Discipline Course-I
Semester -I
Paper: Mechanics IB
Lesson: Momentum of variable mass system
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Ch.4 Momentum of variable mass system

1. **Motion of rocket**
2. **Multistage rockets**
3. **Brief history of Indian space mission**
4. **Summary**
5. **exercise**

Objectives

After completion of this chapter you will understand:

- The concept of variable mass system
- The propagation of a Single stage Rocket
- The basic theory of Multi-Stage Rocket propagation
- The history of Indian space research programme

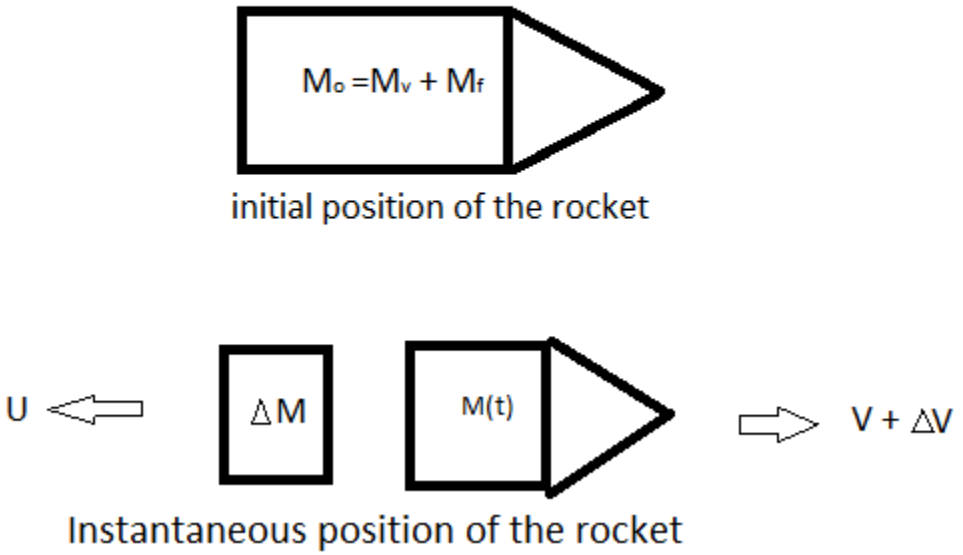
1. Motion of rocket

We now consider the motion of a system when the mass varies with time. Examples of such systems are:

- A drop of water falling through a cloud (will gain mass as it fall down)
- A rocket (will lose its mass its flight as a result of burning of fuel).

We will treat only non-relativistic velocities.
A rocket fired from the earth will always be affected by the gravitational pull of the earth. Other heavenly bodies are at great distances from the rocket and the effect of such objects on the motion of rocket can be ignored. We also ignore the effect of rotation of earth and gravitational force of earth and assume free flight of the rocket. Let us consider the motion of rocket along the x direction and the motion is supposed to be constraint in the x direction only. The rocket is propelled by burning fuel. For the equation of motion, we find the change in momentum of the whole system in time interval Δt.

Let M be the mass of the rocket and v is its speed at time t. Then, in time interval Δt, the mass of the system is reduced by amount ΔM due to burning of the fuel and expulsion of an equal amount of mass of the gas. As a result of reduction in mass, the velocity of the system increase by amount Δv. Let u be the velocity of the exhaust gases relative to the rocket as shown in figure below.



Then, the law of conservation of momentum gives

$$Mv = (M - \Delta M)(v+\Delta v) - \Delta M(u-v)$$

Or

$$Mv = Mv + M\Delta v - \Delta Mv - \Delta M\,\Delta v - u\,\Delta M + \Delta Mv$$

Now simplifying above equation and retaining only first-order infinitesimal quantities, we get

$$M\Delta v = u\,\Delta M$$

Dividing throughout by Δt and taking the limit as Δt tends to zero, we get

$$M\,\dot{v} = -u\,\frac{dM}{dt}$$

Where $\dot{v} = \frac{dv}{dt}$.

The negative sign shows that velocity increases as mass decreases.

Integrating w.r.t. time, we get

$$\int_{Vo}^{V} dv = -u \int_{Mo}^{Mt} dM/M$$

Or

$$v = v_o - u \ln \frac{M_t}{Mo}$$

Where v and M_t are the velocity and mass of the system at the instant t and v_o and M_o are those at t=0.

Let us suppose that the fuel is burnt at constant rate $\frac{dM}{dt} = \beta$ and it lasts for time T. If the mass of the vehicle is M_v and that of the fuel initially at t=0, is M_f,

then

$$M_o = M_v + M_f$$

The mass of the vehicle fuel system at any instant t can be written as

$M_t = M(t) = M_v + M_f\,(1- t/ T) = M_o - M_f \frac{t}{T}$,for $0 \le t \le T$

And $M(t) = M_o - M_f = M_v$ for $t \ge T$

Substituting the value of M_t in velocity equation, we have

$$v=\frac{dx}{dt} = v_o - u \ln (1- \frac{M_f t}{M_o T})$$

Integrating w.r.t time again, we get

$$x = x_o + v_o t - u \int_0^t \ln(1 - \frac{M_f t}{M_o T})dt$$

the last integral can be obtained by parts, so we get

$$\int_0^t \ln(1 - \frac{M_f t}{M_o T})dt = (t - \frac{M_o T}{M_f}) \ln(1 - \frac{M_f t}{M_o T}) - t$$

Thus the distance covered by the rocket in time t is given by

$$x = x_o + v_o t - u\left[(t - \frac{M_o T}{M_f}) \ln(1 - \frac{M_f t}{M_o T}) - t\right]$$

the rocket attains maximum velocity at t= T when all its fuel is burnt out. The maximum velocity is given by

$$v_{max} = v(t=T) = v_o - u \ln(1 - \frac{M_f}{M_o})$$

$$= v_o + u \ln(M_o/M_v)$$

$$= v_o + u \ln(1+M_f/M_v)$$

So from above equation it is clear that the larger the value of ratio M_f/M_v, the greater will be maximum velocity attained by the rocket.

Now we include the gravitational pull of earth, we have the equation of motion for the rocket as

$$M\dot{v} = -u\frac{dM}{dt} - Mg$$

Or

$$\dot{v}dt = -\frac{udM}{M} - gdt$$

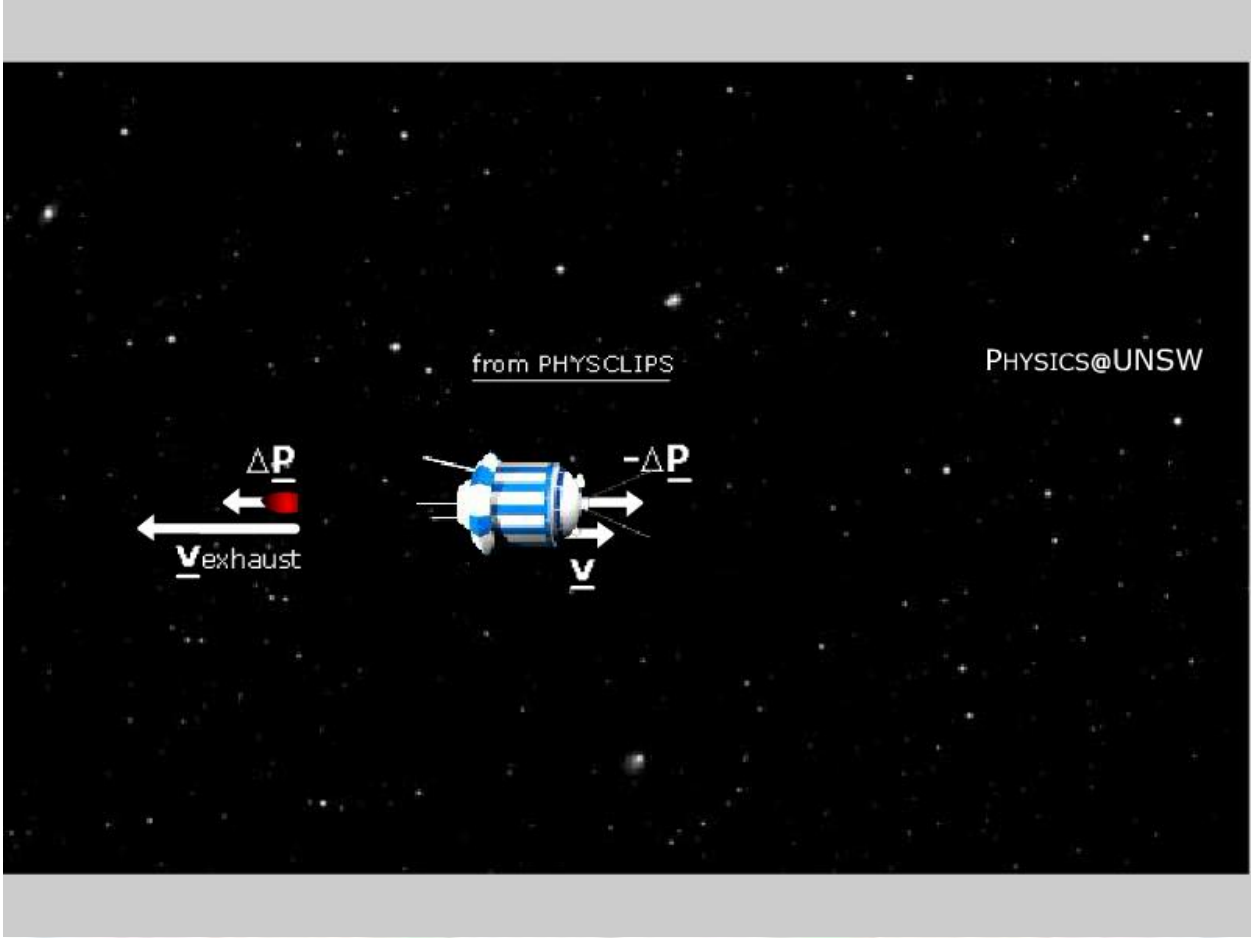
Integrating w.r.t. time, we get

$$v = v_o - u \ln(M_t/M_o) - gt$$

assuming height $x_o=0$ and velocity $v_o=0$, initially, we have the expression for the height attained by the rocket at time t, as

$$x = ut - \frac{1}{2}gt^2 - (t - M_o T/M_f) \ln(1 - M_f t / M_o T)$$

the rocket carries some load, called the payload. Payload may be a satellite to be placed in the orbit of the earth, or a bomb in the case of a missile. The payload and the body of the rocket have a fixed mass so the ratio M_f/M_v has a practical limit, hence the maximum speed of the rocket cannot be increased infinitely, so we have to make multistage rockets to attain high speeds.



To play the movie , click Mechanics with animations and film clips: Physclips.

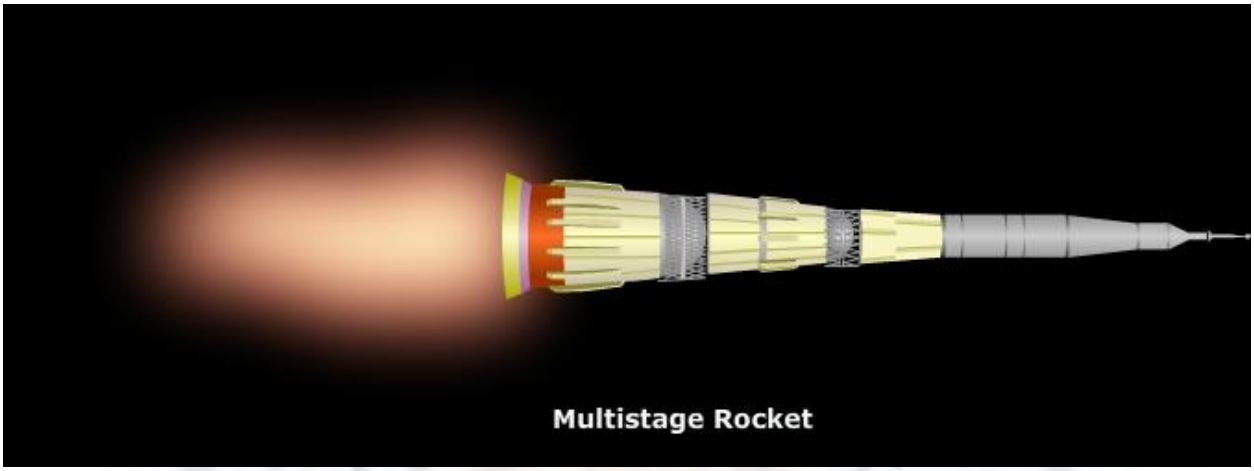
Credits: Authored and Presented by Joe Wolfe

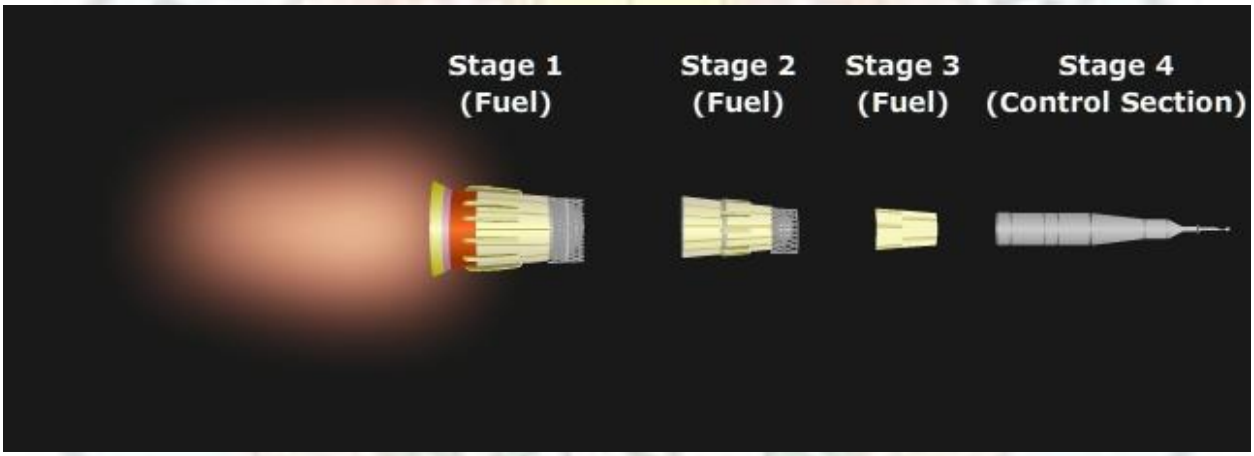
Multimedia Design by George Hatsidimitris

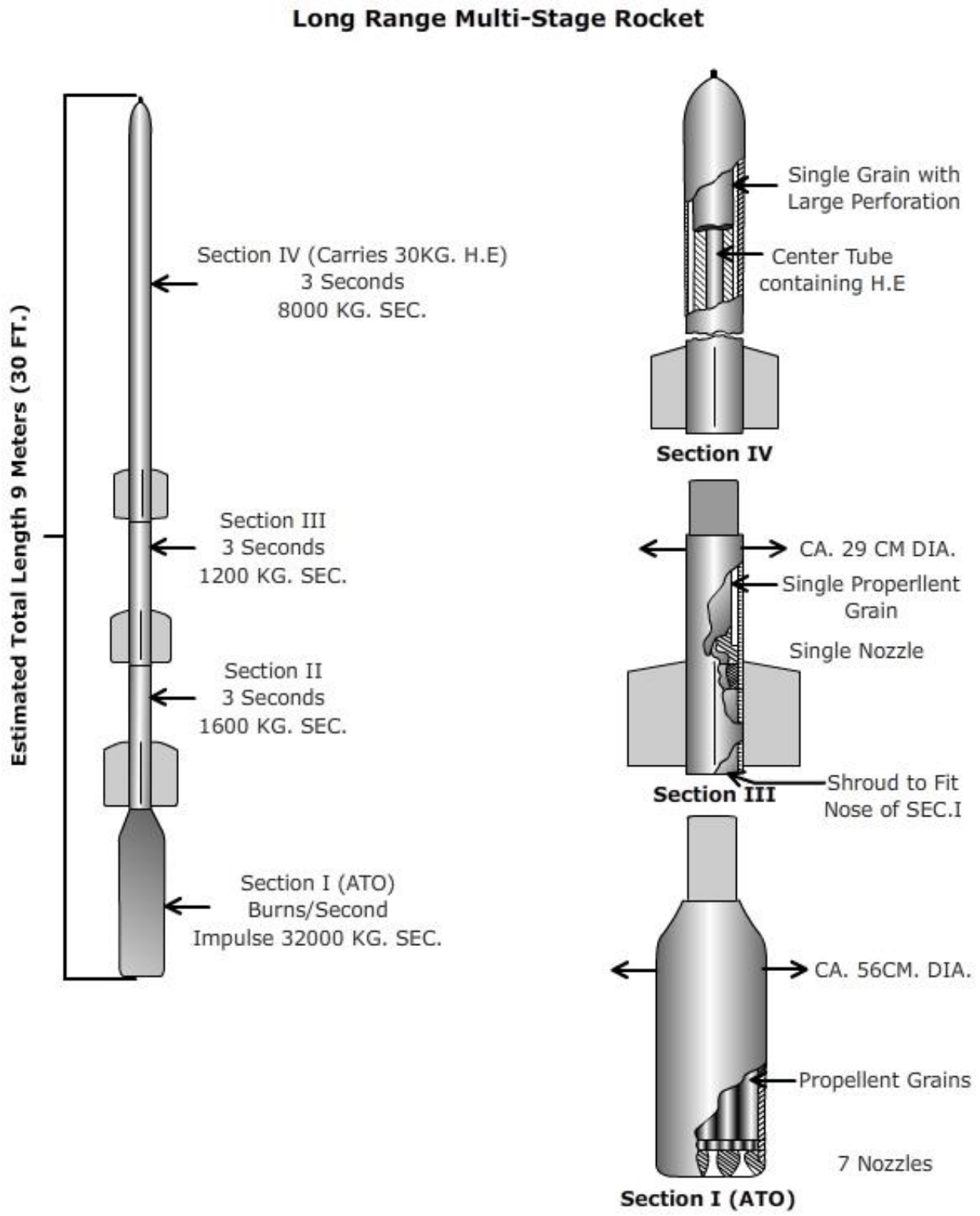
Laboratories in Waves and Sound by John Smith

Multi –stage rockets (optional)

Multi-stage rockets are a group of rockets combined either in sequence of one inside the other, or the rear part of one inside the nozzle of the other. As shown in figures below:


Multistage Rocket

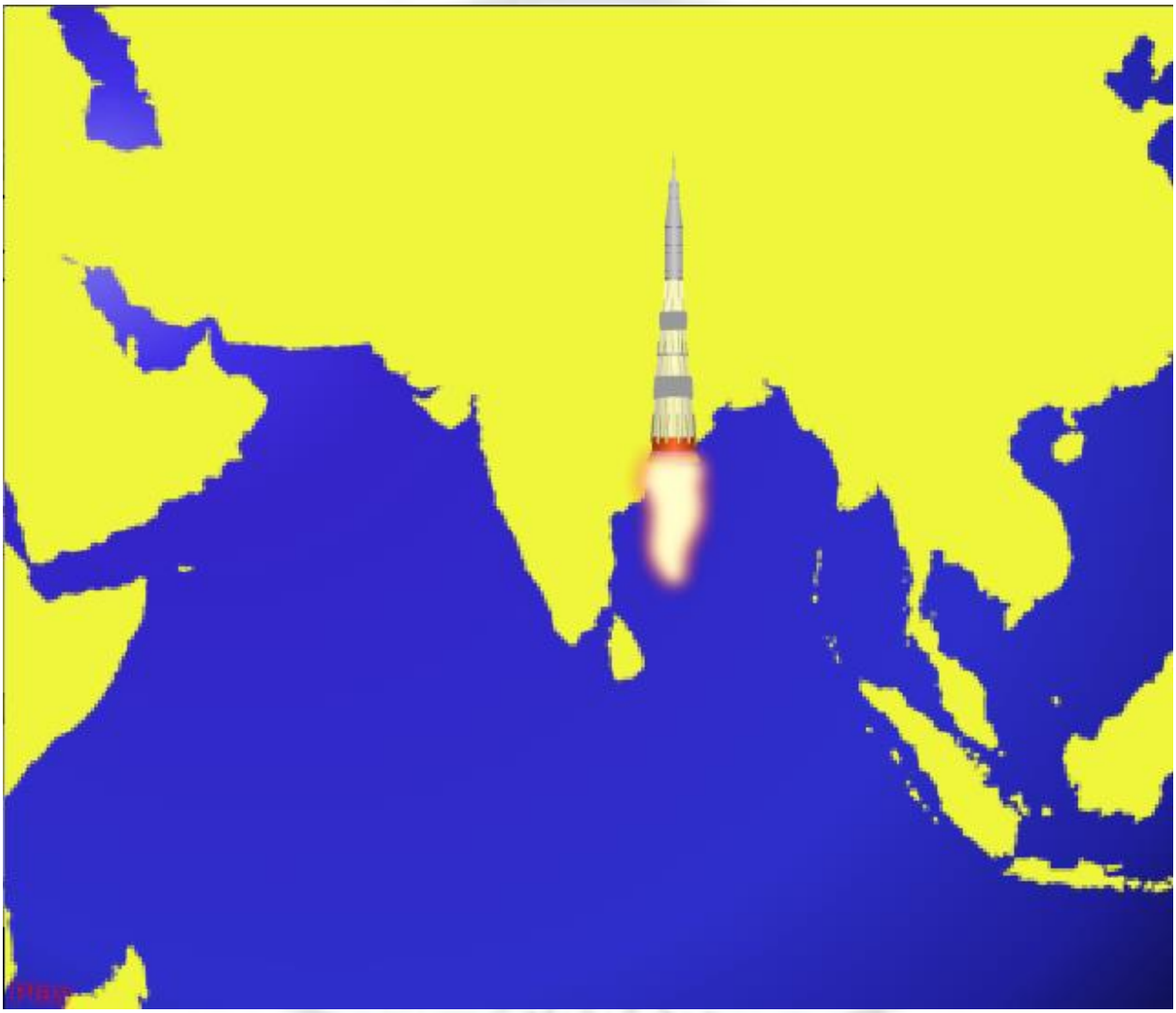

Stage 1
(Fuel)
Stage 2
(Fuel)
Stage 3
(Fuel)
Stage 4
(Control Section)



Credits: **Creator:** David Shoemaker

Date: approx. 1945

In multi-stage rockets, the first stage rocket is used first and when its fuel gets consumed, it gets detached. Then the second stage rocket takes its place and produces further acceleration, when its fuel also gets consumed, the third stage rocket comes into picture and take the position of second stage rocket. This is how the velocity after each stage goes on increasing. The fuel consumption and the thrust for the first stage are about hundred times more than for the third stage and the fuel stock carried by it about 60 times that carried by the third stage. The following animation displays the working of a multi-stage rocket.



Additional websites for information:

Wikipedia: Multi Stage Rocket

Atomic Rockets

The Free Dictionary: Multi Stage Rockets

Science Daily: Multistage Rocket
Think Quest: Multistage Rockets

3. Brief history of Indian space mission (optional)

As Russia (USSR) launch Sputnik in 1957, India too felt the importance of space science and technology for the socio-economic growth of the society. In 1960, India started its space programme with the establishment of Thumba Equatorial Rocket Launching Station near Thiruvananthapuram, for the investigation of ionosphere. It's the efforts of Dr. Vikram Sarabhai, also known as the father of Indian space programme, who started space research programme in India. At beginning, Department of Atomic Energy, carried out space programme, but in June 1972 Department of Space (DOS) was established for the purpose. Now Indian Space Research Organization (ISRO) under DOS executes space programme through its establishments located at different places in India (Ahmedabad in Gujarat, Bangalore in Karnataka, Mahendragiri in Tamil Nadu, Sriharikota in Andhra Pradesh, Thiruvananthapuram in Kerala, etc.). We are the sixth nation in the world, which have the capability of designing, constructing and launching a satellite in an Earth orbit.

Following are the mile stones in the history of Indian space research:

(A) Indian satellites- these are the series of Indian satellites:

1. Aryabhatta - The first Indian satellite was launched on April 19, 1975.

2. Bhaskara - 1

3. Rohini

4. APPLE - It is the abbreviation of Ariane Passenger Pay Load Experiment.

   APPLE was the first Indian communication satellite put in geo - stationary orbit.

5. Bhaskara - 2

6. INSAT - 1A, 1B, 1C, 1D, 2A, 2B, 2C, 2D, 3A, 3B, 3C, 3D, 3E

   INSAT is the short for Indian National Satellite. Indian National Satellite System is a joint venture of Department of Space, Department of Telecommunications, Indian Meteorological Department and All India Radio and Doordarshan.

7. SROSS - A, B, C and D (Stretched Rohini Satellite Series)

8. IRS - 1A, 1B, 1C, 1D, P2, P3, P4, P5, P6

IRS is short for Indian Remote Sensing Satellite-Data from IRS is used for various applications like drought monitoring, flood damage assessment, flood risk zone mapping, urban planning, mineral prospecting, forest survey etc.

9. METSAT (Kalpana - I) - METSAT is the first exclusive meteorological satellite.

10. GSAT-1, GSAT-2 (Geo-stationary Satellites)

(B) Indian Launch Vehicles (Rockets)-following are the Indian Launch Vehicles (LV):

1. SLV - 3 - This was India's first experimental Satellite Launch Vehicle. SLV - 3 was a 22 m long, four stage vehicle weighing 17 tons. All its stages used solid propellant.

2. ASLV - Augmented Satellite Launch Vehicle. It was a five stage solid propellant vehicle, weighing about 40 tons and of about 23.8 m long.

3. PSLV - The Polar Satellite Launch Vehicle has four stages using solid and liquid propellant systems alternately. It is 44.4 m tall weighing about 294 tons.

4. GSLV - The Geosynchronous Satellite Launch Vehicle is a 49m tall, three-stage vehicle weighing about 414 tons capable of placing satellite of 1800 kg.

(C) India's first mission to moon: ISRO sent an unmanned spacecraft to moon in the year 2008. The spacecraft is named as CHANDRAYAAN-1. This programme was for expanding scientific knowledge about the moon, upgrading India's technological capability and providing challenging opportunities for planetary research for the younger generation. This journey to moon was supposed to take 5½ days. CHANDRAYAAN - 1 probed the moon by orbiting it at the lunar orbit of altitude 100 km. This mission to moon was carried by PSLV Rocket.

For more information on Indian space programme please visit the website of ISRO

www.isro.org

http://www.textbooksonline.tn.nic.in/Books/11/Std11-Phys-EM-1.pdf

4. Summary

- The motion of rocket is an example of the variable mass system, the velocity at any time during the flight of the rocket is given by

$$v=\frac{dx}{dt} = v_o - u \ln\left(1- \frac{Mf\, t}{MoT}\right)$$

- The maximum velocity is given by

$$v_{max} = v_o + u \ln(1+M_f/M_v)$$

- The distance covered during this time is given by

$$x= x_o + v_o\, t - u\left[\left(t-\frac{MoT}{Mf}\right)\ln\left(1-\frac{Mf\, t}{MoT}\right) - t\right]$$

- When we take the gravitational effect of earth , the distance is

$$x= ut - \frac{1}{2} gt^2 - (t - M_oT/M_f)\ln(1 - M_ft / M_oT)$$

- For high speed, we have to launch Multi-stage Rockets.

- Dr. Vikram sarabai was the father of Indian space research programme.

- The Indian space Mission is carried by ISRO.

- The First Indian satellite – Aryabhatta was launched on April 19, 1975.

6. Exercise

Q1. A rocket consumes 200 kg fuel per second, exhausting it with a speed of 20km/s.

(a) What force is exerted on the rocket?

(b) If its mass is reduced to $1/10^{th}$ of its initial mass and taking its initial velocity as zero, what is the speed of the rocket at this time?(neglect gravitational effects, other effects)

Q2. If the maximum possible exhaust velocity of a rocket be 3km/s ,

(a) Calculate the ratio, M_o/M for it if it is to achieve the escape velocity 11.2km/s.
(b) How long will it take the rocket(starting from rest) to attain this velocity if its rate of change of mass in terms of its initial mass is $1/20^{th}$?

Q3. (a) a rocket is set for vertical firing has a weight of 40 kg and contain 400 kg of fuel.

If it can have maximum exhaust velocity of 1km/s, what should be its minimum rate of fuel consumption (1) to just lift it off the launching pad,(2) to give it an acceleration of 10m/s?

(b)What will be the speed of the rocket when the rate of fuel consumption is (1)5kg/s,(2)10kg/s,(3)20kg/s,(4)40kg/s ?

Q4. Show that a rocket has thrice the exhaust speed when $M_o/M = e^3$.

Q5. A rocket of mass 30kg has 200 kg of fuel. The exhaust velocity of fuel is 2km/s. Calculate the maximum vertical speed gained by the rocket when the rate of fuel consumption of fuel is 3kg/s. Also calculate the maximum distance covered.

Fill in the blanks:

Q6. The propagation of rocket is based on_____ of Newton.

Q7. The total linear momentum of rocket remains _____.

Q8. The free fall of rain drops is another example of _____ systems.

Q9. The first Indian satellite was launched in the year _____.

Q10. The name of first Indian satellite was _____ .

State whether the following statements are true or false:

Q11. The mass of a particle or the system of particle is a constant of motion always.

Q12. The multi-stage Rocket can achieve higher speed as compared to single stage rockets.

Q13. The first man send on the moon was American.

Q14. The father of Indian space programme was Dr. Vikram sarabai.

Q15. NASA is the Indian space research agency.

Choose the most appropriate option for the following question:

Q16. Which one of the following was the first Indian communication satellite put in geo - stationary orbit:

(A) APPLE

(B) ARAYABHATTA
(C) BHASKAR

Q17. For high speed, we have to launch.

(A) A rocket with large amount of fuel
(B) Single-stage Rockets
(C) Multi-stage Rockets

Q18. Which one of the following is the meteorological satellite?

(A) SROSS
(B) METSAT
(C) GSAT

Q19. The satellites which have same Time period of orbiting as that of Earth are known as

(A) IRS
(B) METSAT
(C) GSAT

Q20. The payload is defined as

(A) The amount of money paid to design a rocket
(B) The weight of fuel in the rocket
(C) Payload may be a satellite to be placed in the orbit of the earth , or a bomb in the case of a missile.

Q21. Show that the velocity at any time during the flight of the rocket is given by

$$v=\frac{dx}{dt} = v_o - u \ln\left(1 - \frac{Mf\, t}{MoT}\right)$$

Q22. Show that the distance covered any time during the flight is given by

$$x = x_o + v_o t - u\left[\left(t - \frac{M_o T}{M_f}\right)\ln\left(1 - \frac{M_f t}{M_o T}\right) - t\right]$$

Q23. Show that when we take the gravitational effect of earth , the distance is

$$x = ut - \frac{1}{2}gt^2 - (t - M_o T / M_f)\ln(1 - M_f t / M_o T)$$

Q24. What are multi-stage rocket? Briefly explain their working principle.

Q25. Write a short note on the History of Indian space mission.