Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Ballistic GalvanometerLesson 7.2: Basic Parameters associated with the Ballistic Galvanometer

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Learning Objective

This lesson focuses on the following student learning objectives.

- Basic parameters of a ballistic galvanometer
- Current sensitivity of a ballistic galvanometer
- Charge sensitivity of a ballistic galvanometer
- Electromagnetic damping of a ballistic galvanometer
- Role of damping key in experiments involving ballistic galvanometer
- The practical and experimental set-up to determine the parameters of a ballistic galvanometer

Introduction

In this lesson we will learn about the basic parameters of a ballistic galvanometer. The main focus will be on current and charge sensitivities, and electromagnetic damping. The importance of critical damping resistance will also be discussed. For a better understanding of these concepts, a detailed experimental setup is also explained which provides information on how to determine the basic parameters of a ballistic galvanometer.

Current Sensitivity

The current sensitivity, of the coil of a moving coil galvanometer, equals the (steady) current needed to produce a (steady) unit deflection of its coil. For a moving coil galvanometer, we know that the (steady) torque acting on its coil, when a steady current, I, flows through it, equals nABi. Here n = number of turns in the coil of the galvanometer, A = area of the coil and B = magnetic field in which the coil is situated.

This deflecting torque is opposed by the (restoring) torque ($=C\theta$), acting on the coil, due to the (angular) twist, θ, produced in the suspension wire supporting the coil. The equilibrium deflection, θ, corresponding to the flow of a steady current, I, through the coil of the galvanometer, is, therefore, given by

$$nABi = C\theta$$

We thus have,

$$i = (C/nAB)\,\theta = k\theta, \text{ where } k = C/nAB$$

Clearly, $i=k$ when $\theta = 1$ unit. Thus k equals, i_s, the current sensitivity of the given moving coil galvanometer.

We see that since i_s ($=k$) $= C/nAB$, the current sensitivity, of a given galvanometer, can be *improved* by making,

i. ‘C’ small
ii. ‘n’ large
iii. ‘A’ large
iv. ‘B’ large

Here it is important to note that the current sensitivity, of a given moving coil galvanometer, improves when its magnitude is small. This is because a galvanometer, needing a smaller current to produce a unit deflection of its coil, is more sensitive (or responsive) to the flow of a current through its coil than the one which needs a larger current for having its coil deflected by a unit amount.

Current Sensitivity of a Ballistic Galvanometer

The ballistic galvanometer, by its very definition, is a galvanometer designed to measure the total charge (or quantity of electricity), flowing through its coil, when a transient current flows through it. Can we then define, and associate, a current sensitivity with it when we know that the 'current sensitivity', of a moving coil galvanometer, is defined as the 'steady' current needed to produce a 'steady' unit deflection of its coil?

The answer to the question is: 'YES'. This is because a ballistic galvanometer, in spite of the (two) basic change in its design, i.e.,

i. Large value of the *free* time period of its coil
ii. Minimization of damping effects

still remains basically, a moving coil galvanometer. We can, therefore, still, use a 'ballistic galvanometer' as a 'moving coil galvanometer' though the reverse is, generally, not true. Such a use of a ballistic galvanometer has, however, to be done with extra care because of the very nature of its design. A ballistic galvanometer would (generally) need much smaller currents than an ordinary moving coil galvanometer, to produce its 'permitted maximum deflection'.

The current sensitivity of a given ballistic galvanometer would then be defined in the same way as that for an ordinary moving coil galvanometer. It equals the current needed to produce a unit angular deflection of its coil. It, therefore, again equals the ratio (C/nAB), i.e., the same as the expression used for an ordinary moving coil galvanometer.

It is to be again noted that a given galvanometer would have a better current sensitivity if i_s (= C/nAB) has a small value. We thus need to keep C small and n, A, B as large for the galvanometer to have a better current sensitivity. Incidentally, a smaller value of C is also needed to increase the free time period of the coil of a galvanometer, which is an essential requirement for the galvanometer to act as a ballistic galvanometer.

Practically Convenient Definitions of the 'Current Sensitivity'

We have already noticed that we usually use the 'lamp and scale' arrangement for observing and measuring the deflection of the coil of a ballistic galvanometer. We have also noticed that, because of the use of this arrangement, we often think of a 'unit deflection' of the coil of this galvanometer, as corresponding to a movement of one mm, of the reflected vertical line on a scale kept at a distance of 1m from the mirror (of radius of curvature = 1m) attached to the coil.

The 'definition' of the 'current sensitivity' of a ballistic galvanometer, is therefore, given in either of the following two 'practically convenient' forms:

i. The current sensitivity of a ballistic galvanometer equals the current (in μA) needed to produce a displacement of 1mm (of the vertical dark line on the scale) on the scale kept at a distance of 1m from the mirror of the galvanometer.

Or

ii. Conversely, the current sensitivity, of a ballistic galvanometer, equals the number of millimeters deflection of the dark vertical line, on a scale kept at a distance of 1m from the mirror of the galvanometer when a steady current of 1μA flows through the coil.

Experimental Determination of Current Sensitivity

The current sensitivity, of a ballistic galvanometer, may be experimentally determined by using a simple circuit of the type shown in figure 1. We use a potential divider and an adjustable (high) series resistor to ensure that the measured current flowing through the ballistic galvanometer is of the order of a few μA only.

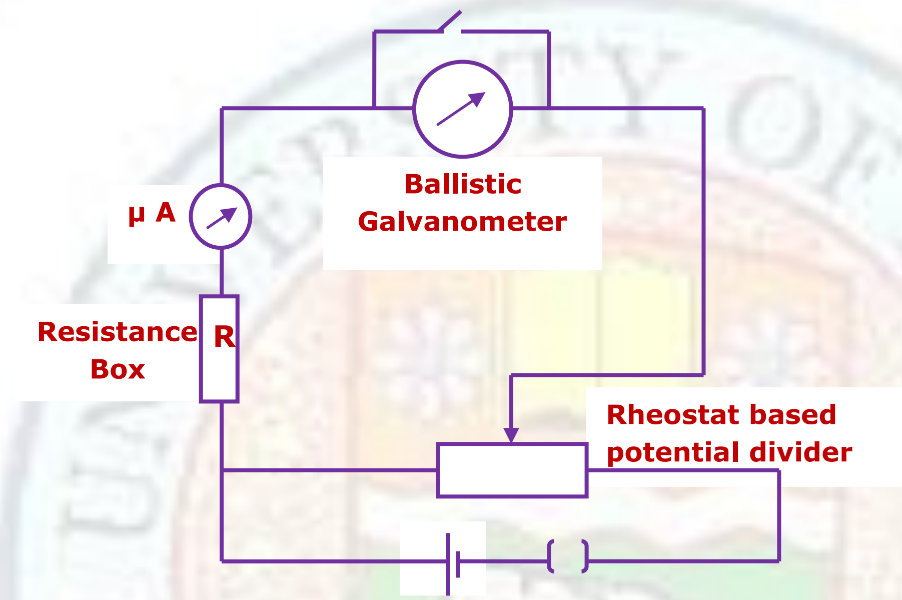


Figure 1

We note (steady) deflection of the coil of the galvanometer for 5 to 6 different values of the current. We can then calculate the mean value of the current per unit deflection and thus calculate the current sensitivity of the given ballistic galvanometer. This value could also be used to find the deflections produced by a current of 1μA. We could thus get a value corresponding to the 'reversed definition' of current sensitivity.

It may be noted that we can avoid the use of the micro-ammeter if we have an arrangement for measuring the potential (V) used to send the current through the galvanometer (of known resistance G). The variable (high) resistance, R, should also have known (or already measured) values. The current would then equal V/(R+G).

Charge Sensitivity

The charge sensitivity of a ballistic galvanometer may be defined as the (transient) charge needed to have a unit value for the first maxima of the deflection of its coil.

As in the case of current sensitivity, we can also define the 'charge sensitivity', in a 'reverse' way. We can say:

The charge sensitivity of a ballistic galvanometer equals the number of millimeter deflections corresponding to the 'first maxima', when a (transient) charge of 1μC flows through its coil.

The word *deflection* used here, could mean,

i. The actual angular deflection of the coil.

Or

ii. The linear displacement, in mm, of the dark vertical line, on the scale, kept at a distance of 1m from the mirror of the galvanometer.

The second of these meanings of deflection is used more often as it is this meaning that is more convenient from a practical point of view.

Another important point needs to be kept in mind here. The *first maximum deflection*, to be used for calculation of *charge sensitivity*, has to be the *corrected value* of the observed first maximum value of this deflection. This *corrected value*, as we have already noticed, is $\theta_1' \approx \theta_1(1+\lambda/2)$, where λ is the 'logarithmic decrement' for the circuit being used in the given experimental setup.

Experimental Determination of Charge Sensitivity

The charge sensitivity, of a given ballistic galvanometer, may be experimentally determined by using a simple circuit of the type shown in figure 2.

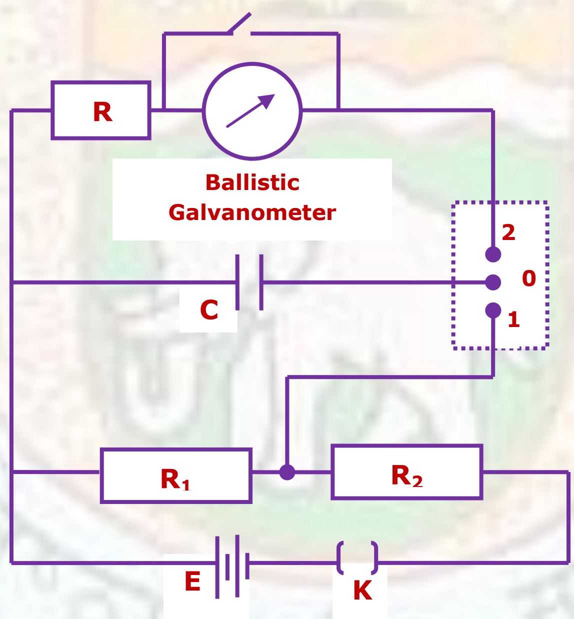


Figure 2

A set of two resistance boxes is used as a potential divider. The potential, V, used to charge the capacitor of (known) capacitance C would equal to

$$V = \frac{E.R1}{R1+R2}$$

Where E is the e.m.f of the cell used. We adjust R_1 and R_2 (keeping $R_1 + R_2$ constant) for taking a number of readings. For each reading, it is desirable to keep R_1 as a small fraction of (R_1+R_2). This is done to ensure that the total charge (Q=CV) that would flow (transiently) through the ballistic galvanometer remains small enough to cause a (first maximum) deflection on the scale of the order of 10-15 cm only. The adjustable resistor R, in series with the ballistic galvanometer can be used to ensure that the B.G. remains in its oscillatory mode and that the deflection of the galvanometer remains within the range of 10-15 cm.

The two-way tapping key, connected to the capacitor, charges it up when this key is pressed down to bring the central stud, 0, in contact with the lower stud, 1. When the key is released, the central stud, 0, 'flies up' to make contact with the stud, 2. This results in a (transient) flow of the charge of the capacitor through the B.G.

Knowing the value of the charge (Q=CV), flowing through the B.G., and the (corrected) value of the first (observed) maximum deflection; one can calculate the charge sensitivity. Its mean value, for all the readings taken, can then be taken as the charge sensitivity of the given ballistic galvanometer, in the given 'setup'.

It may be noted that the charge sensitivity, of a given ballistic galvanometer, is not quite a 'characteristic constant' of that galvanometer. It does depend, somewhat, on the circuit in which the galvanometer is being used. This is because the 'corrected value' of the first (observed) maximum deflection depends on λ, the logarithmic decrement of the circuit. The logarithmic decrement, in turn, depends on the total resistance of the galvanometer circuit.

The reference value, or the quoted value, for the charge sensitivity of a given ballistic galvanometer, is the value corresponding to the galvanometer coil oscillating under open circuit conditions, immediately after the (transient) charge flow, through its coil gets completed. This 'open circuit condition' is achieved in practice by using a special 'two-way' tapping key which breaks the galvanometer circuit immediately after the completion of flow of the transient charge through it.

Under 'open circuit condition', the logarithmic decrement associated with the oscillations of the coil, is practically zero. Hence, under these conditions, the corrected value of the first maximum deflection is almost the same as its actually observed value.

Electromagnetic Damping

Electromagnetic damping, as we know, is used as an effective way to make an ordinary moving coil galvanometer behave in a 'dead-beat' way. This 'dead-beat' action (no oscillations of the coil after getting deflected) is achieved by winding its coil over a soft iron core. The induced (eddy) currents setup in the core cause a (strong) damping torque to act on the coil (while it is getting deflected). This strong torque (because of the relatively large magnitude of the induced currents in the soft iron core) dampens the oscillations of the (deflected) coil to such an extent that it (almost) does not oscillate at all. This makes the ordinary moving coil galvanometer behave in a 'dead-beat' manner.

The ballistic galvanometer, because of the very nature of its action and purpose, is required to have as little damping as possible. The soft iron core is, therefore, either laminated, or replaced by a (insulating in nature) bamboo core or even by an air (or vacuum) core. The damping effects are, therefore, very much reduced and the coil can execute a large number of (practically undamped) oscillations.

It is important, however, to note that the electromagnetic damping effects cannot be completely ignored even for a ballistic galvanometer. This is because the oscillating coil of a ballistic galvanometer is a part of the circuit in which the galvanometer is being used. The coil, oscillating in a magnetic field, has an induced e.m.f, developed across its terminals, as per the phenomenon of electromagnetic induction. This induced e.m.f sends a current through the coil and its associated circuit. The torque, acting on the coil, due to this induced current flow, acts as a damping torque as per Lenz's law.

The interesting thing to note here is the following. The oscillations, of the coil, determine the magnitude of the e.m.f induced in it. The resulting (induced) current would, however, depend on the total resistance (say R_T) of the coil and its associated circuit. The more the value of R_T, the less would be the induced current. The damping torque, associated with this induced current, would, therefore, decrease with an increase in the total resistance_of the galvanometer circuit. This (electromagnetic) damping torque would be practically zero when the galvanometer coil is oscillated under 'open circuit' conditions. The oscillations, under open circuit conditions, are, therefore, practically undamped conditions.

The Role of the Critical Damping Resistance (R_{CDR})

We have already noticed that the coil of a ballistic galvanometer, starts showing its oscillatory behavior only when the total resistance of the galvanometer circuit is more than a minimum value called the critical damping resistance. An increase in the value of the total resistance of the galvanometer circuit decreases the induced current flowing through its coil. This, in turn, reduces the damping torque associated with this induced current. The critical damping resistance can now be viewed as the minimum value of the resistance of the galvanometer circuit for which the damping torque (associated with the induced current) becomes small enough to let the galvanometer coil exhibit its oscillatory behavior.

The Role of the Damping Key

In actual practice, a ballistic galvanometer, when in use, always has a tapping key connected across its coil terminals. This key is known as the *damping key* and it plays a significant role during the practical use of a ballistic galvanometer.

We can understand the role of the damping key as follows. The coil of the ballistic galvanometer, when oscillating, has an e.m.f induced across its terminals as per the phenomenon of electromagnetic induction. The induced current (responsible for the damping torque) has a magnitude dependent not only on the magnitude of the induced e.m.f but also upon the total resistance of the galvanometer circuit. This total resistance, being usually more than the critical damping resistance (electromagnetic), the damping torque on the oscillating galvanometer coil is usually quite small.

During the course of an experiment, it often becomes necessary to stop the oscillations of the coil so that one can go on to take the next reading. The damping key helps to do the needful in such situations. When the damping key is pressed (as in figure 3), the total effective resistance of the galvanometer coil circuit becomes just the resistance of the coil itself. This being the minimum resistance of this circuit, the induced current caused by the induced e.m.f., becomes maximum. The resulting electromagnetic damping torque acting on the coil also has its maximum value and this maximum value is usually sufficient to bring the oscillating coil to rest almost instantaneously. The damping key thus serves as an instant stopping device whenever the oscillating coil needs to be quickly brought to rest. This role of the damping key is of great use to the experimentalist as it helps to save a lot of her/his precious time.

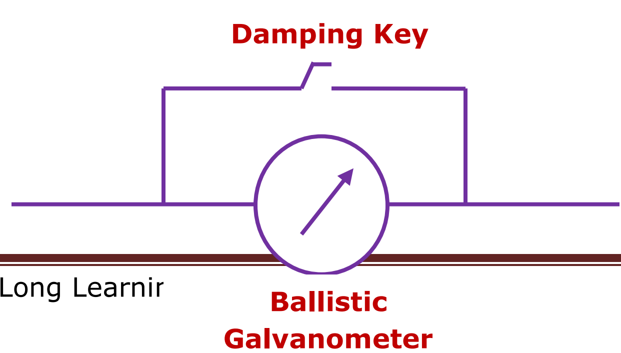


Figure 3

Logarithmic Decrement

The logarithmic decrement (λ) is a useful quantitative measure of the extent of damping in a given ballistic galvanometer circuit. It also provides the correcting factor (≈1+λ/2) needed to correct the value of the experimentally observed first maximum deflection of the coil of the galvanometer.

To understand how λ is defined, we again look at the relation between the deflection (θ) and Q (transient charge) flowing through the galvanometer coil (as derived in the previous lesson). We have,

$$\theta = \frac{\omega 0}{\omega} e^{-lt} \sin(\omega t)$$

As we have seen, θ has its maxima at all time instants, T_n, given by

$$\omega t_{n} = n\Pi + \tan^{-1}(\omega/l)$$

The maxima values of θ, at these time instants are given by:

$$\theta_n = \frac{\omega 0}{\omega} e^{-lt_n} \sin(n\Pi + \tan^{-1}(\omega/l))$$

$$= \pm \frac{\omega 0}{\omega} e^{-lt_n} \sin(\tan^{-1}(\omega/l)) \qquad (n=0, 1, 2, 3 \ldots)$$

$$= \pm \frac{\omega 0}{\omega} e^{-lt_n} \left(\frac{\omega}{\sqrt{l^2+\omega^2}}\right)$$

$$= \pm \frac{\omega 0}{\sqrt{l^2+\omega^2}} e^{-lt_n}$$

But, $l^2 + \omega^2 = l^2 + m^2 - l^2 = m^2$ (therefore, $\omega = \sqrt{m^2 - l^2}$) θ_{n+1}

Therefore, $\theta_n = \pm \frac{\omega 0}{m} e^{-lt_n}$

$$= \pm \omega_0 . \sqrt{I/C} . e^{-lt_n} \quad (\text{since } m^2=C/I)$$

$$= \pm nABQ/I . \sqrt{I/C}\, e^{-lt_n}$$

$$= \pm nAB/\sqrt{CI} . Q . e^{-lt_n}$$

The ratio of the magnitudes of two successive maxima for θ is, is

$$\left|\frac{\theta n}{\theta n+1}\right| = e^{-lt_n} / e^{-lt_{(n+1)}}$$

$$= e^{l(t_{n+1} - t_n)}$$

But, $t_n = 1/\omega\, [n\Pi + \tan^{-1}(\omega/l)]$

Therefore, $t_{n+1} - t_n = 1/\omega\, [(n+1)\Pi - n\Pi] = \Pi/\omega$

Therefore, $|\frac{\theta n}{\theta n+1}| = e^{l.(\Pi/\omega)}$

The ratio, of the magnitudes of any two successive maxima, during the oscillations of the coil, is therefore, a constant for a given circuit. We may call this ratio as the decrement factor because it gives us the factor by which each maximum gets decreased relative to its predecessor. This factor is clearly going to depend on the extent of damping in the circuit.

The logarithm (to the base of e) of this decrement factor is known as the logarithmic decrement (λ) for the given circuit. We thus have,

$$\lambda = \log_e |\frac{\theta n}{\theta n+1}| = \log_e e^{l.(\Pi/\omega)} = \Pi\ l/\omega$$

We often rewrite this equality as:

$$\lambda/\Pi = l/\omega$$

We may define the logarithmic decrement (λ) as follows;

The logarithmic decrement (λ), for a given ballistic galvanometer circuit, equals the natural logarithm of the ratio of the magnitudes of any two successive maxima of the deflection of the oscillating coil of the ballistic galvanometer.

The logarithm decrement (λ), rather than the decrement factor ($=|\frac{\theta n}{\theta n+1}|$), is taken as the practical indicator and measure of the extent of damping in a given ballistic galvanometer circuit. This is because of the relatively more compact form of its expression.

Now, $\lambda/\Pi = l/\omega$

Also, $l = 1/2I\ [b + (nAB)^2/R]$

And, $\omega = \sqrt{m^2 - l^2}$ where $m^2 = C/I$

The term l, as we see, is related to the sum of the viscous damping related term, b, and the electromagnetic damping related term $(nAB)^2/R$. The term l, therefore, is an indicator of the total damping present in a given circuit.

When the galvanometer coil is executing its free or natural oscillations, it is in an open circuit, so that $R \rightarrow \infty$. We can, therefore, take $l \rightarrow 0$ during the free or natural oscillations of the coil of the galvanometer.

Since, $\lambda = \Pi\ l/\omega$, we can take $\lambda \rightarrow 0$ during the free or natural or open circuit ($R \rightarrow \infty$), oscillations of the coil. The very small damping associated with such oscillations is due only to the viscous effects of the air in which the coil is oscillating. For such free oscillations, λ has its minimum value. This minimum value is usually denoted by λ_0. Thus,

λ_0 = logarithmic decrement, for a given ballistic galvanometer coil, when the coil is oscillating under open circuit (or free or natural) conditions.

Further, $\omega = 2\Pi/T$, where t is the time period of oscillations of the galvanometer coil under actual circuit conditions.

Therefore, $\lambda = \Pi\ l/\omega = \Pi.l.T/2\Pi = l.T/2$

Now, $T = 2\Pi/\omega = 2\Pi/\sqrt{m^2 - l^2}$

Also, T_0 = free or natural time period of oscillations of the coil,

$= 2\Pi/m = 2\Pi\sqrt{I/C}$ (to correspond to $l \rightarrow 0$)

Therefore, $T/T_0 = m/\sqrt{m^2 - l^2} = m/m\ (1 - \frac{l^2}{m^2})^{1/2}$

$= (1 - \frac{l^2}{m^2})^{-1/2}$

The term of the right equal nearly $(1+\frac{1}{2}\frac{l^2}{m^2})$ when l << m. For a ballistic galvanometer circuit, l is usually kept very small. Hence the approximation, l << m, is usually a practically valid approximation. We then have,

$(1 - \frac{l^2}{m^2})^{-1/2} \approx (1+\frac{1}{2}\frac{l^2}{m^2}) \approx (1+\frac{l^2}{m^2})^{1/2}$

When l << m, we can also put ω ($=\sqrt{m^2 - l^2}$) as approximately equal to m only. We can, therefore, put $((1+\frac{l^2}{m^2})^{1/2})$ as, approximately equal to $((1+\frac{l^2}{\omega^2})^{1/2})$. But,

$l/\omega = \lambda/\Pi$

Therefore, $(1+\frac{l^2}{\omega^2}) = (1+\frac{\lambda^2}{\Pi^2})$

We, therefore, have

$T \approx T_0\ (1+\frac{l^2}{\omega^2})^{1/2} = T_0\ (1+\frac{\lambda^2}{\Pi^2})^{1/2}$

We had made use of this approximation in our derivation of the relation between the deflection, θ, and the transient charge, Q, flowing through the ballistic galvanometer coil.

A Practically Useful Expression for λ

We have seen above that

$|\frac{\theta n}{\theta n+1}|$=decrement factor=$x=e^{l.\Pi/\omega}$

This has a constant magnitude for a given circuit. We can now write,

$x = |\frac{\theta n}{\theta n+1}| = |\frac{\theta 1}{\theta 2}| = |\frac{\theta 2}{\theta 3}| = |\frac{\theta 3}{\theta 4}|$ =---------= $|\frac{\theta n}{\theta n+1}|$

Now, $|\frac{\theta n}{\theta n+1}| = |\frac{\theta 1}{\theta 2}| X |\frac{\theta 2}{\theta 3}| X |\frac{\theta 3}{\theta 4}|$ X--------X $|\frac{\theta n}{\theta n+1}|$

= (x) X (x) X (x) X------------n times = x^n

Therefore, $\log_e|\frac{\theta n}{\theta n+1}| = n.\log_e(x)$

But, by definition, $\log_e x = \lambda$, the logarithmic decrement.

Hence we can write,

$\log_e|\frac{\theta n}{\theta n+1}| = n\lambda$

Therefore, $\lambda = 1/n\ (\log_e|\frac{\theta n}{\theta n+1}|)$

In practice, it usually becomes difficult to note the magnitude of two successive maxima. However, it is quite convenient to note the magnitudes of, say, the first maxima (θ_1) and the 11th maxima (θ_{11}). In such a case, we can calculate λ from the relation,

$$\lambda = 1/10\ (\log_e|\frac{\theta 1}{\theta 11}|)$$

This way of calculating λ is a useful direct practical way of obtaining the value of the logarithmic decrement. We use this approach whenever we calculate λ in a direct manner based on the definition of λ.

Indirect Experimental Methods of Calculating the Critical Damping Resistance using the Values of λ

We have an indirect experimental method for calculating the critical damping resistance for a given ballistic galvanometer. This method is based on the knowledge of the values of the logarithmic decrement, corresponding to different values of the resistance of the circuit. We first notice that,

$$\lambda = l.T/2$$

We can write, $l = 2\lambda/T$

But, $T \approx T_0\ (1+\frac{\lambda^2}{\Pi^2})^{½}$

Hence, $l = 1/2I\ [b + (nAB)^2/R] = 2\lambda/\ T_0\ (1+\frac{\lambda^2}{\Pi^2})^{½}$

Therefore, we can write,

$$\frac{b}{4}\frac{To}{I} + \frac{To}{4I}\frac{(\mathrm{nAB})^2}{R} = \lambda\ (1+\frac{\lambda^2}{\Pi^2})^{-½}$$

We can regard this equation as giving us the relation between R, the total resistance of the galvanometer circuit and λ, the corresponding logarithmic decrement of the circuit. To make use of this relation for calculating R_{CDR}, the critical damping resistance of the given galvanometer, we notice that we can take,

i. $\lambda \rightarrow \lambda o$ (the minimum value of λ) when $R \rightarrow \infty$, i.e., when the galvanometer coil is oscillating under its open circuit or free or natural condition.
ii. $\lambda \rightarrow \infty$ (i.e. λ has a very large value) when $R \rightarrow R_{CDR}$. This is so because the galvanometer coil just starts coming into its oscillating mode (i.e. is still quite heavily damped) when $R=R_{CDR}$. The proper oscillatory behavior of the coil comes in only when $R>R_{CDR}$. For $R>R_{CDR}$, λ keeps on decreasing with increasing R.

We now again rewrite the above relation between R and λ. We have,

$$\frac{b}{4}\frac{To}{I} + \frac{To}{4I}\frac{(\mathrm{nAB})^2}{R} = \lambda\ (1+\frac{\lambda^2}{\Pi^2})^{-½}$$

When $R \rightarrow \infty$, we have $\lambda \rightarrow \lambda_o$. Hence,

$$\frac{b}{4}\frac{To}{I} \approx \lambda_0\ (1+\frac{\lambda o^2}{\Pi^2})^{-½}$$

Now λ_o, the minimum value of λ, is quite small. We can therefore neglect in comparison to 1. We thus get

$$\frac{b}{4}\frac{To}{I} \approx \lambda_0$$

Hence, $\lambda_o + [\frac{To(\mathrm{nAB})^2}{4I}].1/R = \lambda\ (1+\frac{\lambda^2}{\Pi^2})^{-1/2}$

Here it is important to note that both λo and λ can be easily determined experimentally by observing the oscillations of the coil when it is oscillating in

i. an open circuit, and
ii. in a circuit having a total resistance R.

The total resistance R equal $R_{ex} + R_G$ where R_{ex} is the external resistance put in series with the galvanometer coil and R_G, is the resistance of the galvanometer coil itself. The galvanometer coil resistance (R_G) may be pre-known (say from the data provided by the manufacturer) or may have to be determined experimentally.

Now, when $R \rightarrow R_{CDR}$, λ acquires a very large value. We can then neglect 1 in comparison to $\frac{\lambda^2}{\Pi^2}$. The term $[\lambda\ (1+\frac{\lambda^2}{\Pi^2})^{-1/2}]$, therefore, approximates λ X Π/λ or Π when $R \rightarrow R_{CDR}$.

Also, when $R \rightarrow R_{CDR,}$ the coil has just starting coming into its oscillatory mode, thus, for $R \rightarrow R_{CDR,}$ we can regard T, the time period of oscillations of the coil, as having a very large value.

These approximations imply that for $R \rightarrow R_{CDR,}$ we can write the above relation between λ and R, in the form

$$\lambda_o + [\frac{To(\mathrm{nAB})^2}{4\Pi}].1/R_{CDR} = \Pi$$

Therefore, $R_{CDR} = [\frac{To(\mathrm{nAB})^2}{4\Pi}] / [\Pi - \lambda_0]$

For a given galvanometer, T_0 and λ_0 can be easily measured experimentally. However, the other terms (n, A, B) cannot be measured that easily. We therefore use an approximate graphical method for finding out R_{CDR}.

We again look at the relation

$$\lambda_o + [\frac{To(\mathrm{nAB})^2}{4I}].1/R = \lambda\ (1+\frac{\lambda^2}{\Pi^2})^{-1/2}$$

When R is quite large in comparison to R_{CDR}, the damping is quite small. For large values of R, we can therefore, neglect in comparison to 1 and write

$$\lambda_o + [\frac{To(\mathrm{nAB})^2}{4I}].1/R \approx \lambda$$

A graph of λ against 1/R, would, therefore, be a straight line under these conditions. The slope of this straight line equals the term $[\frac{To(\mathrm{nAB})^2}{4I}]$. Knowledge of this term, along with the experimentally determined value of λ_0, enables us to calculate R_{CDR}.

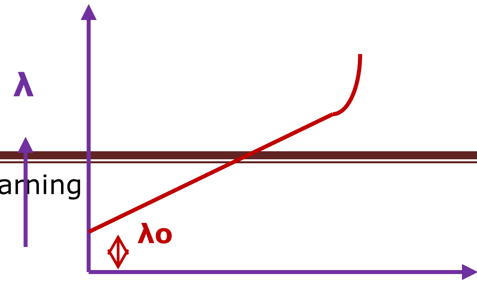


Figure 4

Referring to figure 4, it may be noted that the assumed condition ($\lambda^2/\Pi^2 << 1$) tend to break down for increasing values of (1/R) i.e., for decreasing value of R. this is understandable because the electromagnetic damping effects keep on increasing with decreasing values of R. The experimentally plotted graph, between λ and 1/R, therefore, starts deviating from its straight line nature beyond a certain value of 1/R. It may be noted that the point corresponding to 1/R = 0, i.e., $R \rightarrow \infty$, would correspond to $\lambda = \lambda_0$. This may be checked against the experimentally determined value of λ_0.

Summary

In this lesson we have learned the following concepts.

1. Basic parameters of a ballistic galvanometer, such as, its current and charge sensitivity and the electromagnetic damping.

2. We have learned about the importance of critical damping resistance in the determination of the above mentioned parameters.

3. A detailed experimental set-up has been explained to determine the above mentioned parameters.

4. Role of damping key and the concept of logarithmic decrement has also been discussed.

Exercise

Fill in the Blanks

i. The free time period of the coil of a ballistic galvanometer should be __________ compared to the time of flow of the transient current through it.
ii. The deflection of the coil of a ballistic galvanometer is assumed to be __________ at the instant at which the flow of the transient current through it stops.
iii. The core, used for winding the coil of a ballistic galvanometer, can be either a __________ core or a __________ core.
iv. The induced currents, flowing in the coil of a ballistic galvanometer during its oscillations, cause a ________torque to be exerted on it.
v. The coil of a ballistic galvanometer exhibits an oscillatory behavior only when the total resistance of the circuit of which it is a part, __________ the CDR of the galvanometer.

Answers:

i. Large
ii. Almost zero
iii. Laminated soft iron ; bamboo
iv. Damping
v. exceeds

True/ False

State whether the following statements are 'true' or 'false':

i. The electromagnetic damping torque acting on the coil of a ballistic galvanometer equals (nAB)/R$\frac{d\theta}{dt}$, where the symbols have their usual meaning.
ii. The deflection of the coil and the angular velocity acquired by it at the instant the flow of the transient current stops, are both assumed to be zero for a ballistic galvanometer.
iii. The coil of a ballistic galvanometer exhibits an oscillatory or a non oscillatory behaviour depending on the total resistance of the galvanometer and its associated circuit.
iv. The time period of oscillations of the coil of a ballistic galvanometer, when the coil is a part of a circuit, is more than its free or open circuit time period.
v. The logarithmic decrement for a given ballistic galvanometer has a constant value irrespective of the circuit in which the galvanometer is being used.

Answers:

i. FALSE
The electromagnetic damping torque = induced current x total area of the coil x magnetic field
ii. FALSE
While the deflection of the coil is assumed to be zero, the angular velocity acquired by it due to the impulsive torque has a finite value. It is because of this acquired angular velocity that the coil oscillates even after the flow of the transient current through it stops.
iii. TRUE
The behavior is oscillatory when $R>R_{CDR}$ and non-oscillatory when $R<R_{CDR}$.
iv. TRUE
When the coil is a part of a circuit there is some electromagnetic damping torque acting on it. This slows down its oscillations and increased the time period of these oscillations.
v. FALSE
The logarithmic decrement depends on the extent of damping in the circuit, which, in turn, depends on the total resistance of the circuit.

Multiple Choice Questions

Select the best alternative in each of the following:

Ballistic GalvanometerLesson 7.2: Basic Parameters associated with the Ballistic Galvanometer

i. The amplitude of oscillations for a ballistic galvanometer coil oscillating when present in a circuit
   a. Keeps on decreasing with time with the rate of decrease, increasing with an increase in the total resistance of the galvanometer circuit.
   b. Keeps on decreasing with time with the rate of decrease, decreasing with the increase in the total resistance of the galvanometer circuit.
   c. Remains nearly constant with time irrespective of the circuit of which the galvanometer is a part.
   d. Keeps on increasing with time, with the rate of increase, increasing with an increase in the small time interval for which the transient current flows.

Answer: b

Justification/Feedback for the correct answer:

a. An increase in the total resistance decreases the electromagnetic damping effects which would decrease the rate of decrease of the amplitude of oscillations.

b. same as above

c. same as above

d. the transient current flow affects the impulsive torque exerted on the coil and there by the amplitude of its first (half) oscillation. The subsequent amplitudes have to keep on decreasing with time.

ii. The logarithmic decrement for the oscillating coil of a given ballistic galvanometer present in a given circuit, has a
   a. Constant value for that circuit but this value is more than that for the same ballistic galvanometer when oscillating under open circuit conditions
   b. Constant value for that circuit but this value is less than that for the same ballistic galvanometer when oscillating under open circuit conditions.
   c. Continuously decreasing value with the final value soon approaching zero.
   d. Continuously increasing value with the final value approaching that for the same ballistic galvanometer when oscillating under open circuit conditions.

Answer: a

Justification/Feedback for the correct answer:

a. The logarithmic decrement, by definition, equals the natural logarithm of the ratio of the magnitudes of the amplitudes of two successive oscillations. It is constant for a given circuit with its value decreasing with an increase in the total resistance of the circuit. It, therefore, has its minimum value when a given ballistic galvanometer oscillated under open circuit conditions.

b. same as above

c. the logarithmic decrement for a given ballistic galvanometer used in a given circuit has a constant value for that circuit.

d. same as above

iii. The critical damping resistance (R_{CDR}) for a given ballistic galvanometer has a value,
   a. That mainly depends on the n, A, B, C and I values for that galvanometer and we need to keep the total resistance of the galvanometer circuit just equal to its R_{CDR} for the galvanometer to show an oscillatory behavior.
   b. That mainly depends on the n, A, B, C and I values for that galvanometer and we need to keep the total resistance of the galvanometer circuit less than its R_{CDR} for the galvanometer to show an oscillatory behavior.
   c. That mainly depends on the n, A, B, C and I values for that galvanometer and we need to keep the total resistance of the galvanometer circuit more than R_{CDR} for the galvanometer to show an oscillatory behavior.

d. That does not depend on the n, A and B values for that galvanometer but is a circuit dependent multiple of the resistance of the galvanometer coil itself.

<u>Answer</u>: c

<u>Justification/Feedback for the correct answer:</u>

a. We have

R_{CDR} = $(nAB)^2/(\sqrt{4CI}$ - b)

It thus mainly depends on the n, A, B, C and I values for a given galvanometer. There is a small effect, due to b, the constant determining the effect of air damping. However, we need to keep the total resistance of the galvanometer circuit considerably more than its R_{CDR} for the galvanometer to show an oscillatory behavior.

b. Same as above

c. Same as above

d. Same as above

iv. In the design of a ballistic galvanometer the moment of inertia about the suspension wire axis is kept large and the torsional constant of its suspension wire is kept small. This is done to ensure that

a. The electromagnetic damping effects get very much reduced

b. The critical damping resistance, for the galvanometer has a small value

c. The free time period of the galvanometer coil is much smaller than the duration of the time interval for which the transient current flows through the galvanometer coil.

d. The free time period of the galvanometer coil is much greater than the duration of the time interval for which the transient current flows through the galvanometer coil.

<u>Answer:</u> d

<u>Justification/Feedback for the correct answer:</u>

a. The given factors affect the free time period of the galvanometer coil and not the extent of electromagnetic damping produced in it.

b. The given factors affect the free time period of the galvanometer coil and not the critical damping resistance of the galvanometer circuit.

c. The given factors affect the free time period of the galvanometer coil and not the extent of the electromagnetic damping produced in it. The manufacturer adjusts their values so that this free time period of the galvanometer coil is much greater than the duration of the time interval for which the transient current flows through the galvanometer coil.

d. Same as above.

v. In the lamp and scale arrangement used with a ballistic galvanometer the displacement of the dark vertical line on the scale is proportional, strictly speaking, to,

a. The angular displacement, θ, of the coil

b. 2θ where θ is the angular displacement of the galvanometer coil

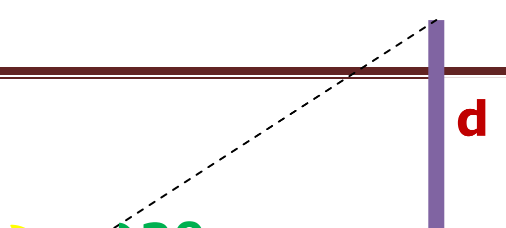
c. tanθ, where θ is the angular displacement of the galvanometer coil

d. tan2θ, where θ is the angular displacement of the galvanometer coil

<u>Answer:</u> d

<u>Justification/Feedback for the correct answer:</u>

a. When the coil of the galvanometer, and, therefore, the concave mirror attached to it turns through an angle θ, the reflected ray turns through and angle 2θ. The displacement, d, of the dark vertical line on the sale, is therefore, given by,



$d/D = \tan 2\theta$

⇨ $D = D \tan 2\theta$

Thus, $d \propto \tan 2\theta$

It is only for small values of θ (and therefore 2θ) that we can approximate tan 2θ by 2θ. However strictly speaking, we have $d \propto \tan 2\theta$

b. Same as above
c. Same as above
d. Same as above

Short Notes

Write short notes on:

1. The main differences in the design of a ballistic galvanometer and an ordinary moving coil galvanometer
2. The critical damping resistance of a ballistic galvanometer.
3. The logarithmic decrement of a ballistic galvanometer under open circuit conditions
4. The charge sensitivity of a ballistic galvanometer
5. The experimental determinations of the current sensitivity of a given ballistic galvanometer.

Essay type questions

1. State, with reason, and with their appropriate expressions, the different torques acting on the coil of a ballistic galvanometer during the duration of the flow of a transient current through it.
2. Explain how the total resistance of a ballistic galvanometer circuit determines the nature of the movement of its coil.
3. Define the term logarithmic decrement. Discuss how it can be determined experimentally for a given ballistic galvanometer being used in a given circuit.
4. Write the equation of motion of the coil of a ballistic galvanometer during the time intervals
   i. $0<t<\tau$
   ii. $t>\tau$.

Here τ is the time for which the transient current flows through the galvanometer coil. Use the first of these equations to obtain an expression for the impulsive angular velocity acquired by the coil at $t=\tau$.

5. Discuss why the electromagnetic damping effects in a ballistic galvanometer depend on the total resistance (R) of the galvanometer circuit. Hence give a qualitative justification for keeping R greater that R_{CDR} (the critical damping resistance) in a practical circuit using a ballistic galvanometer.
6. Discuss the role and significance of the following in the working of a ballistic galvanometer.
    i. Lamp and scale arrangement
    ii. Damping key

References

1. Books:

- Principles of Electromagnetics by Matthew N. O. Sadiku, Oxford University Press
- Introduction to Electrodynamics by David J. Griffiths, Pearson Education
- Electricity and Magnetism by D. L. Sehgal, K. L. Chopra, N. K. Sehgal, Pub: Sultan Chand and sons