Discipline Course-I
Semester -I
Paper: Mechanics IB
Lesson: Linear Momentum

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CH 3.Linear Momentum

Objective

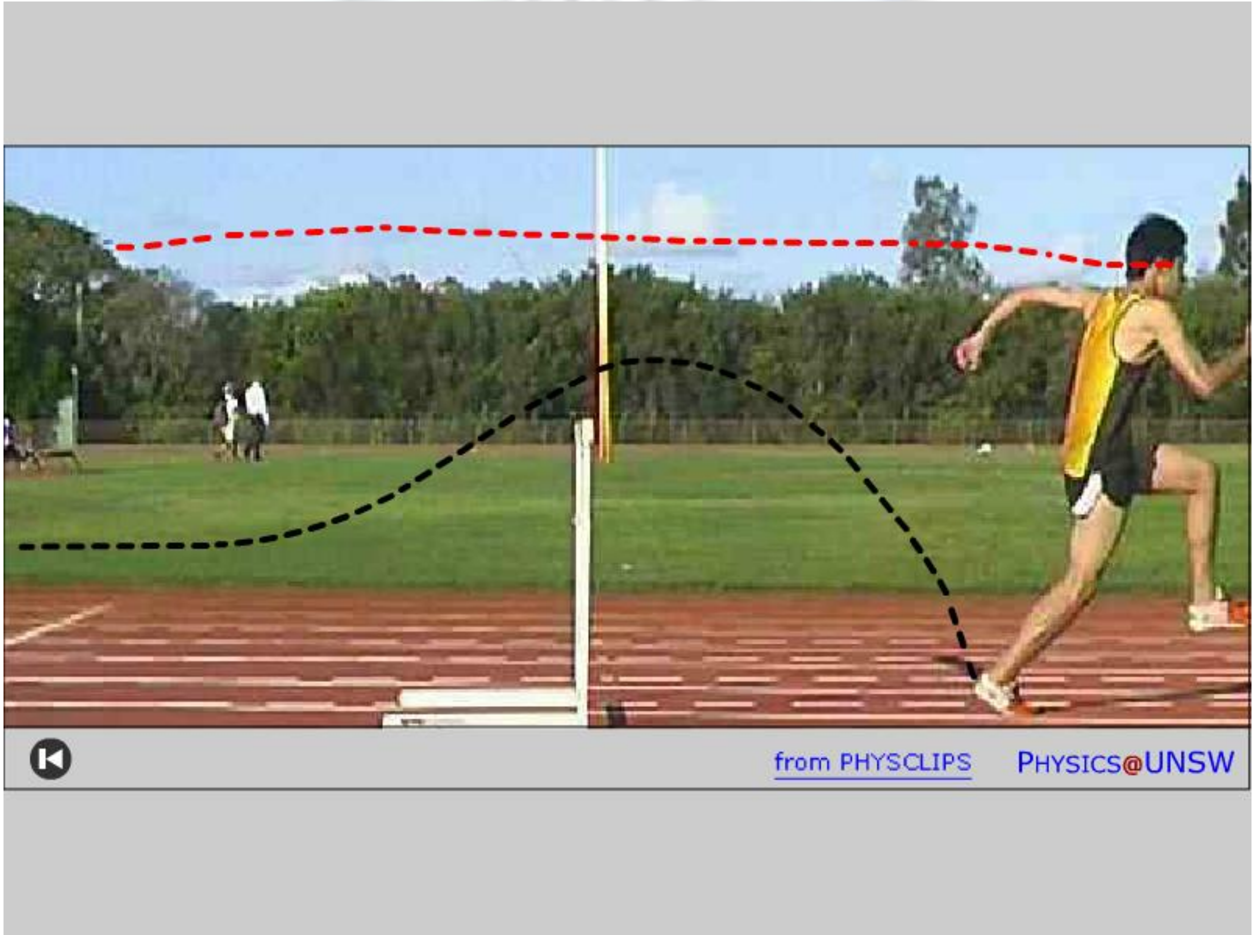
After studying this chapter you will understand:

- The Centre of mass concept and its importance for the system of particles
- The concept of Linear momentum of a particle and the system of particles and its physical importance
- The Conservation of linear momentum and its application
- The concept of Impulse

1. Centre of mass

In chapter 2, we described the motion of system of particles. When we are dealing with the motion of a system of particles, it is always a matter of convenience to look at the problem of the system of particles as if there were only a single particle present. This can be achieved by the concept of Centre of mass.

To get the idea of the Centre of mass, let us first watch following video clips:



To play the movie, click Mechanics with animations and film clips: Physclips.

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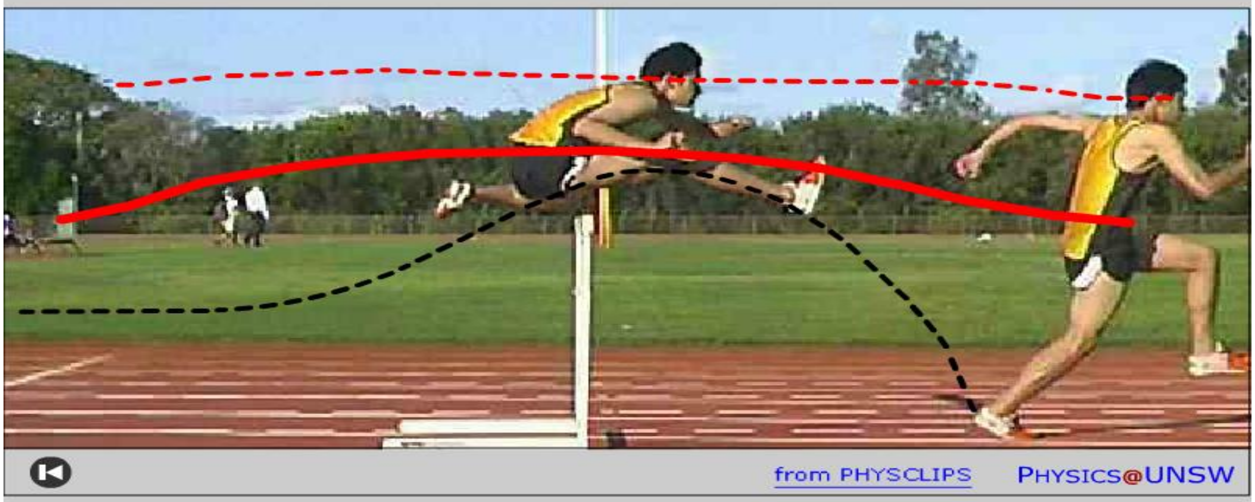

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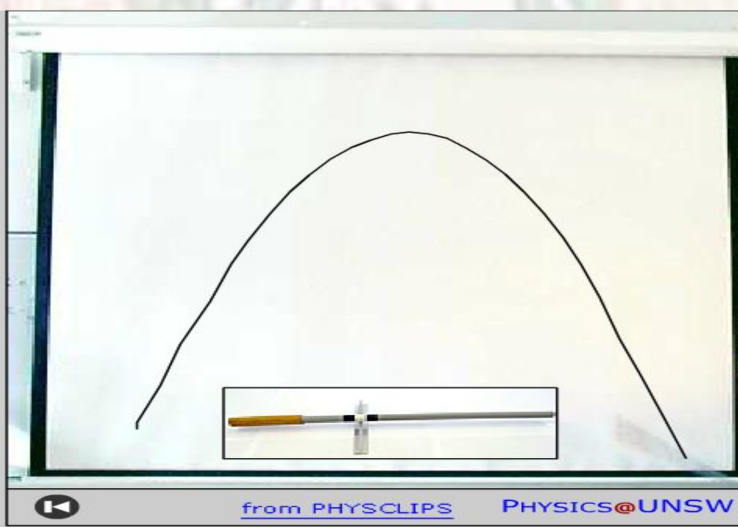

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Now consider there are N particles in a system having masses m_1, m_2, m_3, …m_N, respectively. Now if the position vectors of each particle is given by $\mathbf{R_1}$**,** $\mathbf{R_2}$**,** $\mathbf{R_3}$**, …** $\mathbf{R_N}$ respectively, then we define a imaginary point, called the **Centre of mass**, where we can assume that all the mass of the system is concentrated. The position vector of the center of mass $\mathbf{R_{C.M}}$, is given by

$$\mathbf{R_{C.M}} = \frac{m_1\mathbf{R_1}+m_2\mathbf{R_2}+m_3\mathbf{R_3}+\cdots+m_n\mathbf{R_n}}{m_1+m_2+m_3+\cdots+}$$

$$= \sum_{i=1}^{N} m_i\mathbf{R_i}/M,$$ **where** M is the total mass of the system of particles.

This equation actually is a set of three equations for X- direction, Y-direction and for Z-direction, respectively each for 3-D space, as

$$X_{C\cdot M} = \frac{m_1X_1+m_2X_2+m_3X_3+\cdots+m_nX_n}{m_1+m_2+m_3+\cdots+m_n} = \sum_{i=1}^{N}/m_iR_i$$

$$Y_{C\cdot M} = \frac{m_1Y_1+m_2Y_2+m_3Y_3+\cdots+m_nY_n}{m_1+m_2+m_3+\cdots+m_n} = \sum_{i=1}^{N} m_1Y_1/M$$

$$Z_{C\cdot M} = \frac{m_1Z_1+m_2Z_2+m_3Z_3+\cdots+m_nZ_n}{m_1+m_2+m_3+\cdots+m_n} = \sum_{i=1}^{N} m_iZ_i/M$$

When instead of discrete masses, we have a continuous system of mass, say an irregular shaped body of total mass M and mass density ρ, then we can first choose an infinitesimal mass element dm given by

$$dm = \rho dV ,$$

Where dV is infinitesimal Volume element

and then when we take limit that this mass element reduces to zero, we can extend our summation to integral assuming that the number of particles goes to infinity so that the product Ndm is finite, hence

$$\mathbf{R_{C.M}} = \lim_{N\to\infty} \frac{m_1\mathbf{R_2}+m_2\mathbf{R_2}+m_3\mathbf{R_3}+\cdots+m_n\mathbf{R_n}}{m_1+m_2+m_3+\cdots+m_n} = \lim_{N\to\infty} \sum_{i=1}^{N} m_i\mathbf{R_i}/M$$

$$\Rightarrow \quad \mathbf{R_{C.M}} = (1/M)\int \mathbf{R}\, dm = (1/M) \int \mathbf{R}\rho dV$$

Similarly, we can write for each component,

$$X_{C\cdot M} = \frac{m_1X_1+m_2X_2+m_3X_3+\cdots+m_NX_N}{m1+m2+m3+\cdots+mN} = \sum_{i=1}^{N} m_iX_i/M \longrightarrow X_{C.M} = (1/M)\int X\, dm = (1/M) \int X \rho dV$$

$$Y_{C\cdot M} = \frac{m_1Y_1+m_2Y_2+m_3Y_3+\cdots+m_NY_N}{m1+m2+m3+\cdots+mN} = \sum_{i=1}^{N} m_iY_i/M \longrightarrow Y_{C.M} = (1/M) \int Y\, dm = (1/M)\int Y \rho dV$$

$$Z_{C\cdot M} = \frac{m_1Z_1+m_2Z_2+m_3Z_3+\cdots+m_NZ_N}{m1+m2+m3+\cdots+mN} = \sum_{i=1}^{N} m_1Z_1/M \longrightarrow Z_{C.M} = (1/M) \int Z\, dm = (1/M)\int Z\rho dV$$

2. Linear Momentum

Suppose a particle of mass m is moving with velocity **v,** then we can define a physical quantity called the linear momentum, **p,** of the particle as the product of mass m of the particle and its velocity **v**. So mathematically,

$$\mathbf{p} = m\, \mathbf{v}$$

or for a system of particles of masses m_1, m_2, m_3,…. , moving with the velocities $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$, **…** respectively, we define the total linear momentum **P** of the system of particles as the sum of all individual momenta of the particles. So mathematically,

$$\mathbf{P} = m_1\,\mathbf{v_1} + m_2\,\mathbf{v_2} + \ldots + m_N\mathbf{v_N}\ ,$$

Using summation notation

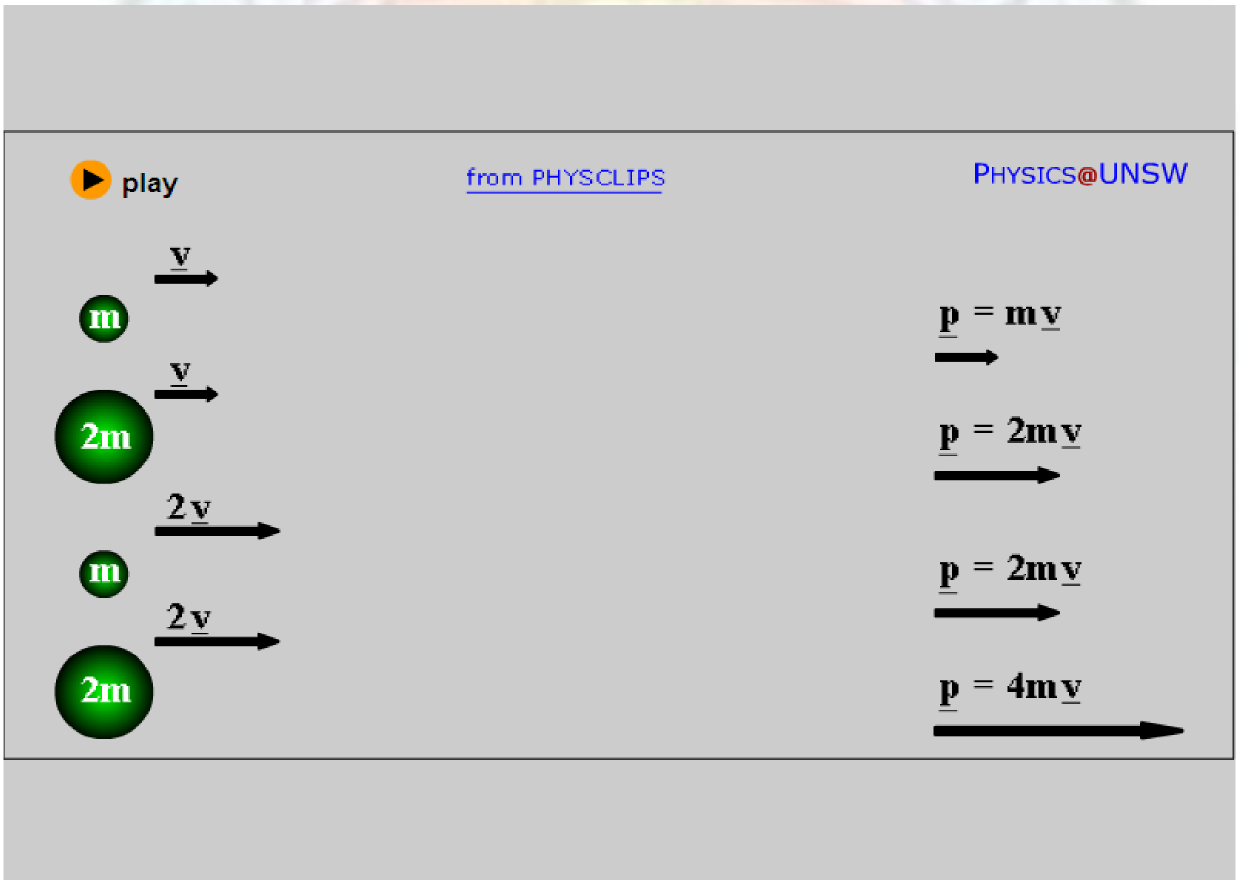
$$\mathbf{P} = \sum_{i=1}^{N} m_i \mathbf{v_i}$$

The momentum, like velocity, is a vector quantity.

It signifies the importance of mass with the motion. We see that momentum is proportional to mass. So having equal kinetic energy, the heavier mass should have large momentum.

For example a man of 50 kg moving with 5m/s have kinetic energy 625J and momentum 250kgm/s ,while a bullet of 20g with a speed of 250m/s have kinetic energy of 625J ,but have smaller momentum 5kgm/s.

The following movie shows the effect of mass in the momentum.



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3. Principle of Conservation of the Linear momentum

The conservation principles are a helping tool for Physicists, they guarantee that a physical quantity can we assumed to be a constant of motion. Also the conserved quantity can be assumed to be associated with some kind of symmetry or invariance, you will study these concepts in your higher studies later, for the time being it is just informative to know that the conservation of linear momentum is associated with the translational invariance.

Now we define the conservation of linear momentum as:

-"For an isolated system of particles, when there is no external force acting on the system (although, there may be internal forces acting on the system), the linear momentum of the system of particles, is conserved or in other words total linear momentum of the system is constant".

The conditional clause of no external force is extremely important. If we are sitting in a chair, our momentum is zero, since our velocity is zero.

But when we stand up and walk away, our momentum is not zero, since we have non-zero velocity. Momentum, in general, is not conserved. When we start to walk, you push against the Earth and it pushes you in the opposite direction. So there is external force acting on us. Momentum is only conserved if the total external force is zero.

This principle demands that when external force **F** =0, the rate of change of linear momentum,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = 0 \rightarrow \ \mathbf{p} = \textbf{constant}$$

So we can have

Total Initial Momentum = Total Finial momentum

or

$$(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \ldots + \mathbf{k}_N) = (\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \ldots + \mathbf{p}_N)$$

Or

$$\sum_{i=1}^{N} \mathbf{k}_i = \sum_{i=1}^{N} \mathbf{p}_i$$

Where $\mathbf{k}_i$' s are initial momentum and $\mathbf{p}_i$'s are the finial momentum of the particles. So we see above equation is a vector equation, it is a set of three equations in x, y and z direction, hence conservation principle is

$$\sum_{i=1}^{N} k_{xi} = \sum_{i=1}^{N} m_i u_{xi} = \sum_{i=1}^{N} p_{xi} = \sum_{i=1}^{N} m_i v_{xi}$$

$$\sum_{i=1}^{N} k_{yi} = \sum_{i=1}^{N} m_i u_{yi} = \sum_{i=1}^{N} p_{yi} = \sum_{i=1}^{N} m_i v_{yi}$$

$$\sum_{i=1}^{N} k_{zi} = \sum_{i=1}^{N} m_i u_{zi} = \sum_{i=1}^{N} p_{zi} = \sum_{i=1}^{N} m_i v_{zi}$$

Where $\mathbf{u_i}$'s are initial velocities and $\mathbf{v_i}$ ' s are finial velocities of the particles.

Example of the conservation of momentum is a bullet fired from a gun. Suppose the mass of bullet is m and it moves forward with a velocity **v** and let the mass of gun is M and it recoils with a velocity **V**. then

Momentum of the bullet in the forward direction, $\mathbf{p_b} = m\mathbf{v}$

Momentum of the gun in backward direction, $\mathbf{p_g} = -M\mathbf{V}$

Now initial total momentum = 0, since both bullet and gun are in rest.

And finial momentum of the bullet-gun system is $= m\mathbf{v} - M\mathbf{V}$

Hence conservation of momentum demands that

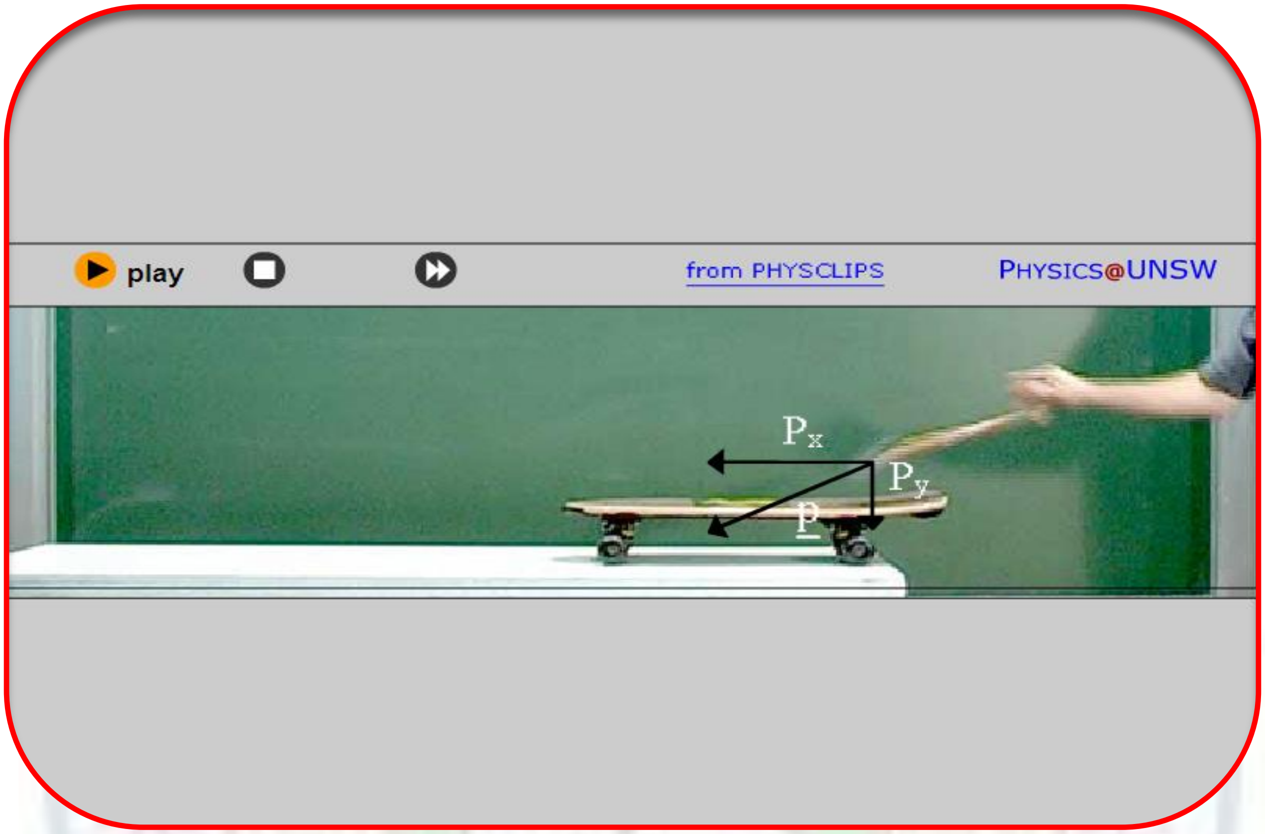
$$m\mathbf{v} - M\mathbf{V} = 0$$

$$\mathbf{v} = (M/m)\,\mathbf{V}$$

Another example of conservation of momentum is the rocket propagation. When the rocket is fired, then hot gases escape from the rear of the rocket. The momentum of those gases is equal to product of the total mass of the gases m_g and their velocity $\mathbf{v_g}$ i.e $\mathbf{p_g} = -m_g\,\mathbf{v_g}$, the negative sign indicates that the gases are escaping backward. The conservation principle then demands that the rocket has to move forward with a momentum $\mathbf{p_R} = M_R\,\mathbf{V_R}$, so that the total momentum remain conserved, since initially rocket was at rest so

$$-m_g\,\mathbf{v_g} + M_R\,\mathbf{V_R} = 0$$

The following multimedia shows some examples of conservation principles. The following example is included to remind us that momentum conservation can apply in only one or two dimensions, and therefore to only some vector component



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Here, the external forces acting on the hammer-skateboard system are gravity, normal force and friction. During the collision, the force between them is much greater than their weight, so weight may be neglected. Notice that skateboard has hardly any vertical acceleration, so the total vertical force on it is close to zero. However, during the collision, there are obviously large vertical forces between skateboard and hammer because the hammer has a large vertical acceleration. So the external normal force acting during this collision cannot be neglected. So momentum is not conserved in the vertical direction. However, this doesn't necessarily prohibit momentum conservation in the horizontal direction. If the mass of the wheels of the skateboard is negligible, then momentum is conserved in the x direction. (In fact, the friction between the wheels and the bench must increase suddenly during the collision,

because the wheels are rolling with different angular velocities before and after (see Wheels and rolling), and this change requires a torque that is supplied by the friction on the bench. However, provided that the mass wheels is small, this force will be small compared to that between hammer and skateboard.)

You can check how well $\Sigma\, p_{x,initial} = \Sigma\, p_{x,final}$ applies here: the mass of the hammer is 2.0 kg, that of the skateboard is 3.5 kg, so conservation of momentum in the x direction predicts that the velocity of the board after collision will be 2.0/(2.0+3.5) = 0.36 times the x component of the hammer's velocity between when it leaves my hand and when it hits the skateboard. The speed is proportional to the number of pixels travelled per frame.

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4. Impulse

Impulse is defined as the change in momentum of a particle or a system of particle.

When we have momentum of a particle or a system of particle changed during the time interval Δt then we have from the second law of newton

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

$$= m(\mathbf{v} - \mathbf{u})$$

Where **v** and **u** are final and initial velocities of the particle.

So we define impulse I as

$$I = \Delta \mathbf{p} = m(\mathbf{v} - \mathbf{u})$$

Now for an infinitesimal interval dt of time we have Impulse I defined as

$$I = \int \mathbf{F} dt$$

5. Summary

- The Centre of mass of a discrete system of particles is defined as

$$\mathbf{R}_{C.M} = \lim_{N\to\infty} \frac{m_1\mathbf{R_2} + m_2\mathbf{R_2} + m_3\mathbf{R_3} + \cdots + m_n\mathbf{R_n}}{m_1 + m_2 + m_3 + \cdots + m_n} = \lim_{N\to\infty} \sum_{\mathbf{i=1}}^{\mathbf{N}} m_i\mathbf{R_i}/M$$

where M is the total mass of the system of particles.

- For a continuous distribution of mass ,we have

$$\mathbf{R}_{C.M} = (1/M)\int \mathbf{R}\, dm$$

$$= (1/M) \int \mathbf{R}\rho dV.$$

- Linear Momentum is defined as the product of mass and velocity. It is a vector quantity.
  So linear momentum $\mathbf{p} = m\mathbf{v}$.

- The principle of conservation of momentum says that total linear momentum is a conserved quantity for an isolated system. Hence

dp/dt =0 so **p =constant**

Total initial momentum = Total finial momentum

- Impulse is the change in the momentum for a small interval of time. It is given by

  I = m(**v - u**)= mΔ**v**

  For infinitesimal time, Impulse is given by

  $$I = = \int \mathbf{F}dt$$

6. Exercise

Q1. A soldier has a rifle that can fire bullets of mass 30g which can speed up to 1km/s.

A 50kg tiger attack the soldier with a speed of15m/s. how many bullets must the soldier fire into the tiger in order to stop it in its path.

Q2. Two boxes of mass m1 and m2 are connected together by a spring and rest on a frictionless table. The boxes are pulled away from each other and then released. Show that their kinetic energies are inversely proportional to their respective masses.

Q3.Find the recoil speed of the500g gun when a shooter fires a 50g bullet with a muzzle speed of 200m/s.

Q4.Find the impulse experienced by the cricketer, when he catch a 20g boll coming towards him with a speed of 10m/s.

Q5.Show that the law of conservation of linear momentum of a system is a direct consequence of the translational invariance of the potential energy of the system.

Fill in the blanks:

Q6. The law of conservation of linear momentum holds when the external______________ is zero.

Q7. The total linear momentum of an isolated system is ______________.

Q8. The Centre of mass of a system of particle is _____________________.

Q9. For an continuous mass system the elemental mass is given by ____________.

Q10. The change of momentum for a small time interval is known as ______________.

State whether the following statements are true or false:

Q11. The total linear momentum of the Universe is constant.

Q12. The Centre of mass of a system of particle always lies inside the system.

Q13. The Linear momentum is a vector quantity.

Q14. The Momentum of a particle is the product of mass and acceleration of the particle.

Q15. The kinetic energy and the linear momentum of a particle is always conserved.

Choose the most appropriate option for the following questions:

Q16. The kinetic energy of two unequal masses is same, then

(A) Their linear momentum is also same.
(B) Lighter mass have larger momentum.
(C) Heavier mass have larger momentum.

Q17. The law of conservation of linear momentum is valid for

(A) All types of systems.
(B) Only for isolated systems.
(C) The non-relativistic systems.

Q18. The Centre of mass of a discrete mass system lies

(A) Always at Centre of the system.
(B) Always outside the system.
(C) Always inside the system.

Q19. The Centre of mass of a uniform regular shaped body is

(A) At its geometrical Centre.
(B) At any point inside the body.
(C) At any point outside the body.

Q20. The Impulse is defined as

(A) The rate of change of linear momentum of the particle.
(B) The change of linear momentum of the particle.
(C) The change in velocity of the particle.

Q21. The distance between the Centres of the two atoms of a dipolar molecule is $1*10^{-10}$ m.

Locate the position of the Centre of mass of the system.

Q22. The cage of parrot is suspended from a spring balance. How does the reading on the balance differ when the parrot flies about from that when it just sits quietly?

Q23. Show that the Centre of mass of two bodies is on the line joining their Centre's is at a point whose distance from each bodies is inversely proportional to the mass of that body.

Q24. Find the position of C.M. of a Solid Cone.

Q25. Find the C.M. of a Solid sphere.