Discipline Course-I
Semester-II
Paper No: Thermal Physics : Physics-IIA
Lesson: Transport Phenomena
Lesson Developer: Seema Dabas
College/ Department: Shyam Lal College, University of Delhi

Table of Contents

Learning Objectives

After reading the lesson, you should be able to

1) understand the concept of transport phenomena.

2) develop the concept of different phenomena like viscosity, conduction and diffusion.

3) Answer the questions of interest to physicists and engineers.

4) Apply your knowledge to the practical life.

Chapter: Title Transport Phenomena

3.1 Introduction

The equilibrium state of a gas is the most probable state but if the gas is not in a state of equilibrium, we may have any of the transport phenomena: Viscosity, Conduction and Diffusion. Transport phenomena is concerned with the evolution of a system in response to a non-equilibrium distribution of the property. The molecules in a particular layer are connected with some particular values of velocity components, temperature and molecular density. Components of velocity may not have same value in all parts of the gas, due to which relative motion of the gas layers with respect to one another arises. Therefore, relative velocity exists between different layers of the gas. This gives rise to the phenomenon of viscosity. The temperature of the gas may not be same throughout. This leads to the transference of thermal energy from regions of higher temperature to lower one. This results to the phenomenon of conduction. The particle density may not be have same value throughout the volume of the gas. This results in the movement of the molecules from regions having higher values of particle density to the one having lower value. This gives rise to the phenomena of diffusion. Diffusion means transport of mass. Actually, in this process, the particle density evolves in response to a spatial gradient in concentration.

All these phenomena are concerned with the exchange of momentum, energy and mass between observed and studied systems or in other words transport of momentum, energy and mass represent viscosity, conduction and diffusion respectively and these are known as transport phenomena.

3.2 Viscosity

Transport of momentum represents viscosity.

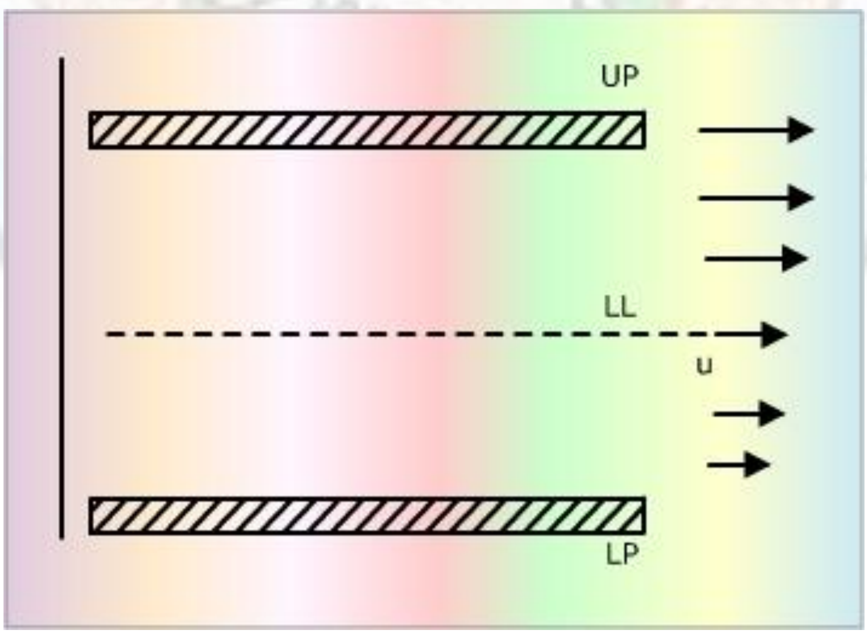

Fig 3.1

Consider two plates upper plate *UP* and lower plate *LP*. Say, plate *UP* is moving with a constant velocity and the lower plate *LP* is at rest. Plates are separated by some space which contains a gas. The viscous force *F* is given by

$$F = \eta A \frac{du}{dy} \tag{3.1}$$

where *A* is area, $\frac{du}{dy}$ is velocity gradient perpendicular to the area and η is the coefficient of viscosity. From both sides, molecules are crossing the dashed surface continously. It is assumed that each and every molecule will get a flow velocity corresponding to the particular height at which last collision was made before crossing the dashed surface *LL*. Net rate of transport of momentum across the surface is present because molecules crossing from above transport a greater momentum (towards the right) across the surface as compared to the molecules crossing from below (flow velocity above the dotted surface is greater than that below the surface). Here, the dashed surface *LL* denotes an imagined surface within the gas at an arbitrary height above the lower fixed plate *LP*. Say *u* is the velocity of the gas and $\frac{du}{dy}$ is the velocity gradient at this height. The velocity *u* is superposed on the random thermal velocities of the molecules, so that the gas is not in thermodynamic equilibrium. We have to calculate the value of average height $\bar{y}$ above (or below) the surface *LL* at which a molecule makes its last collision before crossing.

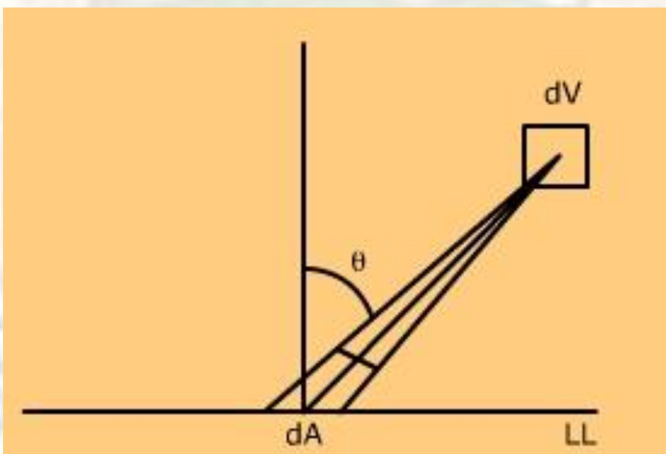

Fig 3.2

If *n* is the number of molecules per unit volume and f is the collision frequency of any one molecule, the total number of molecules within volume element *dV* in time *dt* will be

$$n_f = \frac{1}{2} n f \frac{dV}{dt} \tag{3.2}$$

To avoid counting of each collision twice, the factor $\frac{1}{2}$ is introduced, so that the total number of free paths originating in *dV* in time *dt* is $nf\frac{dV}{dt}$ (Because, at each collision, two new free paths originate). So, the number of paths headed toward area *dA* is $\frac{d\xi}{4\pi}nf\frac{dV}{dt}$ where $d\xi = \frac{dA\cos\theta}{r^2}$ and it is the solid angle subtended at dV by the area dV .

Therefore, the number of molecules N_{dV} which are coming out of volume element *dV* in time interval *dt* and crossing *dA* are

$$N_{dV} = \frac{1}{4\pi} n\, f\, dA\, dt \sin\theta \cos\theta \exp\left(-\frac{r}{\lambda}\right) d\theta\, d\phi\, dr \qquad (3.3)$$

Now, we integrate it so as to find the total number of molecules crossing *dA* in time *dt* from all directions and all distances, and we get

$$\frac{1}{4} nf\lambda\, dA\, dt \qquad (3.4)$$

where collision frequency $f = \frac{\bar{v}}{\lambda}$, $\bar{v}$ is mean molecular velocity and λ is mean free path.

The height of the volume element *dV* above *LL* is $r\cos\theta$. The average height for all molecules is

$$\frac{1}{4\pi} n\, f\, dA\, dt \int_0^{\pi/2} \sin\theta \cos^2\theta\, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} r \exp\left(-\frac{r}{\lambda}\right) dr = \frac{1}{6} nf\lambda^2 dA\, dt \qquad (3.5)$$

Dividing eq(3.5) by (3.4), we get

$$\bar{y} = \frac{\frac{1}{6} n\, f\, \lambda^2 dA\, dt}{\frac{1}{4} n\, f\, \lambda\, dA\, dt} = \frac{1}{6} \times 4\lambda = \frac{2}{3}\lambda \qquad (3.6)$$

At a height $\frac{2}{3}\lambda$ above the plane *LL*, the flow velocity of the gas is $u+\frac{2}{3}\lambda\frac{du}{dy}$. In this case, the rate of change of velocity is assumed to be constant over distances of the order of a free path. The momentum of a molecule with this velocity is $m\left(u+\frac{2}{3}\lambda\frac{du}{dy}\right)$.

Net momentum carried by the molecules crossing the surface from above per unit area and per unit time is given by

$$P_1=\frac{1}{4}n\bar{v}\,m\left(u+\frac{2}{3}\lambda\frac{du}{dy}\right) \tag{3.7}$$

Similarly, the net momentum crossing the surface from below is

$$P_2=\frac{1}{4}n\bar{v}\,m\left(u-\frac{2}{3}\lambda\frac{du}{dy}\right) \tag{3.8}$$

The net rate of transport of momentum, per unit area and per unit time is the difference between these quantities, or

$$P=P_1-P_2=P_1=\frac{1}{4}n\bar{v}\,m\left(u+\frac{2}{3}\lambda\frac{du}{dy}\right)-\frac{1}{4}n\bar{v}\,m\left(u-\frac{2}{3}\lambda\frac{du}{dy}\right)=\frac{1}{3}n\bar{v}\,m\lambda\frac{du}{dy} \tag{3.9}$$

According to Newton’s second law, rate of change of momentum is equal to force. So, comparing eq(3.9) with eq(3.1), coefficient of viscosity η is

$$\eta=\frac{1}{3}nm\bar{v}\lambda \tag{3.10}$$

3.3 Thermal conductivity

In Fig 3.2, consider the upper plate *UP* and lower plates *LP* both are at rest. We assume that these plates are at different temperatures. Let *T* be the temperature at the dashed plane and $\frac{dT}{dy}$ is the temperature gradient.

The mean energy of a molecule at a temperature T is

$E_{mean}=\frac{d}{2}k_BT$ where *d* is the degree of freedom and k_B is Boltzmann constant.

Energy carried across the plane, per unit area and per unit time, by the molecules crossing it from above, is

$$E_1 = \frac{1}{4} n \frac{d}{2} k_B \left(T + \frac{2}{3} \lambda \frac{dT}{dy} \right) \bar{v} \tag{3.11}$$

Energy carried by those crossing from below is

$$E_2 = \frac{1}{4} n \frac{d}{2} k_B \left(T - \frac{2}{3} \lambda \frac{dT}{dy} \right) \bar{v} \tag{3.12}$$

The net rate of flow of energy per unit area is

$$E = E_1 - E_2 = \left[\frac{1}{4} n \frac{d}{2} k_B \left(T + \frac{2}{3} \lambda \frac{dT}{dy} \right) \bar{v} \right] - \left[\frac{1}{4} n \frac{d}{2} k_B \left(T - \frac{2}{3} \lambda \frac{dT}{dy} \right) \bar{v} \right] = \frac{1}{6} n\, d\, k_B \lambda \frac{dT}{dy} \bar{v} \tag{3.13}$$

We know that heat flow per unit time across an area A is

$$H = KA \frac{dT}{dy} \tag{3.14}$$

where $\frac{dT}{dy}$ is temperature gradient and K is thermal conductivity.

Using eq(3.13) and (3.14), we get

$$K = \frac{1}{6} nd\, k_B\, \lambda \bar{v} \tag{3.15}$$

3.4 Diffusion

If a gas contains two or more kinds of molecules whose relative densities vary from point to point, a process known as diffusion takes place until the concentration becomes uniform throughout. For example take a vessel (shown in Fig 3.3) containing two different gases P and Q separated by a barrier.

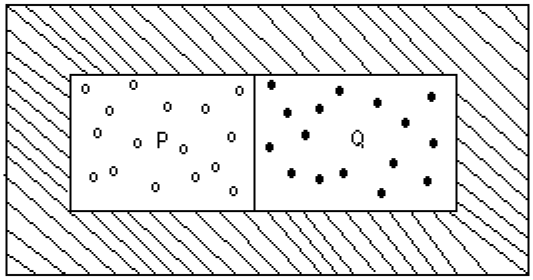

Fig3.3

17.swf

The gases are at the same pressure and temperature so that the number of molecules per unit volume is the same on both sides. When the barrier is removed, initially there is no mass motion of the gas in any of the direction but after some time both gases are found to be distributed uniformly throughout the entire volume. This phenomena is known as interdiffusion of two gases and if we take into account only on one gas, we can say that it diffuses into other gas. The process due to which each gas permeates the other is called diffusion.

Here we have to derive an expression for coefficient of diffusion (ratio of number of molecules across unit area in one second to the concentration gradient). Let us consider a horizontal plane drawn in a gas and let the molecular concentration *c* remains constant along it and vary in the vertical direction (*z*-axis).

$$v = -D\frac{dc}{dz} \qquad (3.16)$$

where *D* is called the coefficient of diffusion.

We assume that two gases *P* and *Q* are arranged in layers one above the other parallel to the *z* axis diffusing through one another. Let at any instant c_1 and c_2 be the concentrations of both gases respectively which vary along *z* axis. As temperature and pressure must be constant throughout the gas, so using Avagadro's Law for perfect gases:

$$c_1 + c_2 = c = const \qquad (3.17)$$

On differentiation w.r.t. *z*,

$$\frac{dc_1}{dz} + \frac{dc_2}{dz} = 0 \qquad (3.18)$$

So, we get

$c_1 = B_1 - \beta z$ (3.19a) and

$c_2 = B_2 + \beta z$ (3.19b)

where $\frac{-dc_1}{dz} = \frac{dc_2}{dz} = \beta$.

B_1, B_2 and β are constants referring to the plane $z = z_0$ across which we will consider the molecular transfers.

Now to find the number of molecules crossing the plane $z = z_0$. Let $\bar{v}_1$ and λ_1 are the mean molecular velocity and the mean free path for molecules of the first type in the gas respectively, whereas $\bar{v}_2$ and λ_2 correspond to second gas. Those molecules which move making an angle θ with the axis of *z* might have suffered their last collision in a plane of which the *z* coordinate is $z_0 - \lambda_1 \cos\theta$. The concentration in this plane may be taken to be

$B_1 - \beta(z_0 - \lambda_1 \cos\theta)$

So, the number of molecules of the first type which cross the plane $z = z_0$ per unit area per unit time in the direction of *z* increasing, as in the case of viscosity, is given by

$$v_1 = \iint \left(B_1 - \beta\left(z_0 - \lambda_1 \cos\theta\right)\right).\frac{1}{2}\sin\theta\cos\theta\, d\theta . \nu\, f(\nu) d\nu = \frac{1}{3}\beta\lambda_1\bar{v}_1 \qquad (3.20)$$

Similarly the number of molecules of the second type in the direction of *z* increasing i.e. upwards is given by

$$v_2 = -\frac{1}{3}\beta\lambda_2\bar{v}_2 \qquad (3.21)$$

Generally sum of mean molecular velocity of two gases is non zero. Therefore, number of molecules $v_1 + v_2$ will tend to move in the upward direction which will set up a difference of pressure in the gas. And this should remain same throughout. So, gas would adjust itself against this tendency by a slow motion of mass with a velocity v_z parallel to z axis. Thus the rate of increase of the number of molecules of the first kind on the positive side of plane $z = z_0$ per unit area is given by

$\tau_1 = v_1 + v_z c_1$

In the same way, we can write the rate of increase of molecules of the second kind $\tau_2 = v_2 + v_z c_2$. Due to steady flow, the total flow of molecules over every plane must be zero i.e. $\tau_1 + \tau_2 = 0$ or

$$v_z = -\frac{v_1 + v_2}{c_1 + c_2}$$

So, expression of τ_1 gets modified to

$$\tau_1 = v_1 - \frac{v_1 + v_2}{c_1 + c_2} c_1 = \frac{v_1 c_2 - v_2 c_1}{c_1 + c_2} \qquad (3.22)$$

Using the value of $\beta = -\frac{dc_1}{dz} = \frac{dc_2}{dz}$, we get

$$\tau_1 = -\frac{dc_1}{dz} \cdot \frac{\lambda_1 \bar{v}_1 c_2 + \lambda_2 \bar{v}_2 c_1}{3(c_1 + c_2)} \qquad (3.23)$$

But if D_{12} is the coefficient of diffusion of two gases, then according to definition, the diffusion current density

$$\tau_1 = -D_{12} \frac{dc_1}{dz} \qquad (3.24)$$

Or

$$D_{12} = \frac{1}{3} \frac{\lambda_1 \bar{v}_1 c_2 + \lambda_2 \bar{v}_2 c_1}{c_1 + c_2} \qquad (3.25)$$

This is the well known equation of diffusion and is symmetrical for the two gases. i.e. $D_{12} = D_{21}$. It is generally known as Meyer's formula for the coefficient of diffusion.

This expression becomes simpler if we consider the molecules of two gases to be approximately of equal size and mass. And also assume λ and $\bar{v}$ to be the same for each gas so that

$$D_{11} = \frac{1}{3} \lambda \bar{v} \qquad (3.26)$$

Now, we compare it with the expression of the coefficient of viscosity $\eta = \frac{1}{3} \lambda \bar{v} \rho$

and we obtain following relation $D_{11} = D = \frac{\eta}{\rho}$

where D_{11} or D may be called the coefficient of self diffusion or inter diffusivity of a single gas.

In the following table, different transport phenomena are shown:

Property transported	**Transport process**
Linear momentum	Viscosity
Energy	Thermal conductivity
Mass	Diffusion
Charge	Ionic conductivity

Value Addition: **Transport Phenomena**

For more information on Transport Phenomena including historical background and related phenomena students are advised to go to the following link:

http://en.wikipedia.org/wiki/Transport_phenomena

Summary

1. Transport of momentum, energy and mass represent viscosity, conduction and diffusion respectively. These are called transport phenomena.

2. Viscosity is given by $\eta = \frac{1}{3} n m \bar{v} \lambda$ where *n* are the number of molecules per unit volume, *m* is mass, λ is mean free path and $\bar{v}$ is mean molecular velocity.

3. Thermal conductivity can be expressed as $K = \frac{1}{6} \frac{\bar{v} f k_B}{\sqrt{2} \chi}$ where *K* is thermal conductivity, k_B is Boltzmann constant, *f* is collision frequency, χ is collision cross section and $\bar{v}$ is mean molecular velocity.

4. Coefficient of diffusion of two gases is given by

$D_{12} = \frac{1}{3} \frac{\lambda_1 \bar{v}_1 c_2 + \lambda_2 \bar{v}_2 c_1}{c_1 + c_2}$ where $\bar{v}_1$ and λ_1 denote respectively the mean molecular velocity and the mean free path for molecules of the first type in the gas whereas $\bar{v}_2$ and λ_2 denote the corresponding quantities for the second type. c_1 and c_2 are the concentrations of two gases. This is the well known equation of diffusion and is symmetrical for the two gases i.e. $D_{12} = D_{21}$. It is generally known as Meyer's formula for the coefficient of diffusion.

5. Transport phenomena have wide application. For example, in solid state physics, the motion and interaction of electrons, holes and phonons are studied under "transport phenomena".

Solved Examples

Q Prove $\frac{K}{\eta C_v} = 1$ for a gas?

Sol. The expression for coefficient of viscosity

$$\eta = \frac{mC}{3\lambda} \qquad (i)$$

The coefficient of thermal conductivity

$$K = \frac{mCC_v}{3\lambda} \qquad (ii)$$

Divide Eq (*ii*) by (*i*)

$$\frac{K}{\eta} = C_v$$

Q2 Calculate the radius of an oxygen molecule if its coefficient of thermal conductivity at $0^\circ C$ is

$K = 20 \times 10^{-3}\ J/m-s-K$ and

$C_v = 12 \times 10^{3}\ J/kilo-mole-K$

mass of an oxygen molecule = $m = 4 \times 10^{-26} Kg$

Sol. $K = \frac{mCC_v}{3(\sqrt{2})\pi\sigma^2}$

Since we know:

$\frac{1}{2}mC^2 = \frac{3}{2}k_BT$ or $C = \sqrt{\frac{3k_BT}{m}}$

$$\sigma^2 = \frac{\sqrt{3k_B T\, m}\,.C_v}{3\left(\sqrt{2}\right)\pi\, K}$$

$$\sigma^2 = \frac{\sqrt{3\times 1.38\times 10^{-23}\times 273\times 4\times 10^{-26}}\times 12\times 10^{3}}{3\times 1.414\times 3.142\times 20\times 10^{-3}}$$

$$\sigma \sim 10^{-9} m$$

Fill in the blanks

1. Transport phenomena are ___________ processes.

2. Heat conduction is _________ transfer.

3. The transport phenomena occurs due to _________ of the molecules.

4. Heat transfer occurs at a ________ rate across materials of high thermal conductivity than across materials of low thermal conductivity.

5. Materials of _________ thermal conductivity are widely used in heat sink applications

6. Materials of _________ thermal conductivity are used as thermal insulation.

7. Thermal conductivity of materials is _________ dependent.

8. The reciprocal of thermal conductivity is called __________.

9. Heat can be transferred from one place to another through three different modes conduction, convection and _________.

10. _________ transfer in a system is governed by Fick's First law.

11. Diffusion flux from higher concentration to lower concentration is proportional to the gradient of the concentration of the substance and _________ of the substance in the medium.

12. Transport phenomena actually encompasses all agents of ________ changes in the universe.

13. The molecular transfer equations of Newton's law for fluid ________, Fourier's law for heat and Fick's law for mass are very similar.

14. Transport phenomena occur only in __________ state of the gas.

Answers

1. irreversible 2. energy 3. thermal agitation 4. higher 5. high 6. low 7. temperature 8. thermal resistivity 9. radiation 10. mass 11. diffusivity 12. physical 13. momentum 14. non-equilibrium

MCQ

Q1. Unit of thermal conductivity is _______.

a) W/s/m/k.

b) J/k.

c) m/k.

d) J/s/m/k.

Q2 Viscosity is ________ transfer.

a) energy

b) mass

c) momentum

d) force

Q3 In steady state, rate of flow of heat through any cross section of slab is directly proportional to ________.

a) length

b) temperature difference

c) area

d) force

Q4 _______ does not require bulk motion.

a) Viscosity

b) Conduction

c) Diffusion

d) None of above

Q5 Coefficient of Self Diffusion is given by

a) $D = 1.543\frac{\eta}{\rho}$

b) $D = 1.342\frac{\eta}{\rho}$

c) $D = 1.711\frac{\eta}{\rho}$

d) $D = 1.200\frac{\eta}{\rho}$

Answers

1. d 2. c 3. b 4. c 5. a

References/ Bibliography/ Further Reading References

1. “Introduction to thermodynamics” Sears

2. “An Introduction to thermodynamics and Statistical Mechanics” A.K.Saxena

3. “Thermodynamics, Statistical Physics and Kinetics” Satyaprakash

4. “Engineering Physics” Abhijit Nayak

5. “Heat and thermodynamics” Brij Lal Subramanyam

6. “Thermal Physics” S.O.Pillai