Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Ballistic GalvanometerLesson 7.1: Basic Theory of the Ballistic Galvanometer

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Learning objective

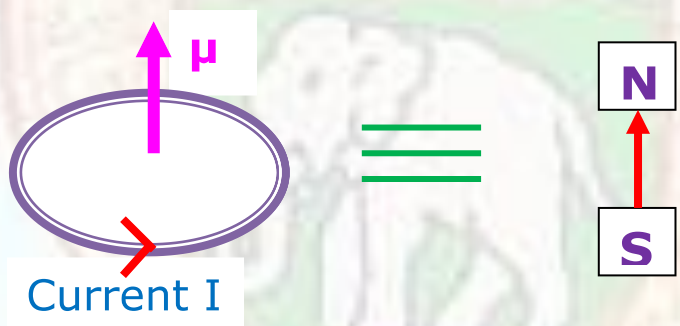
- This lesson is crucial for the basic understanding of a ballistic galvanometer. It aims at the following student learning objectives.
- Basic idea and difference between ordinary moving coil galvanometer and the ballistic galvanometer.
- Concept of transient currents
- Various torques acting on a ballistic galvanometer coil
- Equation of motion of a ballistic galvanometer and its analysis

Introduction

In this lesson we will study the basic theory on a ballistic galvanometer. The lesson starts with torque associated with a current carrying loop, which is necessary to understand as it forms the basis of further study. A brief review of ordinary moving coil galvanometer is presented followed by the effects of transient currents. This paves the way for a detailed analysis of ballistic galvanometer.

Torque on a Current Loop

It is well known that a current loop behaves like a magnetic dipole. The magnitude of the magnetic moment, associated with a current loop, equals the product of the area of the loop and the current flowing through it. Its direction is normal to the plane of the loop and is along the direction of advancement of a right handed screw rotated in the sense of current flow in the loop.



It follows that for a current loop of area A, carrying an anticlockwise current of magnitude I, the equivalent magnetic moment is

$$\vec{\mu} = I\,A\,\hat{n}$$

where $\hat{n}$ is a unit vector normal to the plane of the loop (taken as the plane of the paper here) and directed outwards.

If the same loop were to carry a clockwise current, the direction of $\vec{\mu}$ would be along the inward directed normal to the plane of the paper.

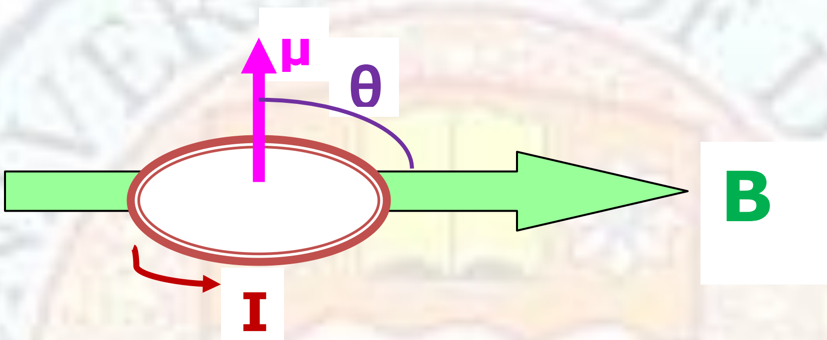
For a loop having n turns, each of area A, the magnitude of its associated magnetic moment would become (n A) I.

When this loop is present in the region of a uniform magnetic field $\vec{B}$, it would experience a torque $\vec{\tau}$ where,

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The magnetic torque tries to align the magnetic moment vector in the direction of magnetic field. Thus, combining the above facts, we have, $|\vec{\tau}|$ = (nAI) B sinθ, where θ is the angle between the directions of $\vec{\mu}$ and $\vec{B}$.

For a loop, having its plane parallel to the direction of $\vec{B}$, θ equals $\frac{\pi}{2}$. Hence the torque, on the current loop in such a case, would have a magnitude (nABI). It is this result, corresponding to $\vec{B}$ directed normal to μ that forms tha basis of the theory of an ordinary moving coil galvanometer.



Moving Coil Galvanometer: A Brief Recapitulation

We all know that the *moving coil galvanometer* is one of the most often used instruments in the Physics Laboratory. It is the usual device for detecting (and deciding) the position of the null point, in a variety of experiments, that make use of a steady d.c. voltage source for providing current to the different parts of the circuit.

The moving coil galvanometer, as we know, is based on the simple fact that, a current carrying coil experiences a torque in a suitable magnetic field. The resulting rotational effect, on the coil, is countered by an elastic restoring torque set up in a wire used to suspend the coil. The coil then comes to an equilibrium position in which the deflecting torque (due to the interaction of the magnetic field with the current carrying coil) gets balanced by the restoring torque (produced in the suspension wire due to its getting twisted by the rotation of the coil). Through suitable adjustment in the nature of the magnetic field used (using a radial magnetic field), it is then ensured that the equilibrium deflection of the current carrying coil, is directly proportional to the current flowing in the coil. The ordinary moving coil galvanometer, therefore, has a linear scale, i.e. the deflection produced is in direct proportion to the steady current flowing through the coil.

Need for Measuring the Effects of Transient Currents

There are a number of practical situations in which the current flows only for a very short duration, i.e., the current flowing is a **transient current**. We have such a transient current flow in situations like the following:

a) A charged up capacitor is made to leak its stored charge by connecting its two plates to a load. The time (τ, say) in which the capacitor loses its charge, is also the time for which the current flows through the load. In usual situations, the time (τ) may be of the order of a fraction of a millisecond or even less. The current, associated with the discharge of a charged up capacitor, is, therefore, a transient current.

b) A momentary induced e.m.f gets set up in an inductive coil when the key in the circuit (containing a steady d.c. voltage source), of which the inductive coil is a part, is 'plugged in' or' taken out'. The current flow, due to such a momentary induced e.m.f, would again be a very short duration current, i.e., a transient current.

It is easy to realize that the very nature of a 'transient current' (i.e., a current that lasts for a very short duration of time) would make it difficult to measure it using devices like the usual moving coil galvanometer. Such devices, as we have noted, produce a steady deflection when a steady (and NOT a transient) current flows through them. It was imperative, therefore, to try and develop devices that could measure the effects of a 'transient current'.

It is important here to note that the product of the transient current, and the time for which it flows, would give the total charge (or total quantity of electricity) that flows through the load, in time τ. The ordinary moving coil galvanometer was subsequently modified so that it could measure the total charge flow (or the total quantity of electricity flowing) associated with a transient current. Such a modified galvanometer came to be known as a **ballistic galvanometer**.

We may, therefore, say that a ballistic galvanometer is a galvanometer designed to measure the charge (or total quantity of electricity) flowing across any cross section of a circuit due to a transient current or voltage.

Ballistic Galvanometer

The basic principle of a *ballistic galvanometer* is the same as that of an ordinary moving coil galvanometer, i.e., a current carrying coil experiences a torque when present in a (suitable) magnetic field. However, the details of its designing are different.

The need for having this difference can be easily appreciated by realizing that the torque, acting on the coil of a ballistic galvanometer, is of the nature of an *impulsive torque* that lasts but for a very short duration of time. In contrast, the torque acting on the coil of an ordinary moving coil galvanometer is a steady and long duration torque. The designing of the ballistic galvanometer is therefore done so as to measure the after effects of this (very short duration) impulsed 'rotational kick' experienced by its coil. It turns out that, through suitable designing, the first maximum deflection of the coil (due to the impulsive kick) can be made proportional to the charge (or total quantity of electricity) that flowed through the coil during the duration of the transient current.

Designing features of a ballistic galvanometer

The fact that the (current related) torque acting on its coil, is but a transient torque, suggests that we now need to ensure that

i. One can safely assume that the deflection of the galvanometer coil during the actual duration (τ) of the transient current is practically zero and we measure only its after effect (first maximum) deflection or its subsequent deflections. This requirement can be met by making the free, or natural, time period of rotation (T_0) of the galvanometer coil as large as possible, and much larger than the time τ. The free or natural time period of rotation of the galvanometer coil is given by,

$$T_0 = 2\pi\sqrt{I/C}$$

here, I = moment of inertia of the galvanometer coil, about the suspension wire as axis

and

C = torsional constant (or restoring torque per unit angular displacement) of the suspension wire.

We need to have $T_0 >> \tau$.

In designing a ballistic galvanometer, care should be taken to have,

a. The moment of inertia, I, of the coil, as large as possible.
b. A suspension wire, made from a material of low torsional constant. However, the material should still have a linear elastic behavior. (the suspension wires, made from silvered quartz, or phosphor bronze, satisfy these twin requirement better than most other material)

ii. The damping effects, acting on the galvanometer coil, are as small as possible. This is necessary because we now have to measure the 'after effects' of a (very short duration) impulsive torque. In the presence of damping effects, the coil would not only be not able to attain a significant 'first maximum' deflection but would also be not able to (or hardly able to), execute any oscillations about its mean position.

In an ordinary moving coil galvanometer, the coil is wound over a soft iron core which produces quite large damping effects on its oscillating coil (due to electromagnetic induction). In a ballistic galvanometer, these damping effects have to be minimized, or eliminated altogether. The coil of a ballistic galvanometer, is therefore, wound on either a laminated soft iron core or a bamboo core. We also have 'air-cored' ballistic galvanometers in which the coil can 'work' without any core. This is ensured by having the coil hardened, or strengthened, by dipping it in a suitable (insulating) fluid.

Basic Theory of the Ballistic Galvanometer

The theory of the ballistic galvanometer can be worked out by writing the 'equation of motion' of its coil and 'solving it'. It is, however, important to note that the torque, acting on the coil due to the flow of a transient current through it, is also a transient torque that lasts but for the time τ (the time for which the transient current flows). We need, therefore, to look at the 'different forms' of the equation of motion of the coil for the two time intervals

i. $0 < t < \tau$
ii. $t > \tau$

During the first of these two time intervals, the coil receives an impulsive rotational kick due to the transient current, and can be said to have acquired a finite angular velocity at $t = \tau$. However, as mentioned earlier, we assume that angular displacement, of the coil, at $t = \tau$, is practically zero.

In the second time interval, i.e., $t > \tau$, the coil keeps on moving because of the angular velocity acquired by it at $t = \tau$. This part of the motion of the coil would be without any deflecting torque acting on it. Hence the oscillations of the coil would keep on having a continuous decrease in their amplitude, because of the damping, that would be inevitably there. It has already been noticed that special precautions are taken in the design of a ballistic galvanometer to minimize the damping effects so that the coil may execute a reasonable number of oscillations (because of the 'impulsive kick' received by it, at the start). It is clear that the amplitude of these oscillations would be maximum for the first half of the first oscillation. It is this maximum (first) displacement that, as we shall see, is proportional to the total charge (or the total quantity of electricity) that flowed through the coil (because of the transient current). We can, therefore, use this first maximum displacement to measure the total (transient) charge that flowed through the coil of the galvanometer.

Different Torques Acting on the Coil

The ballistic galvanometer coil experiences the following torques during its oscillations:

i. A **deflecting torque** due to the transient current (i) flowing through it. If the coil has n turns, each of area A, and if it is situated in a magnetic field **B**, the deflecting torque has a magnitude nABi. For the ballistic galvanometer, this torque acts only over the time interval, $0 < t < \tau$, for which the flow of the transient current through its coil lasts.

ii. A **restoring torque** due to the twist in the suspension wire. If C is the torsional constant, of the material of the suspension wire, and θ is the instantaneous angular displacement, this restoring torque equals Cθ.

iii. A **damping torque** due to the damping effects associated with the oscillations of the coil. The damping is mainly due to two causes:

   a. Due to viscosity of the air surrounding the coil.

      This viscosity based damping torque is taken as proportional to the instantaneous angular velocity of the coil and is, therefore, taken as equal to b $\frac{d\theta}{dt}$ (b = constant).

   b. Due to induced electromagnetic effects

      When the coil oscillates, its oscillations in the magnetic field in which it is situated, set up an induced e.m.f in it which, as per Lenz's law, would tend to oppose or dampen these oscillations.

      The induced e.m.f, in the coil, would equal nAB$\frac{d\theta}{dt}$. The resulting induced current, in the coil, would then equal (nAB/R) $\frac{d\theta}{dt}$, where R is the total resistance of the coil itself and its associated circuit.

      The torque acting on the coil, due to this induced current, would equal (nA) X (induced current) X B.

      The (electromagnetic) damping torque, therefore, has a magnitude nAB X [(nAB/R) $\frac{d\theta}{dt}$] = $(nAB)^2/R \frac{d\theta}{dt}$.

The total damping torque, acting on the coil, therefore, is equal to
$[b + (nAB)^2/R]\frac{d\theta}{dt}$.

We can write it as,

$$\sigma \frac{d\theta}{dt} \quad \text{where} \quad \sigma = b + (nAB)^2/R$$

Hence, if I is the moment of inertia of the coil, about the suspension wire as axis, the equation of motion of the coil would be

$$I \frac{d2\theta}{dt2} = nABi - \sigma\frac{d\theta}{dt} - C\theta$$

$$\text{Or} \quad I \frac{d2\theta}{dt2} + \sigma\frac{d\theta}{dt} + C\theta = nABi$$

This is the form of the equation of motion for the time interval $0 < t < \tau$.

For $t > \tau$, the current I becomes zero. Hence, for $t > \tau$, the equation of motion of the coil, is

$$I \frac{d2\theta}{dt2} + \sigma\frac{d\theta}{dt} + C\theta = 0.$$

Analysis and Interpretation of Equations of Motion

Let us now look at the results and conclusions we get from the equation of motion of the coil in a ballistic galvanometer.

1. For the time interval 0 < t < τ

The equation of motion of the coil is,

$$I \frac{d^2\theta}{dt^2} + \sigma\frac{d\theta}{dt} + C\theta = nABi \quad (1)$$

As per the assumption for a ballistic galvanometer (the coil hardly moves, or gets displaced, from its mean position, during the actual time ($t=\tau$) of the flow of the transient current), we can take $\theta \cong 0$ for this time interval.

Integrating equation (1), over the time interval $t=0$ to $t=\tau$, we get

$$I \int_0^\tau \frac{d^2\theta}{dt^2}\, dt + \sigma \int_0^\tau \frac{d\theta}{dt}\, dt + C \int_0^\tau \theta\, dt = nAB\int_0^\tau i\, dt \quad (2)$$

$$\text{Or} \quad I \left|\frac{d\theta}{dt}\right|_0^\tau + \sigma\, |\theta|_0^\tau + 0 = nABQ \quad (3)$$

Here $Q = \int_0^\tau i\, dt$ is the total charge (or total quantity of electricity) that flows through the coil during the duration of the 'transient current' flow. Since $\theta \cong 0$ even for $t=\tau$, we can take the second term in the above equation as also equal to zero. Hence,

$$I \left[\left(\frac{d\theta}{dt}\right)_{t=\tau} - \left(\frac{d\theta}{dt}\right)_{t=0}\right] = nABQ \quad (4)$$

Now, $\left(\frac{d\theta}{dt}\right)_{t=\tau} = \omega_0$, the angular velocity imparted to the coil by the impulsive rotational kick associated with the flow of the transient current. Also, $\left(\frac{d\theta}{dt}\right)_{t=0}$ can be taken as zero as the coil was at rest to start with. Hence, we get,

$$I\, \omega_0 = nABQ$$

or $\omega_0 = (nAB)Q/I$ (5)

Thus, at the end of the time interval $0 < t < \tau$, the coil is almost at its initial, or rest or mean position but has acquired an angular velocity ω_0 where $\omega_0 = (nAB)Q/I$. It subsequently starts oscillating because of this angular velocity. The amplitude of these oscillations would clearly be maximum during the first half of its first oscillation. Let us denote this maximum amplitude by θ_0. We shall now see that θ0 ∝ Q.

2. For the time interval t ≥τ.

The equation of motion has the form,

$$I \frac{d^2\theta}{dt^2} + \sigma\frac{d\theta}{dt} + C\theta = 0 \quad (1)$$

We rewrite it as,

$$\frac{d^2\theta}{dt^2} + (\sigma /I)\frac{d\theta}{dt} + (C/I)\,\theta = 0 \quad (2)$$

We now put $\sigma/I = [1/I(b+(nAB)^2/R))] = 2l$ (3)

And $C/I = m^2$ (4)

The equation of motion now takes the form,

$$\frac{d^2\theta}{dt^2} + 2l\frac{d\theta}{dt} + m^2\theta = 0 \quad (5)$$

We can now solve this equation. To find the values of the two 'constants of integration' in the solution, we need two boundary conditions. These conditions are

$$\frac{d\theta}{dt} = \omega_0 \quad \text{and } \theta = 0 \quad \text{at } t = \tau$$

However, as τ is a very small time interval (much smaller than the time period, T_o, of free oscillations of the coil of a ballistic galvanometer). We can take $\tau \rightarrow 0$ while applying these boundary conditions. The form of the equation of motion, given above, suggest that we can assume a solution of the form

$$\theta = A\, e^{pt}$$

This would be a solution of the equation of motion provided

$$A.p^2 e^{pt} + Ap.2l\, e^{pt} + m^2Ae^{pt} = 0$$

Or $p^2 + 2lp + m^2 = 0$

Or $p = \frac{-2l\pm\sqrt{4l^2-4m^2}}{2} = -l \pm \sqrt{l^2 - m^2}$

The assumed solution, is therefore, a solution of the equation for two values (say p_1 and p_2) of p where

$p_1 = -l + \sqrt{l^2 - m^2}$ and $p_2 = -l - \sqrt{l^2 - m^2}$

We can, therefore, write the general form of the solution as

$\theta = A_1e^{p_1t} + A_2e^{p_2t}$

Where A_1 and A_2 are two arbitrary constants. We can, however, find their values, relevant to this case, by applying the two boundary conditions noted above. Since we have,

$\theta = 0$ at $t = \tau$ (or at $t \to 0$), we get

$0 = A_1 + A_2$

Again, we have $\frac{d\theta}{dt} = \omega_0$ at $t = \tau$ (or at $t \to 0$)

Now, $\frac{d\theta}{dt} = A_1 p_1 e^{p_1 t} + A_2 p_2 e^{p_2 t}$

Therefore the boundary condition gives

$\omega_0 = A_1 p_1 + A_2 p_2$

The two equations for A_1 and A_2 give their values as,

$A_1 = \frac{\omega 0}{p1 - p2}$ and $A_2 = -\left(\frac{\omega 0}{p1 - p2}\right)$

Thus, $\theta = \frac{\omega 0}{p1 - p2}\left[e^{p_1 t} - e^{p_2 t}\right]$ (6)

A close look, at the expressions for p_1 and p_2, reveals that the presence of the square root term ($\sqrt{l^2 - m^2}$), in them, would make θ have a non-oscillatory or non-periodic nature when l>m and an oscillatory, or periodic nature, when l<m. The condition l=m can, therefore, be marked as making a transition from a non-oscillatory (or aperiodic or dead beat) behavior of the coil to an oscillatory behavior.

Critical Damping Resistance

Let us now see the implications of the condition l=m.

We have (from equations 3 and 4, above),

$2l = 1/I\,[b + (nAB)^2/R]$ and $m^2 = C/I$

We would thus have l = m when,

$1/2I\,[b + (nAB)^2/R] = \sqrt{C/I}$

Or, $b + (nAB)^2/R = \sqrt{4IC}$

Or, $(nAB)^2/R = \sqrt{4IC} - b$

Or, $R = [(nAB)^2 / (\sqrt{4IC} - b)]$

We call this value of R (the total resistance of the coil itself and its associated circuit) as the **critical damping resistance (R_{CDR})** for a given ballistic galvanometer. We thus have

$R_{CDR} = [(nAB)^2 / \sqrt{4IC} - b]$

It immediately follows from above that we would have

l>m when $R < R_{CDR}$

and l<m when $R > R_{CDR}$

It follows that a given ballistic galvanometer set-up would show

a. A non-oscillatory (aperiodic or deadbeat) behavior when the total resistance of its coil and its associated circuit, is less than its (characteristic) critical damping resistance.
b. An oscillatory (or periodic) behavior when the total resistance of its coil and its associated circuit, is more than its characteristic critical damping resistance.

In practice, we would like the ballistic galvanometer to behave in an oscillatory way. This, as we have seen, requires $R > R_{CDR}$.

How to Ensure that $R > R_{CDR}$

We have seen above that

$$R_{CDR} = [(nAB)^2 / \sqrt{4IC} - b]$$

It is, therefore, a characteristic constant of a given galvanometer and can be, theoretically, calculated from the known values of its 'constants' (like n, A, B, C and I). It is, however, easier to determine it experimentally.

For a given experimental setup, the total resistance R equals

$$R = R_{coil} + R_{ext}$$

Where R_{coil} is the resistance of the galvanometer coil and R_{ext} is the external resistance of the associated circuit of the galvanometer.

Suppose the manufacturer adds a resistance (say R') equal to, or slightly greater than R_{CDR} in series with the coil of the given galvanometer. The total resistance, R, of the galvanometer circuit would then be necessarily greater than R_{CDR}. The given ballistic galvanometer would then behave in an oscillatory way irrespective of the circuit in which it is being used.

It is preferable, however, to have a set up in which the resistance R', may or may not be made a part of the circuit of the galvanometer. This can be done by arranging things in the manner suggested here. When the key K is closed, the effective resistance of the parallel combination of R' and the key K, is nearly zero.

Such a 'set-up' enables one to:

i. Study the conditions under which the galvanometer shows a non-oscillatory or aperiodic behavior.
ii. Determine, experimentally, the value of R_{CDR}, the critical damping resistance of a given galvanometer.

For most of the circuits, in which the ballistic galvanometer is used, it is desirable to have the galvanometer in its oscillatory or periodic behavior mode. The key K is then kept open (or plugged out). This ensures that the total resistance of the galvanometer circuit is more than the critical damping resistance of the galvanometer. The galvanometer would then necessarily operate in its oscillatory mode.

<u>Detailed Analysis of the Solution for 'θ'</u>

We have seen that the solution for θ has the form

$$\theta = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

Where, $p_1 = -l + \sqrt{l^2 - m^2}$ and $p_2 = -l - \sqrt{l^2 - m^2}$

1. When l>m (i.e. when $R < R_{CDR}$),

Both p_1 and p_2 are real terms. Also in p_1, since $l > \sqrt{l^2 - m^2}$, we notice that both the terms, in the solution for θ, show an exponential decay with time. Hence, in this case, the coil just gets deflected (because of impulsive (transient) rotational kick at the start) and this deflection decreases exponentially with time. The coil finally comes back to its equilibrium, or mean, position without executing any oscillations.

The ballistic galvanometer is seldom put to use in this type of non-oscillatory, or aperiodic, mode. In practice, we keep l<m (i.e. $R > R_{CDR}$) so that the ballistic galvanometer executes a significant number of slightly damped oscillations before finally coming to rest.

2. When l<m,

The term $\sqrt{l^2 - m^2}$ has the form, $\sqrt{l^2 - m^2} = j\omega$

Where $j = \sqrt{-1}$ and ω= is a real number.

In this case, we would have,

$$\theta = A_1 e^{(-l + j\omega)t} + A_2 e^{-(l + j\omega)t}$$

$$= e^{-l t} [A1\, e^{j\omega t} + A2\, e^{-j\omega t}]$$

Now θ being a real physically observable quantity, we need to have A_1 and A_2 as complex conjugates of each other. We, therefore, put

$A_1 = A' e^{j\delta}$ and $A_2 = A' e^{-j\delta}$

$\therefore\ \theta = A' e^{-l t} [e^{j(\omega t + \delta)} + e^{-j(\omega t + \delta)}]$

$= A\, e^{-l t} \cos(\omega t + \delta)$ (A=2A')

To determine the constants A and δ, we again use the two boundary conditions. We have,

i. $\theta = 0$ at $t = \tau$ (or at $t \to 0$)

hence, $0 = A\cos\delta$

ii. now, $\frac{d\theta}{dt} = Ae^{-lt} . \omega. [-\sin(\omega t+\delta)]$

Hence, $\omega_0 = -A\omega\sin\delta$

The first of these conditions implies that d = ∏/2 or d= 3∏/2. To ensure that the constant A remains a positive quantity, we take d= 3∏/2. We then have

$$A = \omega_0 / \omega = \omega_0 / \sqrt{m^2 - l^2}$$

The solution for θ, therefore, becomes

$$\theta = (\omega_0 / \omega)\, e^{-lt} \cos(\omega t + 3\Pi/2)$$

$$= (\omega_0 / \omega)\, e^{-lt} \sin(\omega t)$$

The galvanometer coil, therefore, shows an oscillatory behavior with the amplitude of its oscillations (=$(\omega_{0/}\ \omega)\ e^{-lt}$) decaying exponentially with time. The time period, T, of these oscillations is

$$T = 2\Pi/\omega = 2\Pi/\sqrt{m^2 - l^2}$$

The deflection (or angular displacement) of the coil would attain its 'maxima' values whenever $\frac{d\theta}{dt} = 0$

Now,

$$\frac{d\theta}{dt} = (\omega_{0/}\ \omega)\ e^{-lt} \cos(\omega t) + (\omega_{0/}\ \omega)\ e^{-lt} (-l) \sin(\omega t)$$

Therefore, $\frac{d\theta}{dt} = 0$ whenever

$\omega \cos \omega t = l \sin\omega t$ or $\tan \omega t = \omega/l$

This requires $\omega t = n\Pi + \tan^{-1} \omega/l$ (n = 0, 1, 2, 3,……)

The first time instant (say $t=t_1$) at which θ attains its first maxima (say $\theta=\theta_1$) corresponds to n=0. We, therefore, have

$$\omega t_1 = \tan^{-1}(\omega/l)$$

Or $t_1 = (1/\omega) \tan^{-1}(\omega/l)$

The value of θ ($=\theta_1$), at $t=t_1$, is given by

$$\theta_1 = (\omega_{0/}\ \omega)\ e^{-lt} \sin \omega t_1$$

Now, $\sin \omega t_1 = 1/ [1+\cot^2\omega t_1]^{1/2} = 1/ [1 + (l/\omega)^2]^{1/2}$ (since $\tan \omega t_1 = \omega/l$)

Also, $\omega_0 = nABQ/I$

We can write,

$nABi = C\theta$ for the steady flow of current through the galvanometer

Hence, i_s (corresponding to θ=1 unit), the current sensitivity of the galvanometer, can be put as

$$i_s = C/nAB$$

Therefore, $nAB = C/i_s$

We can, therefore, write

$\theta_1 = (nABQ/I) .1/\omega .e^{-lt}{}_1.\{1/[1+(l^2/\omega^2)]^{1/2}\}$

$= Q/I.\ C/is\ .\ 1/\omega\ .\ e^{-lt}{}_1.\{1/[1+(l^2/\omega^2)]^{1/2}\}$

The free time period, T_0, of the ballistic galvanometer, is given by

$$T_0=2\Pi\sqrt{I/C}$$

Therefore, $C/I = 4\Pi^2/T_0^2$

Also, $2\Pi/\omega = T$ (the time period of the galvanometer under the actual circuit conditions in which it is being used)

We, therefore, get

$$\theta_1 = Q/i_s . 4\Pi^2/T_0^2 . T/2\Pi . e^{-lt_1}[1+(l^2/\omega^2)]^{-1/2}$$

$$= 2\Pi QT/i_sT_0^2 . e^{-lt_1}[1+(l^2/\omega^2)]^{-1/2}$$

As we shall see, the time period, T, in the actual circuit, is related to the free time period, T_0, through the relation

$$T \cong T_0 (1+\lambda^2/\Pi^2)^{1/2}$$

Where λ, is a characteristic constant for a given circuit is known as the *logarithmic decrement* for the circuit. It is an 'indicator' and a 'measure' of the damping effects present in the circuit. It is related to 'l' the damping related parameter of the circuit through the relation

$$\lambda/\Pi = l/\omega$$

Hence,

$$\theta_1 = 2\Pi Q/I_s . T_0 (1+\lambda^2/\Pi^2)^{1/2}/T_0^2 . e^{-lt_1}[1+(\lambda^2/\Pi^2)]^{-1/2}$$

$$= 2\Pi Q/I_sT_0 . e^{-lt_1}$$

We can now put $lt_1 = \omega.(\lambda/\Pi)t_1$

$= \omega. \lambda/\Pi. (1/\omega) \tan^{-1}(\omega/l) = \lambda/\Pi \tan^{-1}(\omega/l)$

For the ballistic galvanometer circuits, it is usual to keep the damping at a very low level. This implies that we can take both 'l' and λ as very small quantities for a ballistic galvanometer circuit.

Now when 'l' is very small, we can take

$$\tan^{-1}(\omega/l) \approx \tan^{-1}(\infty) = \Pi/2$$

therefore, $lt_1 \approx \lambda/\Pi . \Pi/2 = \lambda/2$

$$\theta_1 = 2\Pi Q/I_sT_0 . e^{-\lambda/2}$$

therefore, $Q = T_0.i_s/2\Pi . \theta_1 . e^{\lambda/2}$

$$= T_0.i_s /2\Pi. \theta_1. (1+\lambda/2 +-----)$$

Since λ is also very small for ballistic galvanometer circuits, we can put

$$Q \approx T_0.i_s /2\Pi. \theta_1. (1+\lambda/2)$$

We thus observe that the transient flow of a charge Q, through the coil of a ballistic galvanometer produces a first throw, or a first deflection, θ_1, which is (nearly) proportional to the charge Q. We say (nearly) because we also have a term (1+λ/2) that depends on λ, a quantity connected with the 'damping effects' present in a given circuit. As we shall see, the quantity λ can be easily determined in any given experimental setup. We can then 'correct' the observed first throw, θ_1, to a value θ_1' where $\theta_1' = \theta_1 (1+\lambda/2)$. The charge Q, is then proportional to the 'corrected first throw', i.e. θ_1'. In terms of the corrected first throw, we have,

$$Q = T_0 . i_s / 2\Pi . \theta_1'$$

We can therefore use this *corrected* first throw to measure the 'transient' charge Q, that has flowed through the coil of the ballistic galvanometer in the very small time interval, τ.

We use the above relation as the basic relation for all measurements done with a ballistic galvanometer.

Example

A capacitor, charged to a potential difference of 5V, is discharged through a ballistic galvanometer. The first throw, observed as a result of flow of this transient charge, equals 0.1 radian. Find the capacity of the given capacitor.

It is given that the free time period, of the coil of the given ballistic galvanometer, equal 6∏ seconds. The current sensitivity, of the coil of this galvanometer equals 2.5μA/radian. The damping effects affecting the motion of the coil of the galvanometer are to be neglected.

<u>Solution</u>: Let C be the capacity of the given capacitor.

We then have Q=CV=5C

$$\therefore 5C = To/2\Pi \; i_s . \theta_1$$

$$= 6\Pi/2\Pi \times 2.5 \times 10^{-6} \times 0.1$$

$$= 7.5 \times 10^{-7}$$

Therefore, $C = 1.5 \times 10^{-7}$ F = 150nF

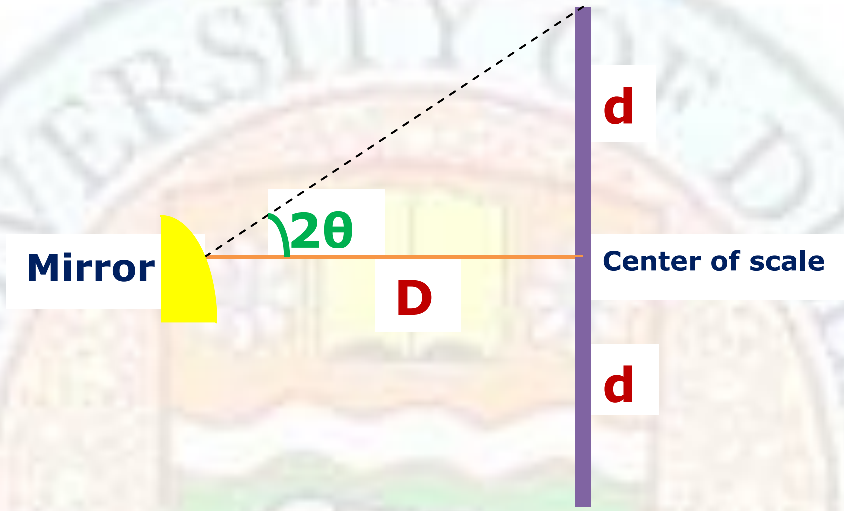
The Lamp and Scale Arrangement

We all know that the rotation of the coil, of an ordinary moving coil galvanometer, is measured by attaching a needle to the coil and letting the needle move over a calibrated scale on a dial. It is easy to appreciate that this needle needs to be as light as possible so that it does not add much to the inertia of the coil and can be regarded as moving along with the coil. Further, a longer needle would mean that one could use a bigger dial and thus could have a larger number of distinguishable subdivisions. This would effectively mean a better least count for the given galvanometer.

The ballistic galvanometer makes use of these advantages of a 'light and long' needle by effectively having a weightless needle whose effective length is usually kept as one meter. It does so by making use of the so called *lamp and scale* arrangement. For using this arrangement the suspension wire, attached to the coil of the ballistic galvanometer, also carries a small concave mirror of radius of curvature of 1m in the usual situations.

In the lamp and scale arrangement, the light from an incandescent lamp is made to fall on this concave mirror using a suitable convex lens along with the lamp. The light reflected by the mirror, is received on a scale kept usually at a distance of 1m (the usual value of the radius of curvature of the conclave mirror) from the mirror. A vertical wire is stretched over the convex lens used along with the incandescent lamp. The image, of the lamp on the scale, is then a circular patch having a dark vertical line in its middle. This vertical line then serves as the reference line for measuring the deflection of the coil of the ballistic galvanometer.

The angular rotation of the coil of the ballistic galvanometer, can be easily related to the linear displacement of the dark vertical line on the scale that has been kept at a distance, D, from the mirror attached to the coil. When the coil rotates through an angle θ, the mirror attached to the coil also rotates through an angle θ. There being no change in the direction of the incident ray coming from the lamp, the ray reflected by the mirror would turn through an angle 2θ with respect to its original direction.



If the linear displacement of the dark line, on the scale, (kept at a distance D from the mirror) is d, we have

$$\tan 2\theta = d/D$$

We usually use the ballistic galvanometer in such a way that ensures that θ remains a small angle (~5^o or so). We can then make the approximation

$$\tan 2\theta \cong 2\theta$$

Therefore, $2\theta \cong d/D$

or $\theta \cong d/2D$

Therefore, $d \cong (2D)\theta$

The linear displacement, d, of the dark line, on the scale, is thus approximately proportional to θ, the angular displacement of the coil. We can, therefore, use 'd' as a measure of the angular displacement (θ) of the coil. In practice, it is usual to think of the coil of the ballistic galvanometer as having a 'unit deflection' when the dark vertical line seen on the scale of the lamp and sale arrangement, moves through a distance of one millimeter. It is, of course, implicitly assumed here than the scale has been kept at a distance of one meter from the concave mirror (of radius of curvature one meter) attached to the coil of the ballistic galvanometer.

Summary

This lesson focuses on the basic principle designing of a ballistic galvanometer. We have learned about the various subtle differences between ordinary moving coil galvanometer and the ballistic galvanometer. We have also studied the effects of transient current on a current carrying coil and the various torques acting on it. The conditions for periodic and aperiodic oscillations have also been discussed, followed by the damping effects affecting the motion of the coil of ballistic galvanometer. We have also learned about the lamp and scale arrangement used for measurement purposes.

Exercise

Fill in the Blanks

1. _____ is a good material for making the suspension wire of the coil of a ballistic galvanometer.

2. Coil of a ballistic galvanometer exhibits _____ motion when its resistance is _____ than the critical damping resistance.

3. Coil of a ballistic galvanometer exhibits _____ motion when its resistance is less than the critical damping resistance.

Answers:

1. phosphor bronze

2. periodic, more

3. aperiodic

True/ False

State whether the following statements are true or false.

1. For a good ballistic galvanometer, moment of inertia of the coil should be large.

2. For a good ballistic galvanometer, the free time period of rotation of the coil should be less than the duration of flow of transit current.

Answers:

1. True

2. False. The free time period should be more than the duration of flow of transient current.

Short answer question

1: How does one make sure that the ordinary moving coil galvanometer shows a deflection which is directly proportional to the current flowing through the coil?

2: What will be the torque acting on a current carrying loop if the applied magnetic field is perpendicular to the plane of the loop?

3: What do you mean by transient current?

4: What precautions are to be taken while designing the core of the coil of a ballistic galvanometer? How is it different from that of an ordinary moving coil galvanometer?

5: Write the equation of motion of the coil of a ballistic galvanometer during the flow of transient current.

Long answer question

1: Explain the principle of working of a moving coil galvanometer.

2: List the differences between moving coil galvanometer and ballistic galvanometer.

3: Describe the various torques acting on the coil of a ballistic galvanometer.

4: What do you understand by the 'critical damping resistance' of a ballistic galvanometer?

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