numbers (in their modulus argument representation)by following the approach suggested below:

(i) Imagine the applied sinusoidal voltage to be replaced by its equivalent complex voltage:

$$V = V_o\ Sin\ \omega t = Im(V_o\ e^{j\ \omega t})$$

Or $V = V_o\ Cos\ \omega t = Re(V_o\ e^{j\ \omega t})$

Here (Im) and (Re) imply that we need to take the imaginary and real part, respectively, of the complex number within the brackets.

(ii) Assign equivalent complex impedances (Z_R = R; $Z_L = j\ \omega L$; $Z_C = \frac{1}{j\ \omega C} = \frac{-j}{\omega C}$;) to the resistors, inductors and capacitors present in the circuit.

(iii) Use the equivalent complex voltage and the assigned complex impedances, to analyze the given circuit. This analysis would use the

  (a) Standard 'series' and 'parallel' circuit rules, of DC circuits, in case the circuit, under consideration, can be viewed as a 'series', or a 'parallel' circuit or a 'mixture' of these two types of circuits. [For example, we would say that the equivalent complex impedance (Z) of a parallel combination of an inductor, L, and a capacitor, C, is given by

$$\frac{1}{Z} = \frac{1}{Z_L} + \frac{1}{Z_C} = \frac{1}{j\omega L} + j\omega C = \frac{1 - \omega^2 LC}{j\omega L}$$

  $\therefore Z = \frac{j\omega L}{1-\omega^2 LC}$]

  (b) Standard Kirchhoff's laws 9the junction law and the loop law), in case the circuit cannot be viewed as a mixture of series and parallel circuits.

(iv) We would finally replace the calculated 'complex current', in each branch of the circuit by its imaginary or real part. This would depend on whether the applied voltage was the 'imaginary' or 'real' part of the equivalent, 'complex voltage' used to represent it. We thus get the current in each branch/ part of the circuit. The circuit, therefore, stands analyzed.

It is important to note that the 'complex number based approach', for analyzing AC circuits uses exactly the 'same' rules and procedures as are used for analyzing similar DC circuits. This approach works because the assignment of the equivalent complex impedances (Z_R = R;$Z_L = j\ \omega L$; $Z_C = \frac{1}{j\ \omega C} = \frac{-j}{\omega C}$;) to resistors, inductors and capacitors, enables us to incorporate their 'magnitude' as well as 'phase' characteristics in a single (complex) number.

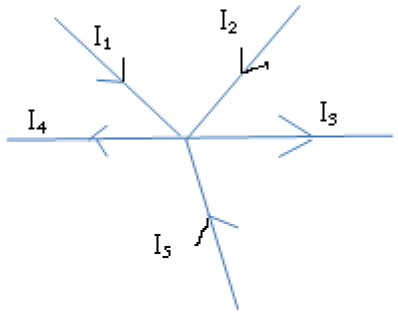
We now illustrate thus approach by using it to first analyze a simple 'series' and a simple 'parallel' circuit using L, C and R. We would then apply the equivalent (complex number based) Kirchhoff's rules to more complex AC circuits and use them to analyze a variety of AC circuits.

<u>Kirchhoff's Rules for AC Circuits:</u>

We now state that the (Complex number based) Kirchhoff's rules for AC circuits. Their basic form and statement, remains the same as for DC circuits. We, however, have to now use the complex equivalents of the voltages, currents and impedances present in the different branches of the circuit.

<u>Kirchhoff's first rule:</u>

The algebraic sum of all the complex equivalents of all the currents, meeting at a junction point, in a circuit, equals zero.



Thus, at the junction point, A, shown here, we would have

$$I_1 + I_2 - I_3 - I_4 + I_5 = 0$$

The sign convention, followed here, is the same as for DC circuits. A complex current is taken as positive if it is flowing towards the junction. It is taken as negative if it is flowing away from the junction.

<u>Kirchhoff's second rule:</u>

Kirchhoff's second rule, as in the case of DC circuits, is stated for a closed path, or a loop in circuit. We can state it in a similar way, as follows:
The algebraic sum of all the (complex) potential drops, across different elements, of a closed path, or a loop, in a circuit is zero.

We need to have a clear cut meaning of the 'potential drop' across any element and the sign convention, to be followed, while using this rule. For this purpose we say that

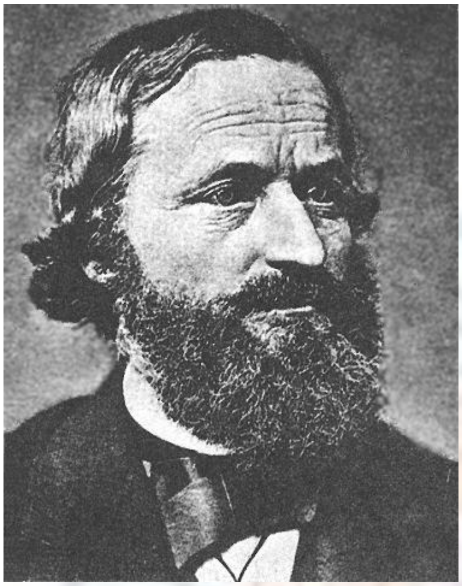
(i) The (complex) potential drop, across any complex impedance, equals the product of the (complex) current through it and the (complex) impedance assigned to it.
(ii) The (complex) potential drop, across any voltage source, equals the complex equivalent voltage of the source.

The sign convention, to be followed, is stated in terms of the sense of traversal of the closed path, or loop, described in the circuit. We say that

(i) The (complex) potential drop, across impedance, is to be assigned a positive sign if this impedance gets traversed in a sense opposite to the assumed sense of flow of the (complex) current through it.
(ii) The (complex) potential drop, across a voltage source, is assigned a positive sign if it gets traversed in the same sense as is the assumed sense of flow of the (complex) current through it.

These two rules, used as per the sign convention suggested here, enable us to write as many equations for the circuit as is the number of unknown currents in its different branches. We can then solve these equations to find the (complex equivalents) of all the unknown currents. These complex equivalent currents can finally be used to find the actual unknown currents in all the branches of the circuit.

Did You Know



Gustav Robert Kirchhoff

(1824 - 1887)

Sir G R Kirchhoff is the German Physicist from the kingdom of Prussia now in Russia. His contribution in the various branches of Physics is remarkable. He was just a student when he formulated the circuit laws to simplify and solve the complex ac and dc circuits. In the branch of thermal Physics, he proposed the laws of thermal radiation and also proved them.

He is the recipient of prestigious Rumford Medal for his contribution in the field of research on the fixed lines of the solar spectrum.

He gives three laws that elaborates the spectral composition of light emitted by incandescent objects, is big contribution in the field of spectroscopy. His work is extended to the field of optics, where his effort provides a solid foundation for the Huygen's principle.

The great theorist died in in 1887.

//Ref: http://th.physik.uni-frankfurt.de/~jr/physpicold.html

Application of Kirchhoff's laws to AC circuits:

We are now in a position to apply the complex number based approach to Ac circuits. We start with the simple cases of

(i) Series LCR (or acceptor circuit) and

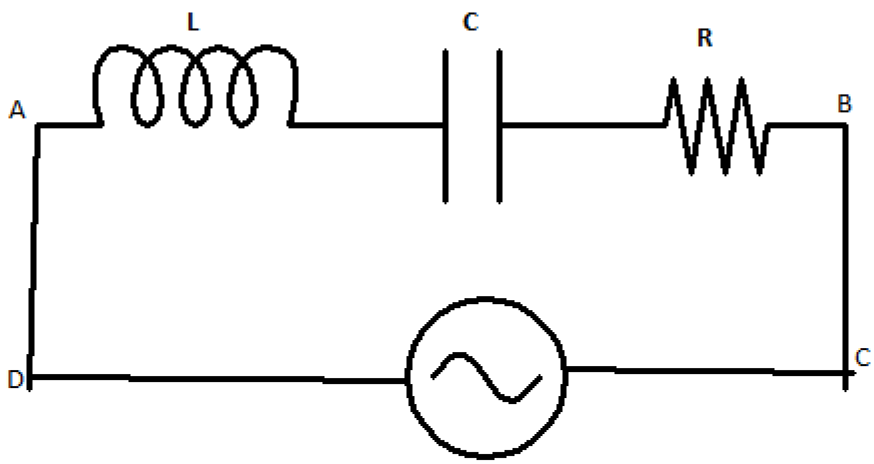
(ii) Parallel LCR (or rejector circuit)

We would then move over to the case of Wheatstone bridge, and Wheatstone bridge like circuits, which find very many interesting practical applications.

It is possible to analyze the first two circuits without explicitly using Kirchhoff's laws. We, however, illustrate the analysis of these circuits by using Kirchhoff's laws.

The Series LCR circuit; An 'Acceptor' circuit

The simple 'series LCR' circuit, has the form shown here.



V = V_o Sinωt

The complex impedances of L, C and R as we have already seen, are $j\,\omega L$; $\frac{1}{j\,\omega C}$ $(= \frac{-j}{\omega C})$ and R respectively.

The sinusoidal applied voltage corresponds to the complex form $V_o\,e^{j\,\omega t}$. Let I represent the (complex) current flowing through the circuit.

$$I\left(j\,\omega L + \left(\frac{-j}{\omega C}\right) + R\right) = V_o\,e^{j\,\omega t}$$

$$I\left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right) = V_o\,e^{j\,\omega t}$$

Now, $R + j\left(\omega L - \frac{1}{\omega C}\right) = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\;e^{j\,\theta}$, *where* $\tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$

$$\therefore I = \frac{V_o\,e^{j\,\omega t}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\;e^{j\,\theta}}$$

$$\frac{V_o\,e^{j\,(\omega t - \theta)}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = I_o e^{j\,(\omega t - \theta)}$$

$$where\;\; I_o = \frac{V_o}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The current, in the circuit, therefore, has the form (using $i = Im\,(I)$)

$$i = I_o \sin\,(\omega t - \theta)$$

The expression for I_o, given above, shows that the impedance, Z, of the circuit, is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The current is seen to lag behind the voltage, in phase, by an angle , where as seen above,

$$\tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

The series LCR Circuit, therefore, has a net impedance

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

And the current, in it, 'lags behind' the voltage, by an angle θ where

$$\theta = tan^{-1}\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

The expression for Z shows that the impedance, of the series LCR circuit, acquires its minmum value, say Z_m, when $\left(\omega L - \frac{1}{\omega C}\right) = 0$. This implies that for

$$\omega L = \frac{1}{\omega C} \ or \ \omega_o = \frac{1}{\sqrt{LC}}$$

$$v_o\left(= \frac{\omega}{2\pi}\right) = \frac{1}{2\pi\sqrt{LC}}$$

The impedance of the circuit becomes equal to R only. Incidently, for $\omega = \omega_o \left(i.e. \omega_o L = \frac{1}{\omega_o C}\right)$, we also see that $\tan\theta = 0 \ so \ that \ \theta = 0^o$. Thus at $\omega = \omega_o$,

(i) The amplitude of the current, flowing in the circuit, becomes maximum $\left(= \frac{V}{R}\right)$.

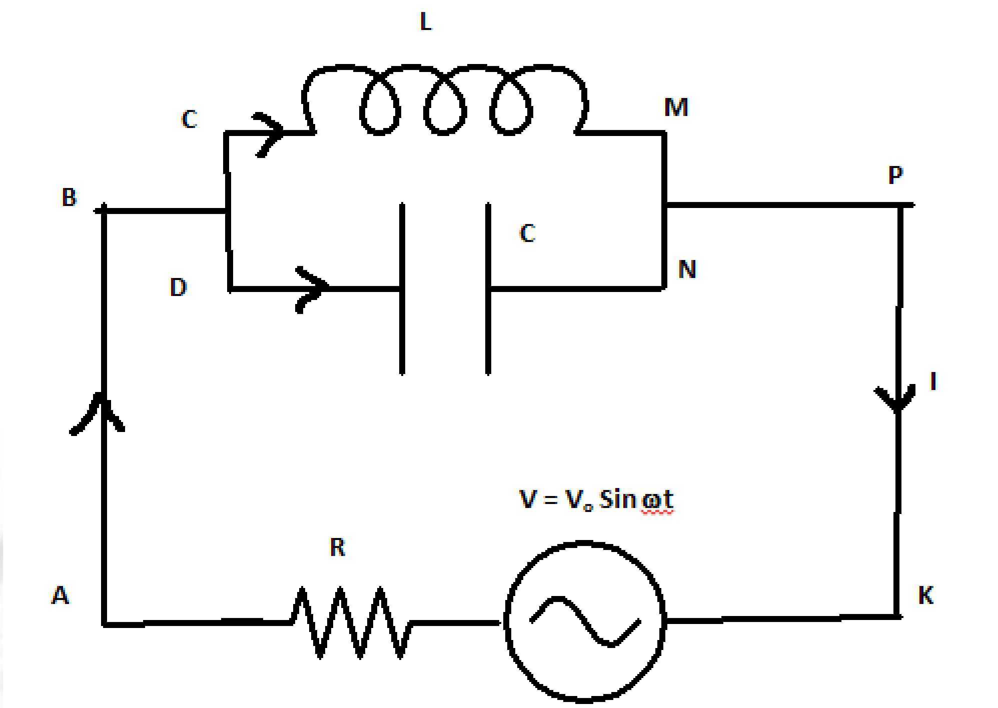
(ii) The current flows in phase $(\because \theta = 0^o, at \ \omega = \omega_o)$ with the applied voltage.

These features, of the series LCR circuit, are expressed by saying that the circuit shows RESONANCE when the (angular) frequency, of the applied alternating voltage, becomes equal to $\frac{1}{\sqrt{LC}}$.

Suppose that a given series LCR circuit were subjected to several ac signals of different (angular) frequencies. The circuit would then have the current flowing through it, attains its peak or maximum amplitude at $\omega = \omega_o \left(= \frac{1}{\sqrt{LC}}\right)$. The response of the circuit, to frequencies other than ω_o, would be very much weaker because of relatively smaller values of the current amplitudes for all these frequencies. The signal, corresponding to the (angular) frequency, ω_o, thus 'stands out', or it is 'accepted' much more 'whole heartedly' than the signals of other (angular) frequencies. We highlight this special feature (of 'accepting' $\omega = \omega_o$ 'much more' than all other values of ω) by saying that the series LCR circuit can be viewed as an <u>acceptor</u> circuit.

<u>Parallel LCR circuit; A 'Rejector' circuit</u>

The parallel LCR circuit is setup by connecting a 'parallel combination' of L and C, in series with R and the source of alternating voltage ($V = V_o \ Sin \ \omega t$).



The sinusoidal applied voltage corresponds to the complex form $V_o\, e^{j\,\omega t}$. Let I be the total (complex) current drawn from the source. This current ' breaks up' into its two (complex) parts at point B. Let the (complex) current flowing through L and C, be I_L *and* I_C. We then have:

$I = I_L + I_C$, as per Kirchhoff's first law.

Also, considering the loop BCMPNDB, we have

$$-I_L\,(j\,\omega L) + I_C\left(\frac{-j}{\omega C}\right) = 0$$

$$\therefore \frac{I_L}{I_C} = \frac{1}{j^2\omega^2\,LC} = \frac{-1}{\omega^2\,LC}$$

$$\therefore I_C = (-\omega^2\,LC)I_L$$

Now consider the loop ABCMPKA. Applying Kirchhoff's second law, we get

$$-I_L\,(j\,\omega L) + V_o\,e^{j\,\omega t} - IR = 0$$

$$But\ I = I_L + I_C$$

$$\therefore I_L\,(j\,\omega L) + (I_L + I_C)R = V_o\,e^{j\,\omega t}$$

Or $$I_L\,(j\,\omega L) + (I_L + (-\omega^2\,LC)I_L)R = V_o\,e^{j\,\omega t}$$

Or $$I_L\,\{R - \omega^2 LCR + j\,\omega L\} = V_o\,e^{j\,\omega t}$$

Or $$I_L\,\{R[1 - \omega^2 LC] + j\,\omega L\} = V_o\,e^{j\,\omega t}$$

Now $R[1 - \omega^2 LC] + j\,\omega L = \{R^2[1 - \omega^2 LC]^2 + \omega^2 L^2\}^{1/2}\,e^{j\alpha}$

Where $\tan\alpha = \frac{\omega L}{R(1-\omega^2 LC)}$

$$\therefore I_L = \frac{V_o}{\{R^2[1-\omega^2 LC]^2 + \omega^2 L^2\}^{1/2}} e^{j(\omega t - \alpha)}$$

Again, $I_C = \ (-\omega^2 \, LC) I_L$

$$\therefore I_C = \frac{-\omega^2 \, LC V_o}{\{R^2[1-\omega^2 LC]^2 + \omega^2 L^2\}^{1/2}} e^{j(\omega t - \alpha)}$$

$$\therefore I = I_L + I_C = \frac{(1-\omega^2 \, LC)V_o}{\{R^2[1-\omega^2 LC]^2 + \omega^2 L^2\}^{1/2}} e^{j(\omega t - \alpha)}$$

This expression can be rewritten in the form

$$I = \frac{V_o}{\left\{\frac{R^2[1-\omega^2 LC]^2 + \omega^2 L^2}{(1-\omega^2 \, LC)^2}\right\}^{1/2}} e^{j(\omega t - \alpha)}$$

$$I = \frac{C}{\left\{R + \left(\frac{\omega L}{(1-\omega^2 \, LC)}\right)^2\right\}^{1/2}} e^{j(\omega t - \alpha)}$$

The actual or real current, flowing in the circuit, is , therefore,

$$I = \frac{V_o}{Z} Sin\,(\omega t - \alpha)$$

Where $Z\left[= \left\{R + \left(\frac{\omega L}{(1-\omega^2 \, LC)}\right)^2\right\}^{1/2}\right]$, is the equivalent impedance of the circuit, as a whole. The current is seen to lag behind the voltage by an angle $\alpha \left(= \tan^{-1}\left(\frac{\omega L}{R(1-\omega^2 LC)}\right)\right)$.

This expression, for the impedance, shows that $Z \to \infty \, for (1-\omega^2 LC\,) = 0$.

Thus for $\omega\,(=\omega_o, say) = \frac{1}{\sqrt{LC}}$, the equivalent impedance of the circuit becomes infinite. This means that for

$$\omega = \omega_o = \frac{1}{\sqrt{LC}}$$

The current, (I), in the circuit becomes zero.

The parallel LCR circuit, therefore, shows the phenomenon of ANTI RESONANCE. The current, I, drawn from the source attains its minimum value (=zero) when the frequency of the applied voltage equals $\upsilon_o \left(= \frac{\omega}{2\pi}\right) = \frac{1}{2\pi\sqrt{LC}}$.

It is interesting to note that this frequency is the same at which the series LCR circuit shows the phenomenon of resonance; the current drawn (by the series circuit) from the source, becomes maximum when the frequency $\upsilon_o \left(= \frac{\omega LC}{2\pi}\right)$ of the applied voltage equals $\frac{1}{2\pi\sqrt{LC}}$.

We can now say that if a given parallel LCR circuit were subjected to several ac signals of different frequencies, the circuit would <u>NOT draw</u> any current from the signal having the (anti resonance) angular frequency $\omega_o \left(= \frac{1}{\sqrt{LC}}\right)$. It would draw (some) current from all the other signals of (angular) frequency other than ω_o. The circuit, therefore, rejects the signal of (angular) frequency ω_o. We can, therefore, (appropriately) say that the parallel LCR circuit acts as a REJECTOR circuit.

It is interesting to note that, each of $I_L and\, I_C$, is, individually, not zero at $\omega = \omega_o = \frac{1}{\sqrt{LC}}$. We see from the above expressions, for $I_L and\, I_C$, that at $\omega = \frac{1}{\sqrt{LC}}$ the (actual or real) currents, through L and C, are

$$I_L = \frac{V_o}{\omega_o L} Sin\left(\omega_o t - \pi/_2\right) = -\frac{V_o}{\omega_o L} Cos(\omega t)$$

$$I_C = -\frac{V_o}{\omega_o L} Sin\left(\omega_o t - \pi/_2\right) = \frac{V_o}{\omega_o L} Sin\left(\pi/_2 - \omega_o t\right) = \frac{V_o}{\omega_o L} Cos(\omega t)$$

We have written these results because at $\omega = \omega_o$, we have

$$\omega^2 LC = \omega_o^2 LC = \frac{1}{LC}.LC = 1$$

And

$$\tan\alpha = \frac{\omega_o L}{R(1 - \omega_o{}^2 LC)} = \frac{\omega_o L}{R(1 - 1)} \to \infty$$

So that $\alpha = \pi/_2$ $(at\, \omega = \omega_o)$.

Thus, even at $\omega = \omega_o$, the currents, $I_L and\, I_C$, are not individually equal to zero. Their sum $I\,(= I_L + I_C)$ is zero as these two currents have equal magnitudes but their phases differ by π. This, incidently, implies that, at $\omega = \omega_o$, the current through R is zero.

The rejector nature of the parallel LCR circuit can be put to appropriate practical uses.

Summary

1. We have learnt the difference between a D C circuit and an A C circuit.
2. Impedance offered by a pure inductor, L, is $j\omega L$.
3. Impedance offered by a pure capacitor ,C, is $\frac{1}{j\omega C}$.
4. Kirchhoff's laws of voltages and currents can also be used to simplify A C circuits as well.
5. Kirchhoff's first rule: The algebraic sum of all the complex equivalents of all the currents, meeting at a junction point, in a circuit, equals zero.
6. Kirchhoff's second rule: The algebraic sum of all the (complex) potential drops, across different elements, of a closed path, or a loop, in a circuit is zero.
7. Series LCR circuit also known as acceptor circuit has circuit impedance given by
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
8. Parallel LCR circuit, also known as rejector circuit has circuit impedance given by
$$Z = \left\{R + \left(\frac{\omega L}{(1 - \omega^2 LC)}\right)^2\right\}^{1/2}$$

Questions

1. Fill in the blanks:

(i) When a given complex number is multiplied by j, ($j = \sqrt{-1}$), there is_______ in its modulus but its argument ____________by _____.

(ii) The _______ _____________, for an inductor of inductance L, equals $j\omega L$.

(iii) The modulus-argument form, for the complex impedance of an inductor, is ____________

(iv) The modulus-argument form, for the complex impedance of a capacitor, is ____________

(v) The equivalent complex impedance of an inductor (L) and a Capacitor ©, put in series (source's angular frequency = ω)is ___________.

Answers

(i) No change; increases;$\frac{\pi}{2}$ (multiplication by j does not change the modulus of a complex number but increases its argument by $\frac{\pi}{2}$)

(ii) Complex impedance

(iii) $\omega L\, e^{j\pi/2}\,(j\omega L = \omega L\, e^{j\pi/2})$

(iv) $\frac{1}{\omega C}\, e^{-j\pi/2}\,(\frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C}\, e^{-j\pi/2})$

(v) $j\left(\omega L - \frac{1}{\omega C}\right)$

True of False

State whether the following statements are 'true' or 'False'.

(i) The equivalent complex impedance, of a parallel combination of an inductor, L, and a capacitor C, equals $(\frac{1}{j\,\omega\,L} + j\omega C)$

(ii) When equivalent complex quantities are used, (for currents, voltage and impedances), Kirchhoff's laws, for AC circuits, have the same form as their corresponding form of DC circuits.

(iii) The complex impedances, assigned to the inductor and the capacitor, 'contain' their phase changing properties in them.

(iv) The 'phasor diagram', for the pure resistor, would have just one line in it.

(v) While using the 'complex number approach', we need to replace the applied sinusoidal voltage by its 'complex equivalent;.

Answers

(i) False (The equivalent complex impedance is the reciprocal of this expression. It is therefore, equal to $\left[\frac{j\omega L}{1-\omega^2 LC}\right]$)

(ii) True (This is correct statement)

(iii) True(The argument of the assigned complex impedance, equals the phase angle for the inductor or the capacitor)

(iv) True (This is correct statement)

(v) True (This is correct statement)

<u>Multiple Choice Questions</u>

Select the best alternative in each of the following:

(i) The series and parallel LCR circuits,

(a) Can both acts as 'acceptor circuits'

(b) Can both acts as 'rejector circuits'

(c) Can act respectively as ‘acceptor’ and ‘rejector’ circuits.
(d) Can act respectively as ‘rejector’ and ‘acceptor’ circuits.

(ii) We can analyze a series parallel AC circuit
(a) Either by using the (dc circuit) rules for ‘series-parallel’ circuits,. or by using Kirchhoff’s laws in their usual dc form
(b) Either by using the (dc circuit) rules for ‘series-parallel’ circuits in their complex number form, or by using Kirchhoff’s laws Ac circuits.
(c) Either by using the (dc circuit) rules for ‘series-parallel’, or by using Kirchhoff’s laws Ac circuits.
(d) Either by using the (dc circuit) rules for ‘series-parallel’ circuits in their complex number form, or by using Kirchhoff’s laws in their usual dc form.

(iii) We can use the complex number approach for the analysis of ac circuits,
(a) Only in circuits in which all the impedances are all connected in series.
(b) Only in circuits in which all the impedances are all connected in parallel.
(c) For those circuits in which the impedances are a mixture of impedances in series and impedances in parallel.
(d) For all types of circuits.

(iv) The equivalent complex impedance of pure resistor and a pure capacitor can be put as
(a) R and $\frac{e^{-j\pi/2}}{\omega C}$.
(b) R and $\omega C\, e^{-j\pi/2}$.
(c) R$e^{j\pi}$ and $\frac{e^{-j\pi/2}}{\omega C}$.
(d) R$e^{j\pi}$ and $\omega C\, e^{-j\pi/2}$. .

(v) For a pure resistor (R) and a pure inductor (L), put in series, the equivalent complex impedance is,
(a) $R + \omega L\, e^{j\pi/2}$
(b) $Re^{j\pi} + \omega L\, e^{j\pi/2}$
(c) $Re^{j\pi} + \omega L\, e^{-j\pi/2}$
(d) $R + \omega L\, e^{-j\pi/2}$

Answers

1. (c)
Justification/Feedback for the correct answer:
(a) The series LCR circuit can be shown to act as the ‘acceptor’ circuit while the parallel LCR circuit can be shown to act as the ‘rejector’ circuit. Hence choice (c) is correct.
(b) The series LCR circuit can be shown to act as the ‘acceptor’ circuit while the parallel LCR circuit can be shown to act as the ‘rejector’ circuit. Hence choice (c) is correct.
(c) The series LCR circuit can be shown to act as the ‘acceptor’ circuit while the parallel LCR circuit can be shown to act as the ‘rejector’ circuit. Hence choice (c) is correct.
(d) The series LCR circuit can be shown to act as the ‘acceptor’ circuit while the parallel LCR circuit can be shown to act as the ‘rejector’ circuit. Hence choice (c) is correct.

2. (b)
Justification/Feedback for the correct answer:

(a) For analyzing any general AC circuit, we need to use either the usual dc circuit rules in which all currents, voltages and impedances are expressed in their equivalent complex form or Kirchhoff's law in their modified AC circuit form.
(b) For analyzing any general AC circuit, we need to use either the usual dc circuit rules in which all currents, voltages and impedances are expressed in their equivalent complex form or Kirchhoff's law in their modified AC circuit form.
(c) For analyzing any general AC circuit, we need to use either the usual dc circuit rules in which all currents, voltages and impedances are expressed in their equivalent complex form or Kirchhoff's law in their modified AC circuit form.
(d) For analyzing any general AC circuit, we need to use either the usual dc circuit rules in which all currents, voltages and impedances are expressed in their equivalent complex form or Kirchhoff's law in their modified AC circuit form.

3. (d)
Justification/Feedback for the correct answer:
(a) The complex number approach, can be used for all types of ac circuits, even when all the impedance are not combines as series or parallel impedances, we can use Kirchhoff's law in their complex form, to analyze such circuits. Hence choice (d) is correct.
(b) The complex number approach, can be used for all types of ac circuits, even when all the impedance are not combines as series or parallel impedances, we can use Kirchhoff's law in their complex form, to analyze such circuits. Hence choice (d) is correct.
(c) The complex number approach, can be used for all types of ac circuits, even when all the impedance are not combines as series or parallel impedances, we can use Kirchhoff's law in their complex form, to analyze such circuits. Hence choice (d) is correct.
(d) The complex number approach, can be used for all types of ac circuits, even when all the impedance are not combines as series or parallel impedances, we can use Kirchhoff's law in their complex form, to analyze such circuits. Hence choice (d) is correct.

4. (a)
Justification/Feedback for the correct answer:
(a) The impedances of a pure resistor, and a pure capacitor, can be put, in their complex equivalent form, as R and $\frac{e^{-j\pi/2}}{\omega C}$. This representation also takes into consideration the phase difference between current and voltage in the two cases. Hence choice (a) is correct.
(b) The impedances of a pure resistor, and a pure capacitor, can be put, in their complex equivalent form, as R and $\frac{e^{-j\pi/2}}{\omega C}$. This representation also takes into consideration the phase difference between current and voltage in the two cases. Hence choice (a) is correct.
(c) The impedances of a pure resistor, and a pure capacitor, can be put, in their complex equivalent form, as R and $\frac{e^{-j\pi/2}}{\omega C}$. This representation also takes into consideration the phase difference between current and voltage in the two cases. Hence choice (a) is correct.
(d) The impedances of a pure resistor, and a pure capacitor, can be put, in their complex equivalent form, as R and $\frac{e^{-j\pi/2}}{\omega C}$. This representation also takes into consideration the phase difference between current and voltage in the two cases. Hence choice (a) is correct.

5. (a)

Justification/Feedback for the correct answer:

(a) The impedances of a pure resistor, and a pure inductor, can be put, in their complex equivalent form, as R and $\omega L\, e^{j\pi/2}$. Hence the equivalent impedance, of their series combination, is $\left(R + \omega L\, e^{j\pi/2}\right)$. Hence choice (a) is correct.

(b) The impedances of a pure resistor, and a pure inductor, can be put, in their complex equivalent form, as R and $\omega L\, e^{j\pi/2}$. Hence the equivalent impedance, of their series combination, is $\left(R + \omega L\, e^{j\pi/2}\right)$. Hence choice (a) is correct.

(c) The impedances of a pure resistor, and a pure inductor, can be put, in their complex equivalent form, as R and $\omega L\, e^{j\pi/2}$. Hence the equivalent impedance, of their series combination, is $\left(R + \omega L\, e^{j\pi/2}\right)$. Hence choice (a) is correct.

(d) The impedances of a pure resistor, and a pure inductor, can be put, in their complex equivalent form, as R and $\omega L\, e^{j\pi/2}$. Hence the equivalent impedance, of their series combination, is $\left(R + \omega L\, e^{j\pi/2}\right)$. Hence choice (a) is correct.

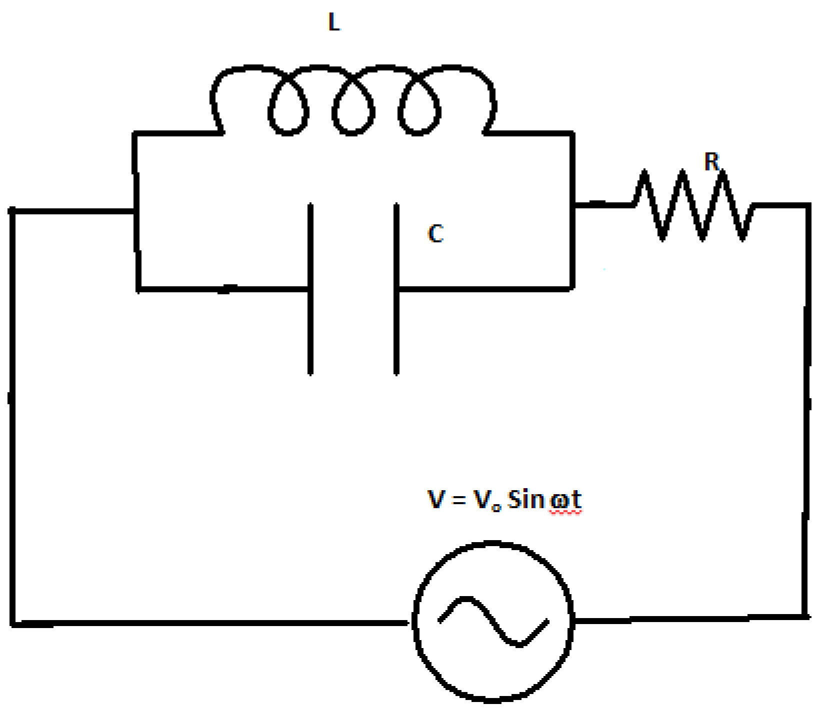
<u>Short note type:</u>

Write short note on

(a) The equivalent complex impedance for an inductor.
(b) The equivalent complex impedance for a capacitor.
(c) Kirchhoff's rules for AC circuits.
(d) The use of Kirchhoff's rules for writing the 'loop equations' in an AC circuits.
(e) The modulus argument representation for a complex number.

<u>Essay Type</u>

(a) Use Kirchhoff's rules, for AC circuits, to show the 'Acceptor circuit' nature of the series LCR circuit.
(b) Use Kirchhoff's rules, for AC circuits, to show why the circuit, shown here, acts as a ' Rejector circuit' for the value of ω given by$\omega = \frac{1}{\sqrt{LC}}$.



(c) Discuss how the complex representation of impedances takes into account their phase effect in ac circuits.
(d) Draw the circuit diagram of the 'rejector circuit' and use the complex number approach to analyse it.
(e) Discuss why it is necessary to assign a complex impedance of $j\omega L$ to an inductor and a complex impedance of $(1/j\omega C)$ to a capacitor.