Discipline Course-I
Semester-II
Paper No: Thermal Physics : Physics-IIA
Lesson: Applications of Second law of thermodynamics
Lesson Developer: Dr. Savita Datta
College/ Department: Physics Department,
Maitreyi College, University of Delhi

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Learning Objectives:

After studying this lesson, you will be able to;

- ✓ **Define Carnot cycle**
- ✓ **Calculate the work done in a Carnot cycle**
- ✓ **Find the efficiency of a Carnot's engine**
- ✓ **Explain the working of a refrigerator**
- ✓ **Calculate the co efficient of performance of a refrigerator**

2.1 Introduction

We can illustrate second law of thermodynamics with an application to the theory of heat engines which are machines that convert heat energy into mechanical work. To be a useful device a heat engine should operate continuously. It works by absorbing heat from a reservoir at a higher temperature and rejecting it to a reservoir at a lower temperature. Thus a heat engine has to operate between two heat reservoirs. Sadi Carnot suggested a theoretical engine which is free from all practical imperfections. It has maximum efficiency which cannot be achieved in the real world. Carnot's engine is a perfectly reversible engine. That means all stages of operation should be carried out infinitely slowly so that there are no dissipative losses. The simplest reversible cycle is due to Carnot. In Carnot cycle any substance can be made to exchange heat from the heat reservoirs. A Carnot cycle has four processes in which two are reversible adiabatic processes and remaining two are reversible isothermal processes. A system undergoing a Carnot cycle is called a Carnot engine, although such a **'perfect' engine** is only a theoretical limit and cannot be built in practice. When the Carnot engine works in the reverse direction it works as a refrigerator. The co efficient of performance of a refrigerator is defined as the ratio of total heat extracted at lower temperature to the amount of input work done.

2.2 Carnot cycle

It is a cycle of expansion and compression of an idealized reversible heat engine that does work without loss of heat.

Every single thermodynamic system exists in a particular state. When a system is taken through a series of different states and finally returned to its initial state, a thermodynamic cycle is said to have occurred. In the process of going through this cycle, the system may perform work on its surroundings, thereby acting as a heat engine.

The Carnot cycle is a theoretical thermodynamic cycle proposed by Sadi Carnot. It is the most efficient cycle for converting a given amount of thermal energy into work, or conversely, creating a temperature difference by doing a given amount of work.

The various stages of the cycle executed by a reversible engine can be represented on the PV indicator diagram (Figure 2.1).

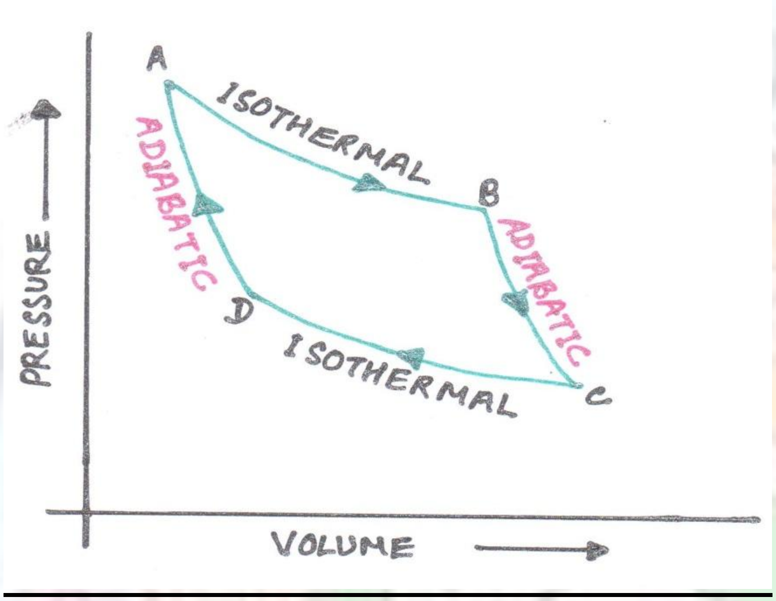


Figure 2.1: Reversible Cycle.

This cycle has four operations : two isothermal processes at two different constant temperatures and two adiabatic processes. All processes are performed **quasistatically.** The first operation from A to B is a reversible isothermal expansion at constant high temperature. The second process from B to C is a reversible adiabatic expansion. The third operation from C to D is a reversible isothermal compression at constant low temperature. The fourth operation from D to A is reversible adiabatic compression. At the end of the Carnot cycle the original state is restored as it is a perfectly reversible cycle.

Ctrl+ click the following link to know more about Carnot Cycle:

http://www.youtube.com/watch?v=MixzZ5F7y1Y

2.3 Carnot's engine and its efficiency

It was there in 1818 that Sadi Carnot saw a steam engine and became hooked on the problem of understanding how it worked. He was struck by the analogy between a water wheel and a steam engine, in which heat (rather than water) flows from a reservoir at low temperature. Carnot's genius was that rather than focus on the details of the steam engine he decided to consider an engine in abstracted form, focusing purely on the flow of heat between two thermal reservoirs. He idealized the workings of an engine as consisting of simple gas cycles and worked out its efficiency. He realized that to be as efficient as possible, the engine has to pass slowly through a series of equilibrium states and that it therefore had to be reversible. At any stage, you could reverse its operation and send it to the other way around the cycle. He was then able to use this fact to prove that all reversible heat engines operating between two temperatures had the same efficiency.

Ctrl+ Click the following link to understand more about Carnot Engine.

http://www.youtube.com/watch?v=kJlmRT4E6R0

A Carnot's engine is an ideal theoretical engine that converts heat into work. It uses a **perfect gas** (e.g. air) as the working substance.

Following is the simplified description of Carnot's engine:

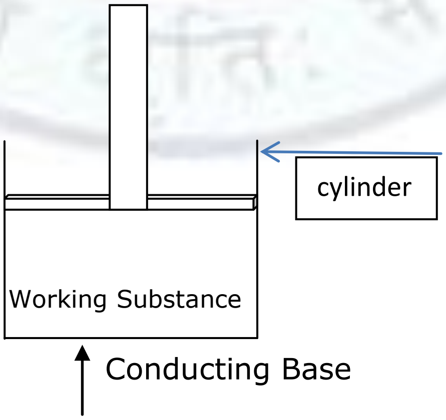
It has four parts:

(1) Working substance- It is one mole of a perfect gas which is enclosed in a cylinder with non-conducting walls but conducting base. The cylinder is fitted with a piston which is frictionless and perfectly insulating.

(2) Source- Source is a reservoir having **infinite thermal capacity** maintained at a high temperature T_1. The top of the source is conducting. The heat engine draws heat from the source.

(3) Sink- Sink is a reservoir having infinite thermal capacity maintained at a lower temperature T_2 and the top of the sink is conducting. The heat engine rejects heat to the sink.

(4) Stand- The stand is a platform which is perfectly non conducting.



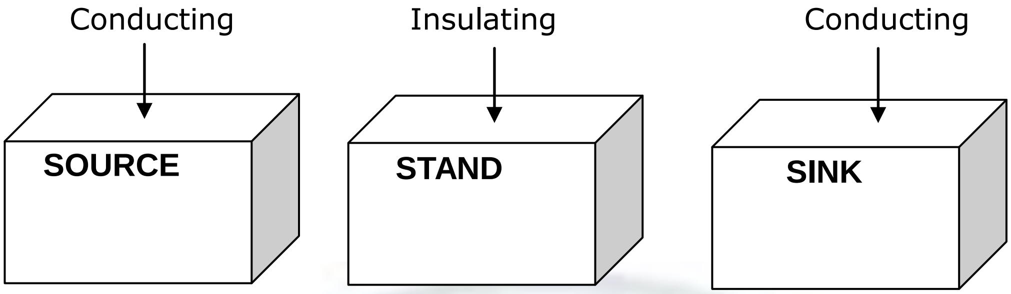


Figure 2.2: Parts of the Carnot's Engine

The operations:

The working substance is placed within the cylinder and is subjected to a cycle of four operations to get a continuous supply of work. The cycles of operations are: two isothermal and two adiabatic which is represented on the PV indicator diagram [Figure(2.3)].

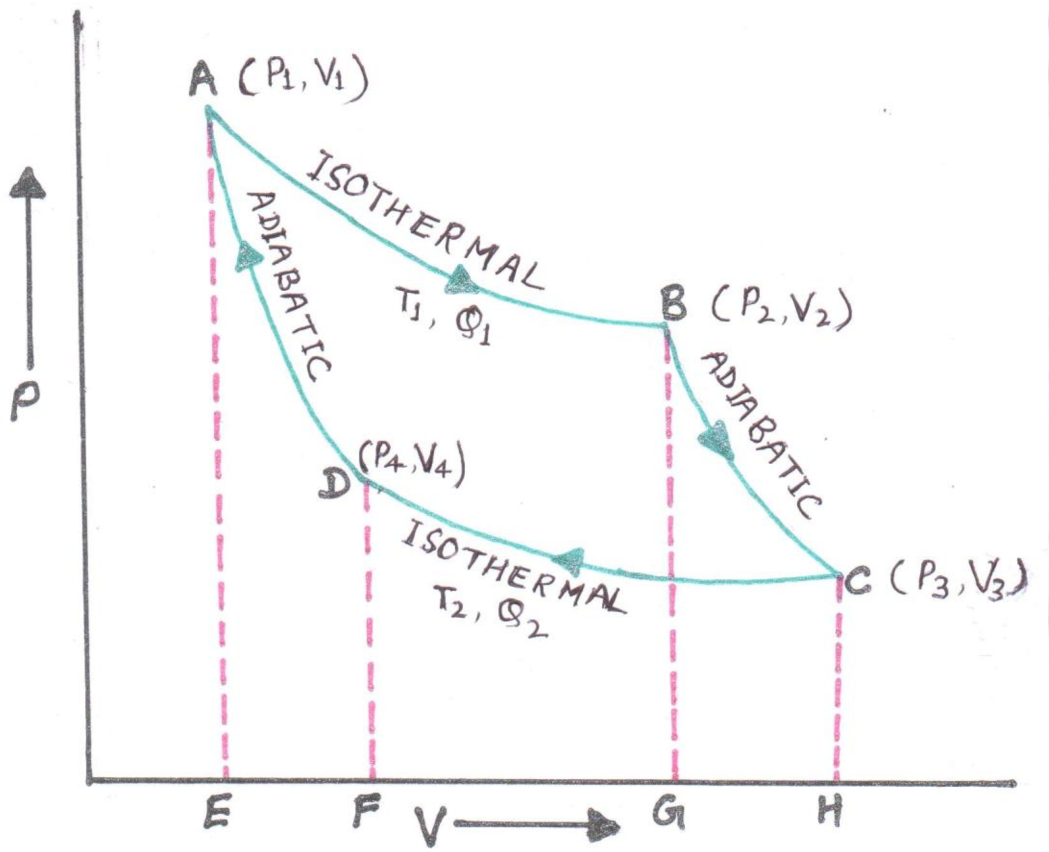


Figure 2.3: Carnot Cycle

Step 1 (Isothermal expansion): The cylinder with the working substance (perfect gas) is kept in thermal contact with the source, the temperature of which is T_1. Let the initial pressure is P_1 and the volume is V_1. This state is represented by the point A in the diagram.

Now the gas is allowed to expand quasistatically at constant temperature(T_1) from A to B i. e the process **A→B** is isothermal. In this process the heat flows from the source to the system. The pressure and volume of the gas change extremely slowly (and becomes P2 ,V2 respectively at point B) but its temperature remains constant throughout the process.

<u>Let during the process from A to B:</u>

Q_1 = the amount of heat that flows from the source into the working substance

W_1= the amount of work done by the system

$$= \int_{V_1}^{V_2} P\, dV$$

For one mole of an ideal gas, $PV = RT$

Or, $P = \frac{RT}{V}$

So, $W_1 = RT_1 \int_{V_1}^{V_2} P\, \frac{dV}{V}$

So, $W_1 = RT_1 ln\left(\frac{V_2}{V_1}\right) - - - - - - - - - - (2.1)$

From the first law of thermodynamics,

$$\boxed{\delta Q = dU + \delta W}$$

For an isothermal process, from A to B, the amount of heat absorbed Q_1 is converted into work W_1 by the engine and the change in internal energy is zero (because the temperature remains constant).

So $\boxed{W_1 = Q_1}$ and $\boxed{dU = 0}$

Now the work done during the isothermal process (from equation 2.1),

$$\boxed{\mathbf{W_1 = Q_1 = RT_1\ ln\left(\frac{V_2}{V_1}\right) = Area\ (A\ B\ G\ E\ A\ )\ ---------------(2.2)}}$$

Step 2 (Adiabatic Expansion): Now the cylinder with the working substance is kept on the insulated stand. The gas is allowed to expand quasistatically from B to C till the temperature falls to the temperature of the sink (T_2). The pressure decreases from P_2 to P_3 and volume increases from V_2 to V_3. The temperature falls from T_1 to T_2. The process **B →C** is reversible adiabatic expansion. There is no heat flow in this process.

Let during the process from B to C:

W_2 = the amount of work done by the system

$$=\int_{V2}^{V3} PdV$$

Now using the adiabatic relation for an ideal gas, $PV^{\gamma} = K$ (a constant)

Here $\gamma = \frac{C_P}{C_V}$ = the ratio of two specific heats.

$$W_2 = K\int_{V2}^{V3} \left(\frac{dV}{V^{\gamma}}\right)$$

$$=\frac{K}{1-\gamma}\left(\frac{1}{{V_3}^{\gamma-1}} - \frac{1}{{V_2}^{\gamma-1}}\right) = \frac{1}{1-\gamma}\left(\frac{K}{{V_3}^{\gamma-1}} - \frac{K}{{V_2}^{\gamma-1}}\right)$$

$$=\frac{K{V_3}^{1-\gamma} - K{V_2}^{1-\gamma}}{1-\gamma}$$

$$=\frac{1}{1-\gamma}(P_3V_3 - P_2V_2) \quad [\text{Since, } P_3V_3^{\gamma} = P_2V_2^{\gamma} = K]$$

$$=\frac{RT_2 - RT_1}{1-\gamma}$$

Or,

$$\boldsymbol{W_2 = R\left(\frac{T_1-T_2}{\gamma-1}\right)} \textbf{ = Area(B C H G B) -----------------------------(2.3)}$$

Step 3 (Isothermal Compression): Now cylinder with the working substance is kept in thermal contact with the sink. The gas is compressed quasistatically from C to D. The pressure increases from P_3 to P_4 and the volume decrease from V_3 to V_4 but the temperature remains the same. The process **C →D** is reversible isothermal at temperature T_2. In this process there is heat flow from the system to the sink.

Let, during the process from C to D:

$Q_2 =$ the amount of heat rejected by the working substance to the sink

W_3 = the work done on the working substance

$$\boldsymbol{W_3 = Q_2 = \int_{V3}^{V4} PdV = -RT_2 ln\left(\frac{V_3}{V_4}\right)} \textbf{= - Area(C H F D C) ------------------------(2.4)}$$

Here the negative sign indicates that the **work** is done **on** the working substance.

Step 4 (Adiabatic Compression): Now the cylinder with the working substance is kept on the insulated stand. The gas is compressed quasistatically from D to A. The pressure increases from P_4 to P_1 and the volume decreases from V_4 to V_1. The temperature increases from T_2 to T_1. The process **D →A** is adiabatic. In this process there is no heat flow. At the end of this process the system comes back to its original state (A).

Let, during the process from D to A:

W_4 = the work done on the system

$= \int_{V4}^{V1} PdV$

$W_4 = -R\frac{T_1-T_2}{\gamma-1}$**= - Area(D F E A D) -----------------------------(2.5)**

So for the complete cycle the total work done by the engine is ,

$W = \oint PdV = W_1 + W_2 + W_3 + W_4$

Since w_2 and w_4 are equal and opposite, they cancel each other.

W = W₁ – W₃ = $RT_1 \ln\left(\frac{V_2}{V_1}\right) - RT_2 \ln\left(\frac{V_3}{V_4}\right)$ -----------------------------(2.6)

The total work done per cycle in terms of area [Figure (2.3)] is,

W = Area (A B G E A) + Area (B C H G B) - Area(C H F D C) – Area (D F E A D)

W = Area(A B C D A)=Area of the Carnot cycle

Thus, the area of the Carnot's cycle represents the net amount of work done per cycle.

In the cyclic process, net heat absorbed = net work done per cycle

So, $Q_1 - Q_2 = W_1 - W_3 = W$ --------------------(2.7)

We use another adiabatic relation for an ideal gas:

$$TV^{\gamma-1} = constant$$

Here $\gamma = \frac{C_P}{C_V}$ = the ratio of two specific heats.

Since B and C lie on the same adiabatic curve,

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

Or, $\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$-------------- (2.8)

Similarly for D and A we can write,

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

Or, $\frac{T_1}{T_2} = (\frac{V_4}{V_1})^{\gamma-1}$-------------- (2.9)

Comparing equations (2.8) and (2.9)

We get,

$$(\frac{V_3}{V_2})^{\gamma-1} = (\frac{V_4}{V_1})^{\gamma-1}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Using this result in equation (2.6), net work done per cycle,

$$\boxed{\mathbf{W = Q_1 - Q_2 = R(T_1 - T_2)\ln(\frac{V_2}{V_1})} \text{ --------------------------(2.10)}}$$

Efficiency:

In general, the "efficiency" or "effectiveness" of a process is calculated by dividing the desired output by the total input. Thus the efficiency should have a larger value.

For a heat engine, the efficiency is the ratio of useful output to the heat energy consumed from the high-temperature reservoir:

$$\text{Eficiency} = \eta = \frac{\text{useful output}}{\text{total input}} = \frac{W}{Q_1}$$

From equations (2.2) *and* (2.10), *we have*,

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{R(T_1 - T_2)\ln(\frac{V_2}{V_1})}{RT_1 \ln(\frac{V_2}{V_1})}$$

$$or, \quad \eta = \frac{T_1 - T_2}{T_2}$$

$$\boxed{\boldsymbol{\eta = 1 - \frac{T_2}{T_1}} \text{ ----------------------------------(2.11)}}$$

Thus the efficiency of the Carnot's engine depends only on the temperature of the source and the sink. It does not depend on the nature of the working substance.

$\boldsymbol{\eta = 1}$ i.e. 100% efficiency is possible only when the temperature of the sink is absolute zero and no heat is rejected to the sink. In practice, these two conditions are unattainable.

Since it is not possible to reach absolute zero (because this would be the violation of second law of thermodynamics) hence it can be concluded that 100% efficient engines are not possible.

This result is the essence of the second law of thermodynamics.

Lord Kelvin showed that with the help of the ideal Carnot engine it is possible to define temperature in terms of energy and the scale so obtained is independent of the nature of the working substance. Such a temperature scale is called as the THERMODYNAMIC SCALE.

Problem: Two Carnot engines E_1 and E_2 are coupled together. The engine E1 draws heat from the source at 1200 K and rejects to a reservoir at temperature T K .The engine E_2 receives the heat rejected by E_1 and in turn rejects to another reservoir at temperature 300 K. What is the value of T if the work outputs of both the engines are equal?

Solution:Let us suppose, Q1—heat absorbed by engine E_1 from the source

Q2---heat rejected by the engine E_1 to the sink

--heat absorbed by engine E_2

Q3—heat rejected by the engine E_2 to the sink

For engine E_1; $W_1=Q_1-Q_2$

For engine E_2; $W_2=Q_2-Q_3$,

Here , $W_1=W_2$

$$Q_1-Q_2=Q_2-Q_3$$

$$\frac{Q_1}{Q_2}-1=1-\frac{Q_3}{Q_2}$$

Also, $\frac{Q_1}{Q_2}=\frac{1200}{T}$ and, $\frac{Q_2}{Q_3}=\frac{T}{300}$

OR, $\frac{Q_3}{Q_2}=\frac{300}{T}$

or, $\frac{1200}{T}-1=1-\frac{300}{T}$

or, **T= 750K**

2.4 Refrigerator and its coefficient of performance

Definition: Refrigerator is a commonly used device that transfers heat from a low temperature medium to a high temperature medium and removes heat from the refrigeration space.

The main objective of the refrigerator is to remove heat from the reservoir at a low temperature.

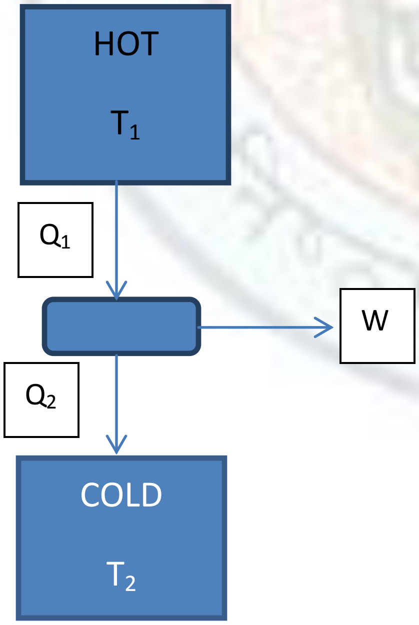
The Carnot cycle is a totally reversible cycle. If one reverses all the processes in the Carnot cycle, we achieve Carnot refrigeration cycle.

Since Carnot cycle is perfectly reversible, it can work as a heat engine as well as a refrigerator.

When it works as a heat engine [Figure 2.4a], it draws heat Q_1 from the source at temperature T_1 and does W amount of work. Rest of the heat Q_2 (Q_1-W) is rejected to the sink at temperature T_2.

When it works as a refrigerator[Figure 2.4b], it absorbs heat Q_2 from the sink at lower temperature T_2 and W amount of work is done on it by some external means. Total heat Q_1 (Q_2+W) is rejected to the source at higher temperature.

This is in accordance with Clausius statement of the second law of thermodynamics.



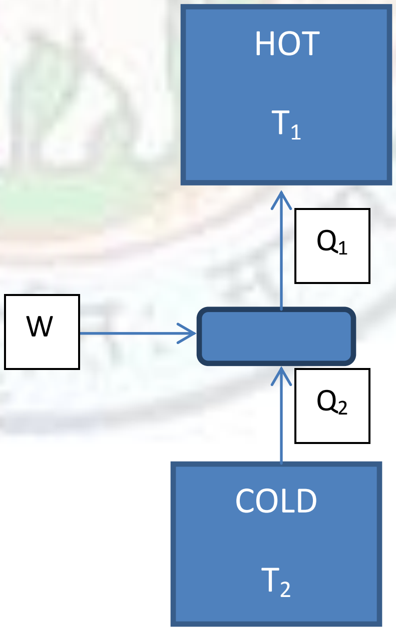


Figure 2.4(a): Heat Engine

Figure 2.4(b): Refrigerator

Co efficient of Performance: This is the measure of the efficiency of a refrigerator.

For a Carnot engine working as a refrigerator, the Co efficient of Performance (COP) is defined as the ratio of the heat extracted from the sink (which is at low temperature) to the external work, which is used by the refrigerator to transfer thermal energy from a low temperature reservoir to a high temperature reservoir.
Alternatively, we can say that it is the ratio of the amount of heat absorbed to the amount of work done on the working substance[Figure 2.4(b)].

i.e. $COP = \frac{Q_2}{W}$

Since: $W=Q_1-Q_2$ (Equation 2.7)

Therefore: $COP = \frac{Q_2}{Q_1-Q_2} ------ (2.12)$

Here, Q_1= heat exchanged with source at higher temperature T_1
Q_2= heat exchanged with sink at lower temperature T_2
The larger is the value of COP, the more efficient is the refrigerator.

Problem: If 400 joules of energy is absorbed by a working substance at a lower temperature and 200 joules of work is done on it by an external agency then find its co efficient of performance.

Solution: Let W=work done on the working substance
Q_2= Heat rejected at higher temperature=400 joules
Q_1=Heat rejected to the source at higher temperature
Here, $Q_1=Q_2+W$
=400+200=600 joules
Now substituting these values in equation (2.12) we get,

$$COP = \frac{400}{600-400} = 2 = 200\%$$

If the value of the coefficient of performance is **1** then the temperature of the source is twice of that of the temperature of the sink. For a refrigerator the maximum amount of heat should be extracted at lower temperature for the least amount of work. The value of COP can be much higher than **1**.

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Or

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Did you know ?

A refrigerant will be most effective when temperature of the things kept inside the refrigerator is nearly equal to the temperature of the surroundings.

[As $T_1 - T_2 \rightarrow 0$; COP$\rightarrow \infty$]

The Co efficient of Performance can also be determined by replacing the heat transfer ratios in the above equations by the absolute temperature ratios. These are,

$$COP = \frac{Q_2}{W}$$
$$= \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

A heat pump is essentially a refrigerator which transfers heat from a low temperature reservoir to a high temperature reservoir but is utilized in a different manner. Its objective is to supply heat to the desired places. In one cycle of the engine, if we need output heat Q_1 from the engine, W amount of work must be supplied in order to accomplish this process. Therefore the efficiency of the heat pump, $\eta = \frac{Q_1}{W}$.

Ctrl+clk the following link to know more about the working of heat pump and refrigerator.

https://www.youtube.com/watch?v=3FoGNvsiT_8

2.5 Summary

- ✓ Hundred percent of energy cannot be transformed into work.
- ✓ A Carnot engine is a hypothetical engine that operates in a Carnot cycle.
- ✓ The maximum efficiency which can be achieved is the Carnot's efficiency.
- ✓ Unattainability of absolute zero is implied by second law of thermodynamics.
- ✓ COP can be much greater than unity.
- ✓ COP measures the performance of a refrigeration cycle.
- ✓ COP is small when the temperature of the sink is low.
- ✓ Quantity of heat is degraded at low temperature.
- ✓ Clausius statement of second law of thermodynamics governs the working of a refrigerator.
- ✓ Kelvin-Plank statement governs the working of a heat engine.

2.6 Exercise

I. Fill in the blanks:

1. Efficiency of perfectly reversible engine is independent of the ________.

2. In a heat engine the process of isothermal and adiabatic expansions and compressions are carried out in a ____________process.

3. Cent percent conversion of heat energy into mechanical work is____________.

4. The efficiency a Carnot's engine is minimum or zero when _______________.

5. The efficiency of Carnot's engine is maximum when ____________.

6. Working substance plays no role in determining ______________.

II. State true/false for each of the following statements:

1. Coefficient of performance is always positive.

2. Self-acting refrigerator is not possible.

3. The coefficient of performance of a refrigerator can be much higher than 100 %.

4. No power can be generated when the temperature of hotter body is equal to the temperature of colder body.

5. Efficiency of a Carnot's engine is independent of nature of working substance.

6. The heat engine does not work when the temperature of the source and the sink are equal.

7. Absolute zero is not attainable.

8. Heat is more useful when it is supplied at higher temperature.

9. 100% efficient engine is possible.

III. Choose the correct option for each of the following statements:

1. The efficiency of Carnot's engine is 50%. If the temperature of the sink is 27^0c, the temperature of the source is,

(a) 600K (b) 600^0C

(c) 500^0C (d) 900K

2. The efficiency of a Carnot's engine working between steam point and ice point is,

(a) 50% (b) 26.8%

(c) 100% (d) 25%

3. The efficiency of a reversible Carnot engine working between T_1 and T_2 ($T_1 > T_2$),

(a) $\frac{T_2}{T_1}$ (b) $1-\frac{T_1}{T_2}$

(c) $1-\frac{T_2}{T_1}$ (d) $\frac{T_1}{T_2}-1$

4. What is the coefficient of performance of a refrigerator working between ice point and room temperature at 27^0C.

(a) 10.11 (b) 12.11

(c) 11.11 (d) 13.11

IV. Answer the following:

1. The cold reservoir of a Carnot engine is at 7^0C. Its efficiency is 30% which is to be increased to 40%. (i)By how many degrees the temperature of the hot reservoir must be increased if the temperature of the cold reservoir is kept constant. (ii) By how many

degrees the temperature of the cold reservoir is decreased if the temperature of the hot reservoir is kept constant.

2. Define efficiency of a heat engine. Obtain an expression for the efficiency of a reversible Carnot's engine with a perfect gas as the working substance.

3. Which among the following is more effective way to increase efficiency of a Carnot's engine: to increase T_1 (temperature of the source) or to decrease T_2(the temperature of the sink)? Comment.

4. What is the principle used in the working of a refrigerator? Define coefficient of performance.

5. Can the coefficient of performance of a refrigerator be greater than one? Explain.

2.7 Glossary

Isothermal process: The process that occurs at constant temperature.

Adiabatic process: The process in which there is no exchange of heat.

Perfect gas: A gas which obeys gas laws.

Perfect engine: The engine which is free from all practical imperfections.

Quasistatically: A process in which the deviation from thermodynamic equilibrium is infinitesimal and all the states through which the system passes can be considered as the equilibrium states.

Infinite thermal capacity: A reservoir having infinite thermal capacity can reject, absorb and retain unlimited amount of heat without the change in temperature.

2.8 References

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