Subject :Physics

Lesson Name: Non-inertial frame of reference

Lesson Developer: Seema Dabas

College/Department :- Shyam Lal College, University of Delhi

Non-inertial Frame of Reference

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Non-inertial Frame of Reference

Learning Objectives

After reading this lesson, you should be able to

a) Understand the concept of linear and rotational motion

b) Define inertial frame of reference

c) Develop the concept of non-inertial frame of reference

d) Answer the questions of interest to engineers and physicists

e) Apply your knowledge to the practical life

f) Develop the concept of centrifugal and Coriolis force

g) Learn the concept of frames of references rotating with constant angular velocity

h) Learn about the consequences of earth's rotation

Non-inertial Frame of Reference

Chapter: Title Non-inertial Frame of Reference

1.1 Introduction The word event is well known in ordinary speech. Anything that happens may be called an event. It has not only position but also the time of occurrence. A system of coordinate axes which describes a particle in two or three-dimensional space is known as a frame of reference. The essential thing about a frame of reference is that it should be quite rigid. As we may consider any number of rigid bodies moving relative to one another, thus any number of frames of reference can be considered. So, first we select one of these along with rectangular axes of coordinates and then assign to any event a set of three numbers x, y, z, the coordinates in the frame of reference of the point where the event occurs.

1.2 Non-inertial Frames The frame of reference in which Newton's laws of motion hold good are known as inertial frames of reference. The basic laws of physics do not get modified or changed in form in such types of frames of reference. In case, when the frame of reference is accelerated relative to an inertial frame, the form of basic physical laws such as Newton's second law of motion becomes completely different. Such frames of reference having an accelerated motion relative to an inertial frame are known as non-inertial frames of reference. For example a uniformly rotating frame has a centripetal acceleration; it is also a non-inertial frame. The rotating frame of reference in which a body, which is at rest in an inertial frame, appears to be moving in a circle (and thus having an acceleration) is not an inertial frame of reference. This indicates that inertial frames are also non-rotating frames.

1.3 Frames of reference having linear acceleration

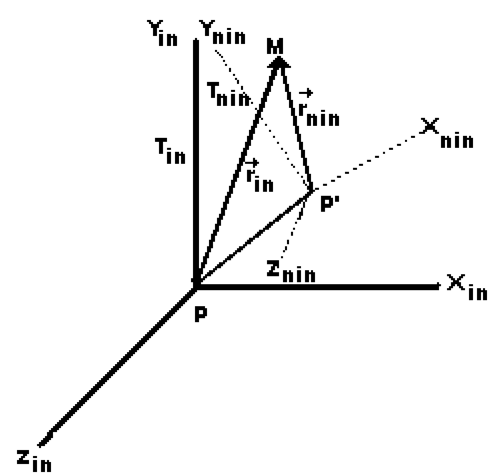


Fig.1.1

Let us consider two frames, one inertial frame T_{in} and other non inertial T_{nin}. Say a frame T_{nin} is moving with a linear acceleration $\vec{a}_l$ with respect to the inertial frame T_{in} (Fig.1.1). Then, any particle (say *M*) at rest with respect to frame T_{in} will clearly appear to be moving with acceleration $-\vec{a}_l$ with respect to frame T_{nin} and therefore a particle having an acceleration $\vec{a}$ with respect to the inertial frame T_{in} will appear to have an acceleration $\vec{a}_{nin}$ in frame T_{nin} which is given by

$$\vec{a}_{nin} = \vec{a} - \vec{a}_l$$

So that, if *m* be the mass of the particle (assumed to remain constant in T_{in} or T_{nin}), we have

Force observed on the particle in frame T_{nin} is given by

$$\vec{F}_{nin} = m\vec{a}_{nin} = m(\vec{a} - \vec{a}_l) = m\vec{a} - m\vec{a}_l \qquad (i)\ \vec{a}_{nin}\ \vec{a}_{nin}$$

where $m\vec{a} = \vec{F}$ is the force on the particle in the inertial frame T_{in}. Therefore, Equation (i) becomes $\vec{F}_{nin} = \vec{F} - m\vec{a}_l$

or Substituting $m\vec{a}_l = \vec{F}_l$ we get $\vec{F}_{nin} = \vec{F} - \vec{F}_l$

And if $\vec{F} = 0$ i.e. no force is acting on the particle in initial frame T_{in} then

$$\vec{F}_{nin} = -\vec{F}_l$$

Or we can say that a force $\vec{F}_l = m\vec{a}_l$ appears to be acting on the particle in frame T_{nin} (moving with respect to T_{in}), which is, therefore a non-inertial frame. This is also known as apparent force in T_{nin}. Because of the fictitious component, a man inside the lift (when moving upward with a uniform acceleration) feels more weight than his real weight and feels less weight when lift is moving downwards with uniform acceleration.

1.4 Rotating frame of reference

Let us suppose an inertial frame of reference T_{in} and another reference frame T_r. Say a particle at *M* (shown in Fig. 1.2) whose position vector is $\vec{r}$ with respect to the origin of either frame of reference. Coordinates of the considered particle are x, y, z in frame T_{in} and x_r, y_r, z_r of frame T_r. Both origin and coordinate axes of T_r are such that they coincide with those of T_{in}. Let frame T_r starts rotating about the common axis of *z*, so that in time *t*, the axes PX_r and PY_r of T_r have turned through an angle ωt (where ω is uniform angular velocity) each with respect to axes *PX* and *PY* of frame T_{in}, in time *t*.

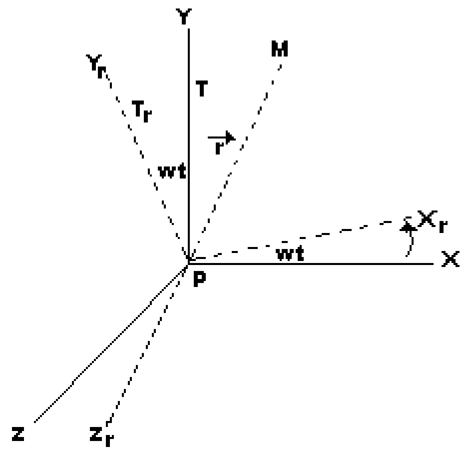


Fig. 1.2

Now we have to find out the relation between the coordinates, *x*, *y*, *z* and x_r, y_r, z_r of a particle *M* in the two frames of reference respectively, its position vector with respect to the origin being the same $\vec{r}$ in either frame.

We have x_r = *sum of the components of x, y, z along the axis* PX_r *i.e.*

Since $\cos XPX_r = \cos \omega t$

$\cos YPX_r = \sin \omega t$

$\cos ZPX_r = 0$

We get $x_r = x\cos\omega t + y\sin\omega t$ $\qquad (a)$

Similarly for y_r :

we have $y_r = -x\sin\omega t + y\cos\omega t$ $\qquad (b)$

and $z_r = z$ $\qquad (c)$

Equations (*a*), (*b*) and (*c*) are therefore the **transformation equations** in the case of the frame of reference T_r which is rotating with a uniform angular velocity ω relative to the inertial frame T_{in}.

The inverse transformation equations (from T_r to T_{in}) will now become

$x = x_r\cos\omega t - y_r\sin\omega t$, $y = x_r\sin\omega t + y_r\cos\omega t$ and $z = z_r$

The particle *M* does not experience any force in frame T_{in} as it is an inertial frame. We, therefore, have $\frac{d^2x}{dt^2}=0$, $\frac{d^2y}{dt^2}=0$ and $\frac{d^2z}{dt^2}=0$

Differentiating expressions (*a*), (*b*), (*c*) for x_r, y_r, z_r respectively w.r.t. *t*, we have

$$\frac{dx_r}{dt}=-\omega x\sin\omega t+\omega y\cos\omega t+\frac{dx}{dt}\cos\omega t+\frac{dy}{dt}\sin\omega t$$

Using Equation (*b*), it becomes

$$\frac{dx_r}{dt}=\omega y_r+\frac{dx}{dt}\cos\omega t+\frac{dy}{dt}\sin\omega t \qquad (d)$$

Similarly,

$$\frac{dy_r}{dt}=-\omega x\cos\omega t-\omega y\sin\omega t-\frac{dx}{dt}\sin\omega t+\frac{dy}{dt}\cos\omega t$$

Or, since $x\cos\omega t+y\sin\omega t=x_r$ [From Equation (a) above], we have

$$\frac{dy_r}{dt}=-\omega x_r-\frac{dx}{dt}\sin\omega t+\frac{dy}{dt}\cos\omega t \qquad (e)$$

$$\frac{dz_r}{dt}=\frac{dz}{dt} \qquad (f)$$

Differentiating expressions (*d*), (*e*), (*f*) once again, with respect to *t*, we have
$$\frac{d^2x_r}{dt^2}=\omega\frac{dy_r}{dt}-\omega\frac{dx}{dt}\sin\omega t+\omega\frac{dy}{dt}\cos\omega t$$

Since from relation (*e*) above, $-\frac{dx}{dt}\sin\omega t+\frac{dy}{dt}\cos\omega t=\left(\frac{dy_r}{dt}+\omega x_r\right)$

We have $\frac{d^2x_r}{dt^2}=\omega\frac{dy_r}{dt}+\omega\left(\frac{dy_r}{dt}+\omega x_r\right)=2\omega\frac{dy_r}{dt}+\omega^2x_r \qquad (g)$

Similarly, $\frac{d^2y_r}{dt^2}=-\omega\frac{dx_r}{dt}-\omega\frac{dx}{dt}\cos\omega t-\omega\frac{dy}{dt}\sin\omega t$

Using the Equation (*d*), we get

$$\frac{dx}{dt}\cos\omega t+\frac{dy}{dt}\sin\omega t=\left(\frac{dx_r}{dt}-\omega y_r\right),$$

We have $$\frac{d^2y_r}{dt^2}=-\omega\frac{dx_r}{dt}-\omega\left(\frac{dx_r}{dt}-\omega\, y_r\right)=-2\,\omega\frac{dx_r}{dt}+\omega^2 y_r \qquad (h)$$

and $$\frac{d^2z_r}{dt^2}=\frac{d^2z}{dt^2} \qquad (i)$$

From relations (*g*) and (*h*) above, we come to the conclusion that even though no force is acting on a particle *M* in frame T_{in}, a force seems to be acting on it in frame T_r, producing an acceleration in it. Frame T_r is, therefore, a non-inertial frame of reference.

1.5 Pseudo forces (or Fictitious forces)

Consider a particle of mass *m*. According to Newton's second law, the force acting on a particle in an inertial frame T_{in} is given by $\vec{F}=m\vec{a}$. The force acting on it in a reference frame T_{nin}, moving with an acceleration $\vec{a}_l$ with respect to T_{in} will be $\vec{F}_{nin}$. Since Newton's laws are not obeyed in frame T_{nin}, so it is a non-inertial frame. Now, put $-m\vec{a}_l=\vec{F}_l$ and $m\vec{a}=\vec{F}$, we have $\vec{F}_{nin}=\vec{F}+\vec{F}_l=0+\vec{F}_l=-m\vec{a}_l$.

This force $\vec{F}_l$ does not actually exist but appears to come into picture as a consequence of the acceleration of frame T_{nin} with respect to T_{in}. Therefore, it is termed as false (pseudo) or fictitious force or can be obtained as the product of mass with the acceleration of the non-inertial frame, with its sign reversed.

It is the force which, when added to the true force $\vec{F}$ in the inertial frame *T*, gives the observed force $\vec{F}_{nin}$ in the non-inertial or accelerated frame T_{nin}. This means, in other words, that Newton's second law of motion will also hold in the non-inertial frame T_{nin} which will, therefore, also behave as an inertial frame of reference, if we add to the true force $\vec{F}$ a fictitious force $\vec{F}_l=-m\vec{a}_l$.

For e.g. Consider a particle at rest with respect to a lift which is going downwards with an acceleration *g*. If *m* is the mass of the particle, then fictitious force acting on it is given by $F_l=-mg$. So that, the resultant force on it, as observed by an observer in the lift (the moving frame) will be F_{nin}= true force acting on it (i.e. mg) + the fictitious force, $-mg$, or $F_{nin}=mg-mg=0$, i.e. the particle is weightless and thus remains suspended in air.

We can find out whether or not a given frame of reference is accelerated with help of fictitious force. For, if two frames were in uniform relative motion (zero acceleration), with respect to each other, they are obviously inertial frames, and it is very complex to detect which one is at rest and which one in motion. If, however, one of them be accelerated with respect to the other, fictitious forces come into picture inside the accelerated frame. These forces are not true or real forces. For, unlike the latter, which go on diminishing rapidly with distance and vanish at a large distance from a body, the former arises precisely due to the accelerated motion of the frame of reference and keeps on increasing with its acceleration. So that, whereas the real forces on particles or bodies at large distances from other bodies are negligible, the fictitious forces may have significant values. So if a body or a particle is found quite distant from other material bodies and it is acted upon by an appreciable force, we can conclude that its frame of reference is an accelerated one.

1.6 Earth (A non-inertial frame of reference) Earth goes round the sun and it also spins about its own axis. Say ω is the angular velocity. Centripetal acceleration (at the equator) is given by $a_c = \omega^2 R \qquad (i)$

Angular velocity $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{24\times 60\times 60} \qquad (ii)$

Radius = $R = 6.4\times 10^8\, cm$.

Substituting the values of ω and R, centripetal acceleration comes out to be $a_c = 3.4 cm/s^2$. This value is quite small for most of our daily life activities, and so can be neglected. The earth is taken to be a satisfactory inertial frame of reference. We must now regard as equally satisfactory the interior of any vehicle, which moves over the earth with constant velocity. This is in accordance with common experience: we are not conscious of the smooth uniform motion of a train when we are travelling in it; we become alert or conscious about the motion only when the train brakes or executes a bend round a corner.

So, now the point is about the location of the frame of reference. We can take any rigid object, which is located in distant space as an inertial frame of reference. Now, since we know that motion is always described in a relative frame of reference, i.e. in a frame in which the position of a particle or a body is specified in relation to other material objects which may themselves be in motion relative to other material objects, Newton insisted the presence of a fundamental frame of reference, which he called absolute space. With respect to this frame, all motion must be measured.

For most of our purposes, the reference frame, stationary with respect to the fixed stars, is good enough as our fixed or absolute inertial frame of reference. All other frames of references having uniform motion relative to it, naturally, also act as equivalent inertial frames, as also any reference frame whose origin coincides with that of an inertial frame even though its coordinate axes may be inclined to those of latter.

1.7 Frames of reference rotating with constant angular velocity

Let T is a Newtonian frame of reference and T_r a frame of reference rotating about a point O of T with constant angular velocity ω. Say $\hat{i}$, $\hat{j}$ be perpendicular unit vectors fixed in T_r. Suppose M be a moving particle,

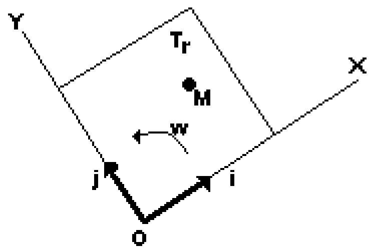


Fig. 1.3

taking axes *OX* and *OY* in T_r, in the directions of $\hat{i}$, $\hat{j}$, the position vector of *M* is

$$\vec{r} = x\hat{i} + y\hat{j} \qquad (a)$$

Now $\frac{d\hat{i}}{dt} = \omega\,\hat{j} \qquad (b1) \qquad \frac{d\hat{j}}{dt} = -\omega\,\hat{i} \qquad (b2)$

(In order to derive the above relation write the unit vectors $\hat{i}$, $\hat{j}$ of the frame T_r in terms of unit vectors of the frame *T*)

Differentiating Equation (a) gives, for the velocity of particle *M* (relative to *T*),

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt} - \omega y\right)\hat{i} + \left(\frac{dy}{dt} + \omega x\right)\hat{j} \qquad (c).$$

Differentiating once again the Equation (a), for the acceleration of *M* (relative to *T*),

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d\vec{v}}{dt} = \left(\frac{d^2x}{dt^2} - 2\omega\frac{dy}{dt} - \omega^2 x\right)\hat{i} + \left(\frac{d^2y}{dt^2} + 2\omega\frac{dx}{dt} - \omega^2 y\right)\hat{j} \qquad (d)$$

Thus, if *X* and *Y* are the components of true force in the directions of $\hat{i}$, $\hat{j}$ respectively, we have the equations of motion

$$m\left(\frac{d^2x}{dt^2} - 2\omega\frac{dy}{dt} - \omega^2 x\right) = X \qquad (e)$$

$$m\left(\frac{d^2y}{dt^2}+2\omega\frac{dx}{dt}-\omega^2 y\right)=Y \qquad (f)$$

These Equations can be rewritten as

$$m\frac{d^2x}{dt^2}=X+X'+X'' \qquad (g) \qquad m\frac{d^2y}{dt^2}=Y+Y'+Y'' \qquad (h)$$

where $X'=2m\omega\frac{dy}{dt}$ $\quad Y'=-2m\omega\frac{dx}{dt}$

$X''=m\omega^2 x \; Y''=m\omega^2 y$

Therefore, the particle moves relative to the rotating frame of reference in accordance with the Newton's law of motion, provided that we add to the true force the two fictitious forces (*X′*,*Y′*) and (*X″*,*Y″*).

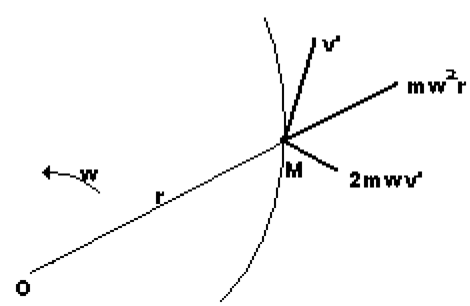


Fig. 1.4

The fictitious force (*X′*,*Y′*) is known as the Coriolis force. Its magnitude is proportional to the angular velocity of T_r and to the speed *v′* of the particle relative to T_r; its direction is perpendicular to the velocity $\vec{v}'$ relative to T_r and is obtained from the direction of $\vec{v}'$ by rotation through a right angle in a sense opposite to the sense of the angular velocity (Fig.1.4). The fictitious force (*X″*, *Y″*) is named as the centrifugal force. Its magnitude is proportional to the square of the angular velocity of T_r and to the distance of the particle from the centre of rotation which is directed radially outward from the center of rotation.

1.8 Consequences of earth's rotation

Since the earth is rotating uniformly about its own axis, from west to east, a reference frame fixed on it is obviously a rotating frame of reference. This causes the two fictitious forces, viz., the Centrifugal force and the Coriolis force acting on a particle at rest and in motion respectively, relative to the rotating frame of reference. Earth is the frame of reference, which we employ in ordinary life. It rotates relative to the astronomical frame with an angular velocity of 2π radians per sidereal day; since one sidereal day contains 86,164.09 sec, the angular velocity of the earth is 7.29×10^{-5} radians per sec. This value is very small, and hence the Coriolis force and the centrifugal force arising from the earth's rotation are almost invisible in our daily lives. They are important geographically, however; the centrifugal force is responsible for the equatorial bulge on the earth, and the Coriolis force is responsible for the trade winds. Let us see the effect of these two forces.

a)Centrifugal force

Suppose a particle *M* is at rest on the surface of the earth, in latitude ψ. Since the particle is at rest in the rotating frame of reference of the earth, hence no Coriolis force is acting on it.

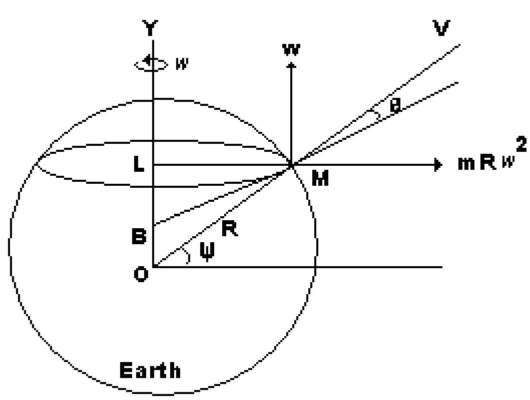


Fig. 1.5

Centrifugal force is the only fictitious force acting on the particle given by $m\omega\times(\omega\times R_n)$ where R_n is the component of the radius *R* of the earth, perpendicular to its axis of rotation (or the perpendicular distance of the particle *M* from its axis). Hence, if the observed or apparent acceleration of the particle, in latitude ψ, be directed towards *B*, and its true acceleration, *g*, towards the centre (*O*) of the earth, we have

$$g_\psi = g - \omega \times (\omega \times R_n) \qquad (a)$$

Considering the axes, *OY* and *OX*, in the direction and perpendicular to ω respectively, with $\hat{i}$ and $\hat{j}$ as the unit vectors along *OX* and *OY* respectively, we have $\vec{g} = -g(\cos\psi\,\hat{i} + \sin\psi\,\hat{j})$, $\vec{\omega} = \omega\hat{j} - g(\cos\psi\,\hat{i} + \sin\psi\,\hat{j})$ and $\vec{R}_n = R\cos\psi\,\hat{i}$

Putting this in the Equation (*a*), we get

$$\vec{g}_\psi = -g(\cos\psi\,\hat{i} + \sin\psi\,\hat{j}) + \omega^2 R\cos\psi\,\hat{i}$$

i.e. the magnitude of the apparent acceleration is

$$g_\psi = \sqrt{(g\cos\psi - \omega^2 R\cos\psi)^2 + g^2\sin^2\psi} \qquad (b)$$

Now, the value of angular velocity ω is small, hence neglecting terms involving ω^4 we have

$$g_\psi = \sqrt{(g^2\cos^2\psi + g^2\sin^2\psi - 2g\omega^2 R\cos^2\psi)} = \sqrt{g^2 - 2g\omega^2 R\cos^2\psi}$$

$$g_\psi = g\left(1 - \frac{2\omega^2 R\cos^2\psi}{g}\right)^{1/2} \approx g\left(1 - \frac{\omega^2 R\cos^2\psi}{g}\right)$$

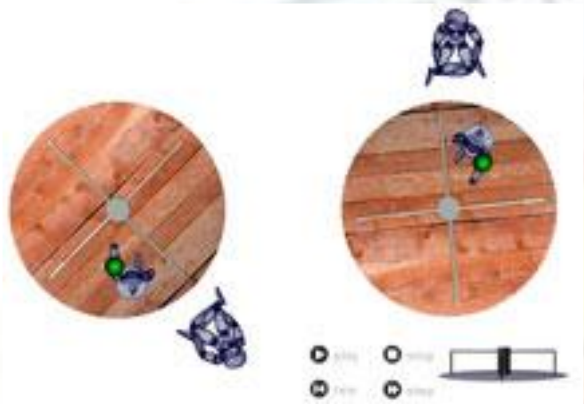
using binomial expansion and neglecting the higher order terms. Very nearly, $g_\psi = g - \omega^2 R\cos^2\psi$. This acceleration is directed towards *B* instead of *O*, the centre of the earth. So that, if the angle that this apparent direction *MB* makes with the true direction *MO* be θ, we have

$$\theta = \tan^{-1}\left[\frac{g\cos\psi - \omega^2 R\cos\psi}{g\sin\psi}\right] = \tan^{-1}\left[\left(1 - \frac{\omega^2 R}{g}\right)\cot\psi\right] \qquad (c)$$

The centrifugal force, due to the rotation of the earth reduces the effective value of g on its surface. θ comes out to be maximum when latitude $\psi = 45^\circ$.

Centrifugal Forces

Centrifugal Forces are a kind of Fictitious forces which have to be invoked to retain Newton's laws in non-inertial frames of reference particularly which are spinning.



Kindly click on the link given below to play the animation given above and also for understanding the concept of Centrifugal Forces.

http://www.phys.unsw.edu.au/einsteinlight/jw/module1_Inertial.htm#IR

b) Coriolis force Consider the case when the body is in motion relative to the rotating frame of reference of earth. Now, the fictitious Coriolis force comes into picture and two special cases are discussed below:

i) If the body is just dropped from rest so that it falls freely under the action of the gravitational force and

ii) when it is provided a large horizontal velocity (case of a projectile).

In the first case the horizontal component of the Coriolis force acting on the freely falling body deflects it a little from its truly vertical path while the vertical component, obviously, produces no such deflection but only affects the value of g. Let us determine this deflection or displacement of the falling body. Let the axes of x, y and z be along the directions east, north and vertically upwards respectively and $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along these axes. Say, *v* be the velocity acquired by the body in time *t* taken by it to fall through a height *h* and given by

$$\vec{v} = -v\vec{k} \qquad (a)$$

(- sign is due to downward direction)

$$\vec{w} = w\cos y\,\hat{j} + w\sin y\,\hat{k} \qquad (b)$$

where ψ is the latitude at the place. So, Coriolis acceleration,

$$\vec{a}_c = -2\vec{\omega}\times\vec{v} = -2\omega(\cos\psi\,\hat{j} + \sin\psi\,\hat{k})\times(-v\hat{k}) = 2\,\omega v(\cos\psi\,\hat{i} - 0) = 2\omega v\cos\psi\,\hat{i} \qquad (c)$$

i.e. the Coriolis acceleration on the body in latitude ψ is $2\omega v\cos\psi$ along the positive direction of the axis of *x* or is directed towards the east.

The equation of motion of the body may, therefore be written as

$$\frac{d^2x}{dt^2} = 2\omega v\cos\psi$$

Now, since *v* is the velocity acquired by the body in time *t*, we have, $v = 0 + gt = gt$, since its initial velocity is zero and the acceleration due to gravity, *g*. So that, $\dfrac{d^2x}{dt^2} = 2\omega\, gt\cos\psi$

So, *x*-component of the velocity of the body, say

$$v_x = \frac{dx}{dt} = \int 2\,\omega\, g\, t\cos\psi\, dt$$

Or, $v_x = 2\,\omega\, g\cos\psi\dfrac{t^2}{2} + C$ where *C* is the constant of integration. In the beginning, v_x is zero; when *t=0*, we have *C=0*

So, $v_x = \omega\, g\cos\psi\, t^2 \qquad (d)$

Integrate Equation (*d*) once again, we have

Displacement along to displacement, i.e. *x=0*, we have *C'=0*

Hence, $x = \dfrac{1}{3}\omega\, g\cos\psi\, t^3 \qquad (e)$

Now, *t* is the time taken by the body to fall through a height *h*. So that its initial velocity being zero, where height *h*

$$h = \frac{1}{2} g t^2$$

or $t = \sqrt{\frac{2h}{g}}$

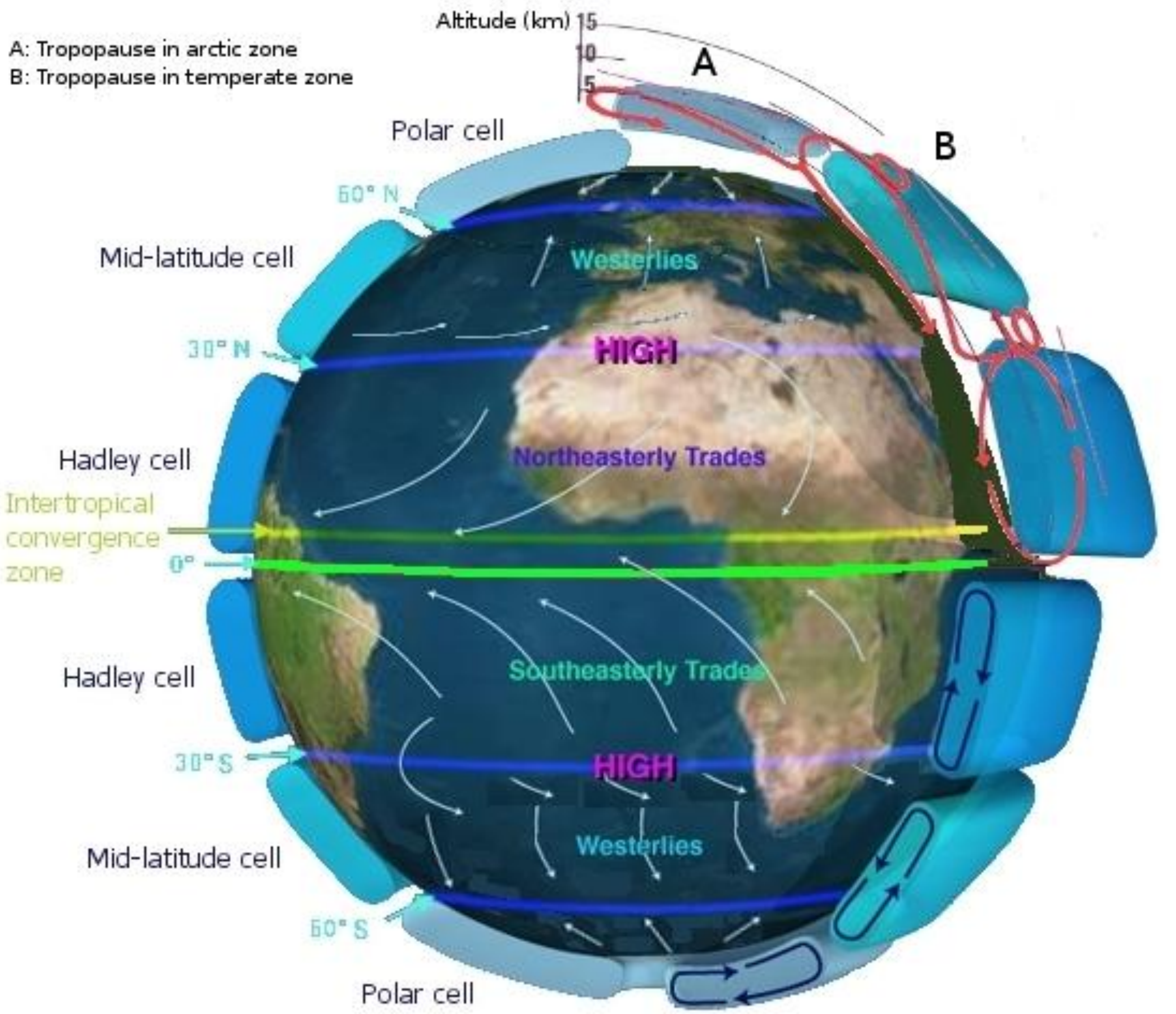
Put this value of *t* in the expression for *x* above, we have $x = \frac{1}{3} \omega g \cos\psi \left(\frac{2h}{g}\right)^{3/2} = \left(\frac{8}{9g}\right)^{1/2} h^{3/2} \omega \cos\psi$

Thus, the horizontal displacement of the body due to the coriolis force in latitude ψ is equal to $\left(\frac{8}{9g}\right)^{1/2} h^{3/2} \omega \cos\psi$.

At the equator $\psi = 0$, or $\cos\psi = 1$, it is equal to $\left(\frac{8}{9g}\right)^{1/2} h^{3/2} \omega$

i.e. the maximum. And it is always directed along the positive direction of the x-axis or towards the east.

ii) If the horizontal velocity of the body, (or the projectile) be sufficiently large, so that it covers large horizontal distances, the small Coriolis force gets sufficient time to act upon it, which makes the position vector to turn at a constant rate of $-\omega \sin\psi$. Since in the Northern hemisphere, $\psi = 0$ is positive, this rotation is clockwise (and hence the projectile gets deflected towards the right) and in the Southern hemisphere, it is anticlockwise (and the projectile thus gets deflected towards the left). This is known as Ferel’s law.

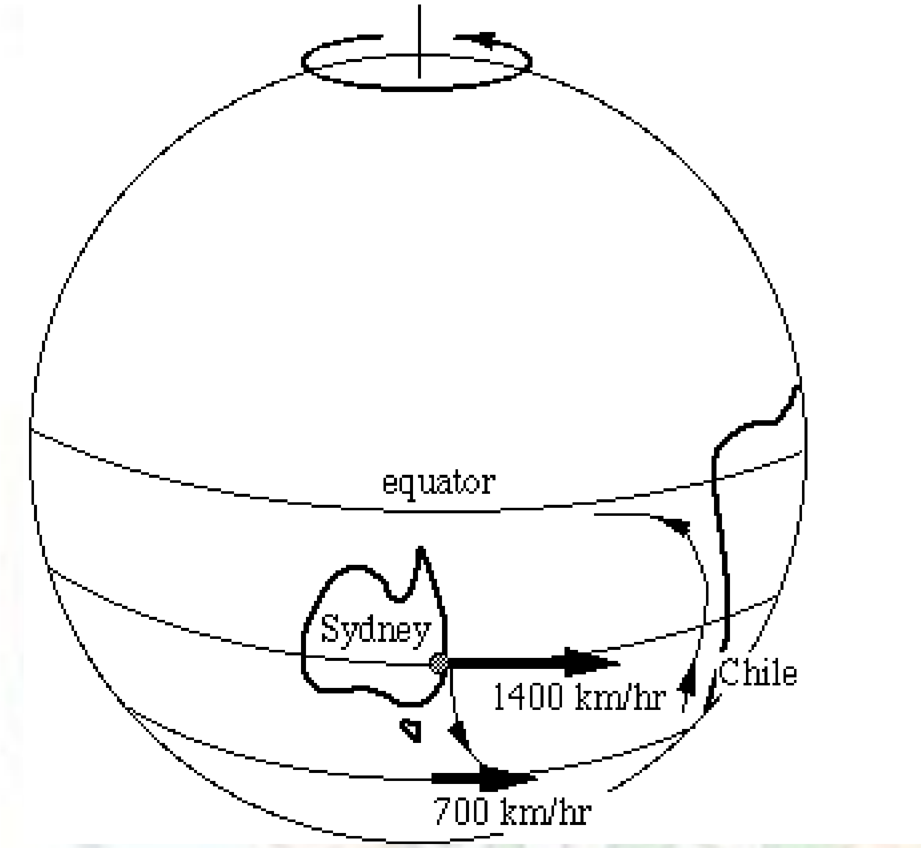


A: Tropopause in arctic zone
B: Tropopause in temperate zone
Altitude (km)
15
10
5
A
B
Polar cell
60° N
Mid-latitude cell
Westerlies
30° N
HIGH
Hadley cell
Northeasterly Trades
Intertropical convergence zone
0°
Hadley cell
Southeasterly Trades
30° S
HIGH
Mid-latitude cell
Westerlies
60° S
Polar cell

Description Global circulation of Earth's atmosphere displaying Hadley cell, Ferrell cell and polar cell.
Date
Source http://sealevel.jpl.nasa.gov/overview/climate-climatic.html
Author *This file is in the*

Coriolis Force

Coriolis forces are responsible for wind and ocean current direction causing them to move in clockwise direction in Northern hemisphere and anti-clockwise in the Southern hemisphere.



For detailed explanation of the Coriolis Forces and the above mentioned phenomena kindly go to the link below:
http://www.animations.physics.unsw.edu.au/jw/coriolis.html

Credits: J.Wolfe@unsw.edu.au, phone 61- 2-9385 4954 (UT + 10, +11 Oct-Mar).School of Physics, University of New South Wales, Sydney, Australia.

Non-inertial Frame of Reference

Summary

1. A frame of reference is a system of coordinate axes describing a particle in two or three-dimensional space. It should be quite rigid.

2. Newton's laws of motion hold good for inertial frames of reference. The basic laws of physics remain invariant in form in these types of frames of reference.

3. If the frame of reference is accelerated relative to an inertial frame, the form of basic physical laws such as Newton's second law of motion gets changed. Such frames of reference having an accelerated motion relative to an inertial frame are non-inertial frames of reference.

4. The rotating frame of reference in which a body, which is at rest (say) in an inertial frame, appears to be moving in a circle (and thus having an acceleration) is not an inertial frame of reference. This indicates that inertial frames are also non-rotating frames.

5. We know that Newton's laws of motion are obeyed only in inertial frames of reference, it follows as a natural consequence that, subject to some constraints like mass remaining constant, the relation $\vec{F} = m\vec{a}$ holds good only in inertial frames, not in non-inertial ones.

6. With the help of fictitious force, we can find out whether or not a given frame of reference is accelerated. If two frames be in uniform relative motion (zero acceleration), with respect to each other, they are obviously inertial frames, and it is not easy to detect which one is at rest and which one in motion.

7. If one of them were accelerated with respect to the other, fictitious forces come into account inside the accelerated frame. These forces are not as the true or real forces. For, unlike the latter, which go on diminishing rapidly with distance and vanish at a large distance from a body, the former arises precisely due to the accelerated motion of the frame of reference and keeps on increasing with its acceleration.

8. Whereas the real forces on particles or bodies at large distances from other bodies are negligible, the fictitious forces may have significant values. If, therefore, we find a body or a particle, far away from other material bodies, to be acted upon by an appreciable force, we can conclude that its frame of reference is an accelerated one.

Some illustrative examples

Ex1 An airplane is flying at 500 mph along a straight horizontal path in the polar region. Find the angle at which the plane is banked against the Coriolis force.

Sol. The frame of reference is the earth, which rotates 2π rad in 24 hr.

$$\omega = \frac{2\pi}{24 \times 60 \times 60} = .0000727 \text{ rad/sec}$$

Velocity = $\frac{500 \times 88}{60} = 733\, ft/\sec$

Coriolis force= $m2v\omega = \frac{2W \times 733 \times .0000727}{g} = .0033W$

where *W* is the weight of the plane. This is the horizontal component of the lift. The vertical component is the weight. Angle of bank is $\tan^{-1} .0033$.

Ex2 A body of mass 10 kg in a frame of reference, is moving vertically downwards, with an acceleration of 5m/s^2. Determine the fictitious force and the observed (or total) force (Take g=9.8 m/s^2).

Sol. Considering the earth to be an inertial frame of reference. Since the body is moving vertically downwards,

True force exerted on the body, $\vec{F} = mg = 10(-9.8) = -98.0\, newton = 98.0\, N$ downwards

And, the pseudo (or fictitious) force acting on the body, $\vec{F}_0 = m(-a_0) = 10[-(-5)] = 50.0N$ upwards

So, the observed or total force on the body $\vec{F}' = \vec{F} + \vec{F}_0 = -98.0 + 50 = -48.0\, N$ = 48.0N downwards

Thus, here fictitious force on the body is 50N upwards and the observed (or total) force on it is 48N downwards.

Ex3 A freely falling body of mass 8kg with reference to a frame is moving with a downward acceleration of 3m/s^2. Calculate the amount of total force exerted on it?

Sol. In a non-inertial frame, force $\vec{F}'$ acting on a body is given by $\vec{F}' = \vec{F}_i - \vec{F}_l$ where $\vec{F}_i$ is the force on the same body in an inertial frame and $\vec{F}_l$ is the fictitious force due to the accelerated motion of the non-inertial frame. Since the body is falling freely, downward force on it in the inertial frame of the earth is $\vec{F}_i = 0$.

So, $\vec{F}' = -\vec{F}_l$ or $\vec{F} = -m\vec{a}_l$ where $\vec{a}_l$ is the acceleration of the non-inertial frame and *m* the mass of the body. As the reference frame is moving downward with an acceleration of 3m/s^2 i.e. $\vec{a}_l = -3m/s^2$.

$$\vec{F}' = -m\vec{a}_l = -(-3 \times 8) = 24N$$

The positive sign implies the upward direction of the fictitious force.

Ex4 Find the effective weight of an astronaut ordinarily weighing 80kg when his rocket moves vertically upwards with 2g acceleration.

Sol. As the rocket moves vertically upwards with an acceleration 2g, it is a non-inertial frame and therefore the total force on the astronaut is given by $\vec{F}' = \vec{F}_i - \vec{F}_l$

where $\vec{F}_i$ is the force on the astronaut in an inertial frame and $\vec{F}_l$ is the fictitious force on the astronaut due to the acceleration of the rocket.

Now $\vec{F}_i = 80 kg\, wt = 80.g\ N$

And $\vec{F}_l = -m\vec{a}_l = -80 \times 2.gN = -160gN$

So, the effective weight of the astronaut

$\vec{F} = \vec{F}_i - \vec{F}_l = 80g - (-160g) = 240gN = 240kg$

Ex5 A bus is running due north at 150km/hr at a place where latitude is $30^\circ N$. Find the magnitude and direction of coriolis force that acts on a 2000kg

Sol. Mass of bus *m*=2000kg

Velocity $v = 150 km/hr = \dfrac{150 \times 1000}{60 \times 60} = 41.66\, m/s$ due north

Latitude of the place $\lambda = 30^\circ N$

So, $\sin\lambda = \sin 30^\circ = \dfrac{1}{2}$

Angular velocity for earth $\omega = \dfrac{2\pi}{24 \times 60 \times 60} rad/\sec$

The *X*-axis is taken towards the east, *Y*-axis towards the north and *Z*-axis in the vertically upward direction.

Coriolis force is given by $\vec{F}_{cor} = -2m(\vec{\omega} \times \vec{v})$

Horizontal component of coriolis force $(\vec{F}_{cor})_{horizontal} = +2m\omega \sin\lambda (v_y \hat{i} - v_x \hat{j})$

And the vertical component $(\vec{F}_{cor})_{vertical} = 2m\omega v_x \cos\lambda\, \hat{k}$

As the bus is running due north (along + Y-axis) the component of its velocity due east (along +*x*-axis) i.e. $v_x = 0$ and $v_y = \vec{v}_a = \vec{v}$

Therefore, Vertical component of coriolis force=0

Horizontal component of coriolis force

$$(\vec{F}_{cor})_H = +2m\omega\sin\lambda\, v_y\hat{i} = 2\times 2000\times \frac{2\pi}{24\times 60\times 60}\times\frac{1}{2}\times 41.66 = 6.05\ Newton$$

This force will act towards east.

Ex6 Find the rate of rotation of the plane of oscillation of a pendulum at latitude 60° and also calculate the time taken to turn through full right angle.

Sol. At a latitude $\lambda = 60^\circ$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$

The period of rotation $T = \frac{2\pi}{\omega\sin\lambda} = \frac{2\pi}{\omega\times\frac{\sqrt{3}}{2}} = 27.7\,hr$

Hence rate of rotation= $\frac{2\pi}{27.7}\,rad/hr$

and time taken to turn through full right angle or $\frac{\pi}{2}$ rad= $\frac{\pi/2}{2\pi/27.7} = 6.9\,hr$

Questions for practice

1. What are non-inertial frames of reference. Explain with an example.

2. Describe the fictitious forces and why they are called so? Under what conditions will an accelerated frame of reference act as an inertial frame.

3. What is a coriolis force. Explain the effect of coriolis force produced as a result of earth's rotation.

4. Calculate the values of coriolis forces on a mass of 45g placed at a distance of 5cm from the axis of a rotating frame of reference, if the angular speed of rotation of the frame be 25rad/sec.

References/ Bibliography/ Further Reading References

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