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Superposition of perpendicular harmonic oscillations by V. S. Bhasin

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Subject Paper No and Name Unit No and Chapter No and Name Name Physics PHYS 202 Oscllations and Waves IV Superposition of perpendicular harmonic oscillations Superposition of harmonic oscillations perpendicular Author and Reviewer Details (Reviewer to add the details when this document reaches him/her) Author Reviewer Photograph Name Prof. V.S. Bhasin (Retd.) Professor in Physics Dr. Amit Kumar Sh. N. K. Sehgal (Retd.) Associate Professor College/Department Deptt. Of Physics & Astrophysics, University of Delhi, Delhi-110007. Hans Raj College Contact No. 25281631 9811998182 e-mail id vsbhasin@yahoo.com nk_sehgal@yahoo.com Date of Submission December 13, 2010 January 21, 2011 Date of Second submission (pl add if any more) Oscillations and Waves Unit 4 Superposition of perpendicular harmonic oscillations 4.1 Introduction Our discussion has so far been confined to the superposition of two harmonic oscillations in one dimension. Let us consider the situation where two harmonic oscillations are perpendicular to each other. The familiar example is the motion of a simple pendulum whose bob can swing in the x - y plane. We displace the pendulum in the x direction and as we release it, we give it an impulse in the y- direction. How does such a pendulum oscillate? Clearly this would be a composite motion whose maximum x - displacement occurs when y-displacement is zero and y-velocity is maximum and similarly maximum y- displacement occurs when x -displacement is zero and x -velocity is maximum. We call such an arrangement a spherical pendulum. The frequency of the superposed simple harmonic motions will obviously be the same as it depends only on acceleration due to gravity and the length of the cord. Let us study in detail the general trajectory followed by such a composite motion. Value Addition: Animation Heading: Animation on Spherical Pendulum: Body: To view the composite motion of a spherical pendulum, the reader is recommended to visit the following

13**web site: Link: www.youtube.com/watch?v=**

7oRm4CtYn0Q Value Addition: Animation Heading: Projections of Spherical pendulum Body:

3**Link: http://www.youtube.com/watch?v=** L4_mFNUdBrg **&NR=1**

Value Addition: Animation Heading: Projections of Spherical pendulum Body:

3**Link: http://www.youtube.com/watch?v=** 6hCLkTENfSA **&NR=1**

4.2 Superposition of two perpendicular harmonic oscillations having equal frequencies: Consider two mutually perpendicular oscillation having the same frequency w0 and amplitudes a1 and a2 such that a1 > a2. These are represented by the equations, x = a1 cos wot (4.1) y = a2 cos (wot + j) , (4.2) where the initial phase along x -axis is zero and along y-axis is j . Let us find out the resultant oscillation for some specific values of phase difference j . (A) Analytical Method Case I. When j = 0 or ? . For j = 0 , x y = = a1 cos wot a2 cos wot i.e y = a a1 2 x (4.3) Similarly, for j = p , x = a1 cos wot y = - a2 cos wot i.e y = - 2 x a a1 (4.4) Thus we find from eqns. (4.3) and (4.4) that the resultant motion of the particle is along a straight line passing through the origin. [Recall, the equation of a straight line is y = m x + c, where in the present case, c = o and the slope, m, is positive in the first case and negative in the second case] For j = 0 , the motion is along the diagonal AOB shown in figure (4.1 (a)) and for j = p , the motion is along DOE shown in figure

7**(4.1 (b)) Fig.4.1 (a) Fig.4.1**

(b) Case II. When j = p / 2 In this case the two vibrations are given by x = a1 cos wot (4.5) and y = a2 cos(wot + p / 2) = - a2 sin wot (4.6) On squaring and adding the two expressions, we get x2 y2 a12 + a22 = cos2 wot + sin2 w0t = 1 (4.7) Fig.4.2 This is the standard equation of an ellipse-- showing that the resultant motion of the particle is along an ellipse whose principal axes lie along x - and y-axis. The

10**semi major and** semi- **minor axes of the ellipse** are a1 **and**

a2. From eqns. (4.6) and (4.7) we can see that as time increases, x decreases from its maximum value a1 but y becomes more and more negative. Thus the ellipse is described in the clockwise direction as is shown in the figure (Fig. 4.2). However, if we analyze the case for j = 3 p / 2 or j = - p / 2, we shall get the same ellipse but the motion will now be in anticlockwise direction (See Fig. 4.3). Fig.4.3 When the two amplitudes a1 and a2 are equal, i.e. a1 = a2 = a, equation (4.7) reduces to x2 + y2 = a2 (4.8) This equation represents a circle of radius a. Thus the ellipse reduces to a circle in this case. Let us now come back to the general case. Considering the Eqs. (4.1) and (4.2), we write eqn. (4.2) as y a2 = cos (wot +j) = cos wot cos j - sin wot sin j (4.9) From eqn. (4.1), coswot=xa,sothat sinwot = 1-x2a12 Substituting for coswot and sinwot in Eq. (4.9), we get y a2 = xcao1sj - 1-x2a12 sinj or x a1cosj- a2 = 1-x2 sinj y a 2 1 On squaring both the sides of this equation, we find x2 + y 2 a12 a22 - 2 a1 a2 x y cos j = sin2 j (4.10) This is an equation describing an ellipse whose axes are inclined to the coordinate axes. It would be instructive to show the resultant trajectories for some typical values of j . These trajectories can be easily demonstrated on a cathode ray oscilloscope (CRO). A few trajectories are shown below: Fig.4.4 (B) Graphical Method: We can also use graphical method by employing the rotating vector technique to obtain the above results. Here the locus of all the points lying on the intersection of the projection of the two rotating vectors, denoting the two perpendicular SHMs respectively, on the x-y plane represents the resultant motion. Consider two perpendicular SHMs given by x = a1 cos wot (4.11) y = a2 cos (wot + j) . (4.12) Draw two circles, one whose radius is equal to the amplitude of the first SHM denoted by Eq. (4.11), i.e., a1 and the other whose radius is equal to the amplitude of the second SHM denoted by Eq. (4.12), i.e., a2 as is shown in Fig. (4.5). Suppose O1P1 represents the position of the rotating vector at a certain instant t and its projection (OX) on the x-axis gives the instantaneous displacement ‘x’ of the rotating vector O1P1, denoted by Eq. (4.11). Fig. 4.5 Geometrical representation of the Superposition of two SHMs at right angles to each other. Similarly, the projection of the rotating vector O2 P2 on the y-axis (OY) gives the instantaneous displacement ‘y’ of the rotating vector O2P2, denoted by Eq. (4.12). Note that at any instant of time t, the displacements x=OX and y=OY represent two perpendicular SHM’s respectively. When a particle is subjected to both the SHMs simultaneously, the resultant displacement at time t would be OP where the vector OP is the vector addition of OX and OY. The intersection of the two lines, produced from points P1 and P2 parallel to the y-axis and x-axis respectively, yields the point P. The trajectory followed by the point P as a function of time t describes the resultant motion. Let us now consider a few specific cases to construct the resultant motion by choosing some values of the phase difference ? . Case (a) Phase Difference ? =0: Here the two perpendicular SHMs are given by x = a1 cos wot y ? a2 cos ?ot Fig. 4.6 Superposition of two perpendicular SHMs of the same frequency and zero phase difference One starts by dividing the circumference of each reference circle into ‘n’ equal parts. Here we have taken n=8 as is shown in Fig. (4.6) (Please note that as the value of n is increased the trajectory of the resultant motion will be more accurate but for convenience one should choose a reasonable value of ‘n’ where the Physics is also not lost and the process does not become very tedious). Since the frequency of the two SHMs is the same, hence the rotating vector in each reference circle will traverse each part in the same time, i.e., 1 8(2p /w0)= T 8 where T is the time period of one complete rotation. The points are numbered 0, 1, 2, … where the point numbered 0 denotes the time t=0 when the rotating vector O1P1 lies along the x-axis and the rotating vector O2P2 lies along the y- axis, as the phase difference ? between two SHMs is zero. The projections from the corresponding points P1 and P2 parallel to the y-axis and x-axis respectively, then result in a set of intersections representing the instantaneous positions of the point P within the rectangle of sides 2 a1 and 2a2 . The locus of these points describes a straight line AB with a positive slope given by a2/a1. Case (ii); Phase ? = ? / 4 Fig. 4.7 Superposition of two perpendicular SHMs of the same frequency and phase difference of ?/4. In this case, the points 0, 1, 2, … are numbered starting with time t=0 when rotating vector O1P1 is parallel to the x-axis denoting zero phase difference while the rotating vector O2P2 is at an angle ? = ? / 4 from the y=axis denoting a phase difference of ? / 4 , measured in counterclockwise sense as shown in Fig. (4.7). As discussed before, for obtaining the resultant motion of the particle, locus of all points of intersection of the two lines drawn parallel to the y-axis and x-axis from the points P1 and P2 respectively, at successive time intervals, is taken. In the present case the locus of these points is an ellipse, described in the clockwise sense, and it makes an angle of ? / 4 with the x-axis as shown in Fig. (4.7). The point P moves within the rectangle of sides 2 a1 and 2a2 . Value Addition: Activity Heading: Graphical Activity resultant trajectory. Body: In a cathode ray oscilloscope, the deflection of electrons by the superposition of two mutually perpendicular electric fields given by x ? 3cos (? ot) and y = 3cos (wot + p 3) Draw the resultant trajectory of the electrons. Use also the graphical method to draw the Solution: Using Eq. (4.10)

8**x2 + y2 2** x y **a 2 a 2 - a 2** x2 + y2 **x**

y 9 9 - 9 x2 + y2 - x y = 27 4 cos p 3 = sin2 p 3 3 = 4 j = p / 3 This is again an equation of ellipse whose axes are inclined to the coordinate axes by an angle of p . Follow the procedure described above to draw the trajectory using graphical 3 method. 4.3 Superposition of Two (Rectangular) Mutually Perpendicular Harmonic Oscillations of Different Frequencies: Lissajous Figures Let us now study the case when the two orthonormal harmonic oscillations havedifferentfrequencies. Heretheresultingmotionismorecomplex. Thisisbecausethe relative phase, j=w2t-w1t+jobetween the two vibrations will now be time dependent and would therefore gradually change with time. This makes the shape of the figure to undergo a slow change. The patterns which are thus traced out are called Lissajous figures. Lissajous figures can be traced on a cathode ray oscilloscope when two different alternating sinusoidal voltages are applied on the defection plates, XX and YY, of the CRO. The electron beam would then trace the resultant trajectory on the fluorescent screen. Frequencies in the ratio 2:1 (A) Analytical Method: Let us study the situation when two perpendicular harmonic vibrations represented by x = a1 cos(2 wot + j1) (4.13) and y = a2 cos wot , (4.14) where the two frequencies are in the ratio 2:1. To find the resultant motion, we consider threecases: Case(i);when j1=0: x=a1cos2wot= a1(2cos2wot-1) and y=a2coswot Thisgives x a1 = 2ay222 - 1. Let us arrange this equation as y2 = 2aa221 (x + a1) (4.14) This is an equation of a parabola (y2=ax+b), which can be traced as is shown in the figureFig.(4.8). Fig.(4.8) Case (ii) when j1 = p / 2 x = - a1 sin 2 wot , or - a1 x = 2 sin wot cos wot and y = a2 cosw0t or coswot = y a 2 and sinwot = 1- y2 a22 Substituting for cos wot and sin wot in the equation for x , we get - x a1 = 2 y a2 1- y2 a22 On squaring both the sides, we have 4 ay222 ay222 - 1) + ax122 = 0 (This is an equation, which is fourth order in y and second order in x . Therefore, it is expected to have, in general, four roots in y (intersecting parallel to y-axis at 4 points) and two roots in x (intersecting two points on lines parallel to x -axis). Thus, this equation represents a figure of ‘8’ (Fig. (4.9)). Fig.4.9 Case(iii) when j1 = p : x = -a1 cos 2 wot and y = a2 cos wot i.e. - x a1 = 2 cos2 wot -1 = 2 y2 a22 -1 2 or y2 = - 2 (x - a1) a 2a1 Fig.4.10 This equation again represents a parabola (Fig.4.10) but just opposite to the case shown in Fig. (4.8) when j1 = 0 . (B). Graphical Method: The analysis given above shows clearly that the analytical method becomes much involved for values of phase constant j1 other than zero. Using the graphical method we shall find, that the resultant motion can be constructed quite conveniently. Let us consider the case when the two SHMS are given by y = a2 cos wot Fig. (4.11) given below shows how the rotating vector technique is used to obtain the shape of the Lissajous figure when j1 = p / 4 and the two frequencies are in the ratio 2:1. The rotating vector O1P1 makes an angle ? / 4 at time t=0 with the x-axis to show that x oscillation has an initial phase of ? / 4 . However, at this instant of time the rotating vector O2P2 just coincides with the y-axis to represent that the y oscillation has initial phase zero. Since the x oscillation has frequency twice that of the y- oscillation, therefore the circumference of the reference circle with origin O1 and radius a1 is divided into 8 equal parts and the circle with origin O2 and radius a2 is divided into 16 equal parts. In this way a frequency ratio of 2:1 of the two perpendicular SHMs is achieved. The time period in which rotating vector O2P2 completes one rotation the rotating vector O1P1 will execute two complete rotations. Fig. 4.11 Superposition of two perpendicular SHMs with the frequency in the ratio 1:2 and phase difference equal to ?/4. It is now a simple exercise to construct the resultant motion corresponding to other phase differences. The following figure (Fig. (4.12)) shows the sequence of these motions for values of phase j1 in the range 0 to ? . Fig. 4.12 Lissajous figures: ?1=2?2 with various initial phase differences Value Addition: Biographical Sketch Heading: Biography of Jules Antoine Lissajous Body: Jules Antoine Lissajous mathematician who was born on March 4,1822 at Versaillas, France. Lissajous graduated from the Ecole Normale Superieure in 1847 in Paris. He is known for many innovations but perhaps the best known is a special apparatus – the Lissajous apparatus--- that creates the variety of figures which are now named after him as the Lissajous figures. This apparatus made him realize, in a way, his idea of making 'sound waves visible'!. This apparatus uses two tuning forks with mirrors attached to them , mounted in mutually perpendicular directions. By making the light reflected from the mirror of the first tuning fork , to fall on the mirror of the second fork and then getting the light reflected from this on the screen, one can see the resulting Lissajous figures on the screen. It is possible to vary the frequency ratio and the phase difference between the two oscillations and see the effects of these on the patterns of the Lissajous figures obtained. Lissajous was a French physicist and Lissajous apparatus led to the invention of other apparatus such as the harmonograph. It is interesting to note that many of the designs, used by fabric and cloth designers, can be correlated with Lissajous figures corresponding to different mutual orientations and to different frequency, phase and amplitude ratios of the two superimposed simple harmonic motions. Image Coutsey: en.wikipedia.org/wiki/Jules_Antoine_Lissajous Value Addition: Animation Heading: Lissajous Figures Activity For studying the Lissajous figures on an oscilloscope with variation of the ratio of the two orthonormal frequencies the reader is suggested to visit the link given below:

> 6**http://www.animations.physics.unsw.edu.au/jw/** Lissajous **.htm** Credits: Authored **and**

Presented by Joe Wolfe ,

> 4**School of Physics, Faculty of Science,** The **University of New South Wales, Sydney, Australia. Multimedia**

> 5**Design by George Hatsidimitris. Laboratories in Waves and Sound by John Smith.**

Animation Idea Show a mirror attached to one of the prongs of a tuning fork. Let a beam of light fall on this mirror. Let the reflected beam fall on the prong of a second tuning (identical) fork oriented perpendicularly and again having a mirror attached to it. Let the reflected beam now fall on a screen. Make both the tuning forks vibrate simultaneously. See the figure on the screen. Change the figure by (i) making one fork vibrate and (ii) letting the other fork vibrate a little late (vary this interval and get different figures). The animation idea given above can be viewed by visiting the following webpage:

> 2**http://www.hps.cam.ac.uk/whipple/explore/acoustics/lissajoustuningforks/**

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whipple/explore/copyrightinformation/ Summary In this chapter, the reader learns to ‣ Apply the principle of superposition to two mutually perpendicular harmonic oscillations of equal frequency for different phases using analytical method; ‣ Use graphical method to study the resultant motion for different phases; ‣ Study superposition of two mutually perpendicular SHMS of frequencies in the ratio 1:2 and of different phases analytically. This enables the reader to study the formation of Lissajous figures; ‣ Apply graphical method to trace the trajectory of the resultant motion of two perpendicular SHMs of frequencies in the ratio1:2 for phase differenc zero and ? / 4 . Problems 1. Two tuning forks A and B of frequencies close to each other are used to obtain Lissajous figure. It is observed that the figure goes through a cycle of changes in 20s. If A is loaded slightly with wax, the figure goes through a cycle of changes in 10s. If the frequency of B is 256 Hz, what is the frequency of A before and after loading? Solution : Period of 20 s corresponds to the frequency of 0.05 Hz. nA-nB=±0.05Hz On loading the prong of A with wax, the frequency of A will decrease. But the cycle of changes is completed in 10 s, i.e the frequency difference increases to 0.1Hz. Therefore the frequencies of A before and after loading are respectively 256 -0.05 = 255.95 Hz and 256 – 0.1 = 255.9 Hz. 2. When two simple harmonic motions x=a1cosw0t and y=a2cos(w0t+j) are superimposed, what is (a) the basic difference in the two trajectories when j = p / 2 and -p /2; and (b) when a1 > a2 and a1 < a2 ? Solution. (a) In both the cases the figure is an ellipse. However, when j =p /2, the ellipse is described in the clockwise direction and when j =-p /2, it is in the anti- clockwise direction. (b) When a1>a2 it implies

12**semi major x-axis** a1 is greater **than** the **semi- minor y-axis**

a2 and for a1<a2 semi major axis is

11**along the** y **-axis and semi** minor **axis** is **along the** x- **axis.**

3. In a cathode ray oscilloscope, the deflection of electrons by the superposition of two mutually perpendicular fields is given by x=4cosw0t , y=4cos(w0t+j). Analyze the resultant trajectories followed by the electrons when j=p /4 and j=5p/4. Solution: Using the equation (4.10) , the resultant trajectory follows the path given by x2 a12 + y 2 a22 - 2 ax1 ay2 cos j = sin2 j For a1 = a2 = 4 and j = p / 4, we get x 2 + y 2 - 2 xy cos p / 4 = 16 sin 2 j i.e. x2 + y2 - 2xy =8. 1 2 In the second case, when j = 5p / 4 x2 + y2 -2xycos5p / 4

=16sin2j i.e, x2 + y2 + 2xy =8. 1 2 The two trajectories follow the paths as given in Fig.4.4 for the cases j = p / 4 and j = 5p / 4 . 4. Use the graphical method to trace the trajectories of the resultant motion of problem 3 given above. 5. Consider the superposition of two harmonic waves given by the equations x = a sin (wot) y = a sin(3 wot + j1) , when j1 = p 2 Draw the Lissajous figure representing the resultant motion. Solution: y a ? sin(3?0t ?? /2)?cos(3?0t) ? (3cos?0t ? 4cos3 ?0t) or ? 1?sin2 ?0t(3?4(1?sin2 ?0t)) y a ? 1?x2 /a2 (4x2 /a2 ?1) On squaring both sides, we get ay22 ?(1?ax22)(4ax22 ?1)2 ThecorrespondingLissajousfigureisasshown. 6. How would the figure drawn above change, if we consider the superposition of two harmonic waves given by the equations x = acos(3w0t +j1), y =acos(w0t) , when j1 =p / 2? Solution:

14**It is easy to** show **that the** roles of **x-** and **y-**

axes have interchanged. That means the above curve would now be rotated by 90degress. Prof. V. S. Bhasin Paper No. PHYS 202 Unit No. I Chapter No. Prof. V. S. Bhasin Paper No. PHYS 202 Unit No. I Chapter

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