Discipline Course-I
Semester -I
Paper: Mechanics IB
Lesson: Rotational Dynamics / Mechanics-III
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Table of Contents

Unit IV: Rotational Dynamics

Lesson 3

3.1 Introduction

In the preceding lesson (Lesson 2, Rotational Dynamics), we studied how a torque acting on a rotating object about a given axis can be expressed in terms of its moment of inertia. We shall here extend this study to establish the relation between angular momentum and moment of inertia of rigid bodies. This lesson also highlights a few examples from everyday life demonstrating the principle of conservation of angular momentum.

Objectives

After studying this lesson you should be able to

- derive the relationship between angular momentum and moment of inertia of a rigid body in terms of its angular velocity of rotation
- describe an example when the direction of angular momentum vector is not along the direction of the angular velocity vector
- define the radius of gyration of a rigid body
- state the condition under which angular momentum of a body is conserved
- describe various examples demonstrating the conservation of angular momentum
- know how a gyroscope works

3.2 Relationship between Angular Momentum and Moment of Inertia of a Rigid Body

Consider a rigid body of any arbitrary shape rotating with an angular velocity $\vec{\omega}$ about an axis AB passing through a point O, as shown in Fig.3.1. All the particles of the body will move in circular path about the axis AB. Let us focus on the particle P located at any distance $\vec{r}$ from the point O. Let the perpendicular drawn from P on AB be PC= r_0. Then C will be the centre of the circle described by the point P. The linear velocity of the particle P is given by

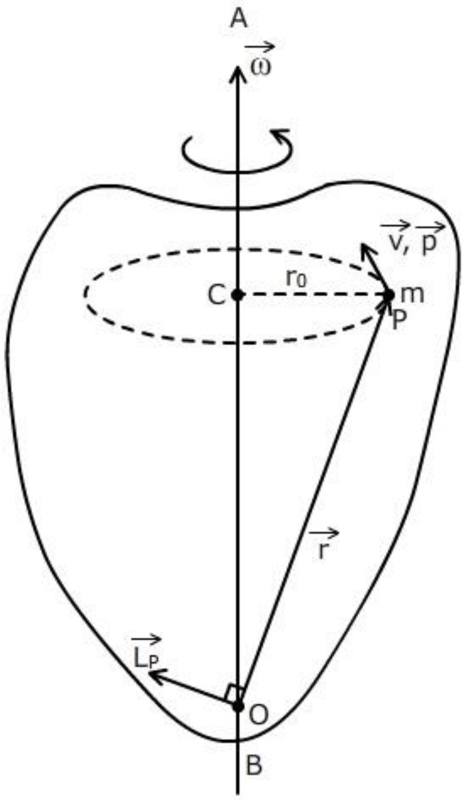

Fig.3.1

$$\vec{v} = \vec{\omega} \times \vec{r} \;. \qquad (3.1)$$

Its magnitude is $\omega r \sin\theta = \omega r_0$ and direction at any instant is perpendicular to the position vector $\vec{r}$ and tangential to the circular path.

The angular momentum of the particle P about the point O is given by

$$\vec{L}_P = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \quad , \qquad (3.2)$$

whose direction is perpendicular to $\vec{r}$ *and* $\vec{v}$.

The angular momentum $\vec{L}$ of the entire body about the point O will be obtained by the vector sum of the angular momenta for all the particles of the body i.e.,

$$\vec{L} = \sum \vec{L}_P = \sum m\vec{r} \times \vec{v} = \sum m\vec{r} \times (\vec{\omega} \times \vec{r}) \quad . \qquad (3.3)$$

The direction of the angular momentum $\vec{L}$, in general, will not be along $\vec{\omega}$.

The Eq.(3.3) can be further simplified, using the standard identity,

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

which reduces Eq.(3.3) to

$$\vec{L} = \sum m\left[(\vec{r}.\vec{r})\vec{\omega} - (\vec{r}.\vec{\omega})\vec{r}\right] \tag{3.4}$$

Or
$$\vec{L} = \sum m\left[(r^2)\vec{\omega} - (r\omega\cos\theta)\vec{r}\right] \tag{3.5}$$

Since θ is the angle between vectors $\vec{r}$ and $\vec{\omega}$, the magnitude of the component of $\vec{r}$ along $\vec{\omega}$ will be r cos(θ). So the magnitude of the component(L_ω) of the angular momentum $\vec{L}$ along the axis of rotation will be given by

$$L_\omega = \sum m(r^2\omega - \omega r\cos\theta(r\cos\theta)) = \sum m r^2\omega(1-\cos^2\theta)$$

$$= \omega\sum m r^2 \sin^2\theta = \omega\sum m r_0^2 \tag{3.6}$$

Notice that the distance $r_0 = r\sin\theta$, i.e., the perpendicular distance of the particle from the axis of rotation will be different for different particles. That is why it is within the summation sign. Further, since the angular velocity, $\vec{\omega}$, about the axis of rotation is the same for all the particles in the rigid body, it is outside the summation sign.

The sum $\sum m r_0^2$ in equation (3.6) represents the moment of inertia, I, of the body about the axis of rotation. Eq.(3.6), thus, gives an important relation between the component of angular momentum $\vec{L}$ along the axis (OA) and the angular velocity $\vec{\omega}$ about the axis of rotation, viz.,

$$\vec{L}_0 = I\vec{\omega} \tag{3.7}$$

In the case of symmetrical object which is allowed to rotate about the axis of symmetry, the component of $\vec{L}_P$ perpendicular to OA will be cancelled by an equal amount of the angular momentum of another particle on the opposite side of the dotted circle. In such cases, the net result is that the total angular momentum of the body will be along the axis of rotation, giving

$$\vec{L} = I\vec{\omega} \tag{3.8}$$

Note that the relations given by Eqs.(3.7) or (3.8) for rotational motion are just the counter part of the relation $\vec{p} = m\vec{v}$, expressing linear momentum in terms of velocity for translational motion.

3.2.1 Value Addition*:* *Example when angular momentum vector is not along the direction of angular velocity vector of a rotating body*

From Eq.(4.3) obtained above it has been noticed that the direction of the angular momentum vector $\vec{L}$ of a rotating body may not necessarily be along the angular velocity vector $\vec{\omega}$. Do you know of any example which can illustrate such a situation, where the angular momentum vector is not along the angular velocity vector?

Consider a wheel which is fixed to a shaft in a lopsided manner, but ensuring that its axis is passing through its centre of gravity as shown in the figure.

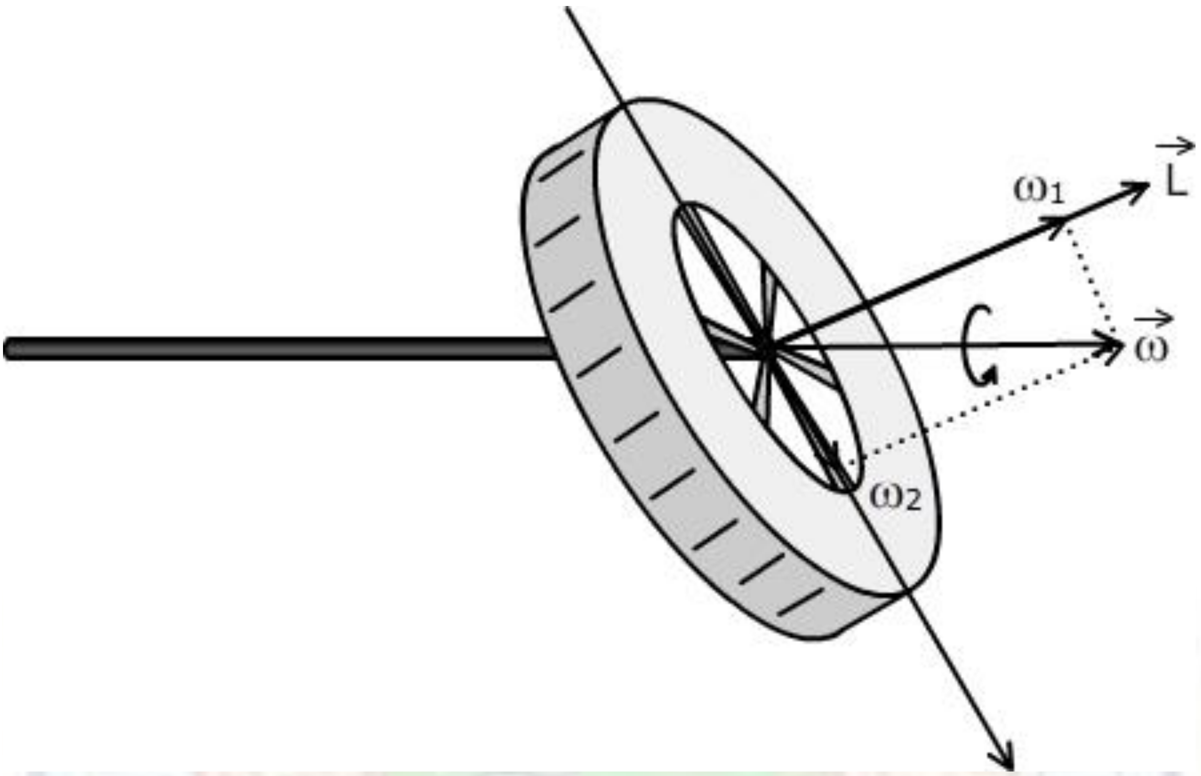

Fig. 3.1(A)

When you spin the wheel around the axis, you will find that there will be shaking at the bearings because of the lopsided way it has been mounted. You know that in the rotating system there is a centrifugal force acting on the wheel which tries to push its mass away from the axis. This would tend to bring the plane of the wheel perpendicular to the axis. To resist this, a torque is exerted by the bearings. How has a torque been generated here? To answer this, let us resolve the angular velocity vector into two parts ω_1 *and* ω_2 perpendicular and parallel to the plane of the wheel. Now, since the moments of inertia of the wheel about these axes are different, the corresponding angular momenta about these axes would also be different. And the torque is nothing but the rate of change of angular momentum. Thus when you turn the wheel, you have to turn the angular momentum vector in space, thereby exerting torque on the shaft.

3.3 Radius of Gyration

It is always possible, independent of the shape of a body, to find a distance from the axis of rotation at which whole mass of the body can be taken to be concentrated so that its moment of inertia about the axis remains the same. Thus if K is the distance from the axis of rotation to the point where whole mass of the body is supposed to be concentrated, then

$$I = MK^2 = \sum mr^2 \qquad (3.9)$$

Or $$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\sum mr^2}{M}}$$

This quantity, K, is called the **radius of gyration** of the body about the axis of rotation. **It is defined as the distance from the axis of rotation, the square of which when multiplied by the total mass of the body gives the moment of inertia of the body about that axis.**

3.4 Conservation of Angular Momentum

We have already seen that the rate of change of angular momentum gives us the torque (cf., Eq.(1.11) in Lesson 1, Rotational Dynamics), i.e.,

$$\vec{T} = \frac{d\vec{L}}{dt} \qquad (3.10)$$

Using this general relation and substituting for $\vec{L}$ from Eq.(3.8), we get

$$\vec{T} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} \qquad (3.11)$$

This is the same expression as obtained earlier (cf., Eq.(2.14) in Lesson 2, Rotational Dynamics). According to this, torque acting on the object is equal to the time rate of change of angular momentum of the object. This is the rotational analogue of Newton's second law $\vec{F} = d\vec{p}/dt$.

When the net torque acting on the system is zero, we see from Eq.(2.14)

$$\frac{\Delta\vec{L}}{\Delta t} = 0,$$

implying thereby that the angular momentum remains constant in time. In other words,

$L_i = L_f$ if net torque acting on the system is zero.

The angular momentum of the system is conserved when the net external torque is zero.

You must have studied in your earlier classes various examples of conservation of angular momentum, which applies to both macroscopic objects such as planets as well as to atoms and molecules.

A simple but well known example demonstrating the conservation of angular momentum is a man standing on a turn table, holding its arms extended (see Fig.(3.2(a)) with a weight in his each hand. The turn table is free to rotate. To start with, suppose his friend sets him in slow motion. Now as he brings his hands inwards close to his sides, he finds that he starts rotating much more rapidly (Fig.3.2(b)). As he draws inward the two weights, the moment of inertia of the weights gets considerably reduced since the distance of the weights from the body is much smaller than that in the earlier case. A reduction in the value of moment of inertia I has brought about a corresponding increase in the value of ω. From the relation, L=Iω(cf., Eq.(3.8)), it is easy to infer that the increase in ω as a result of decrease in the value of moment of inertia I is such so as to maintain the angular momentum constant .

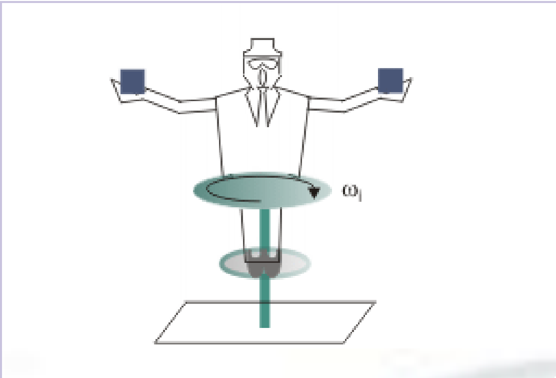

Fig. (3.2(a))

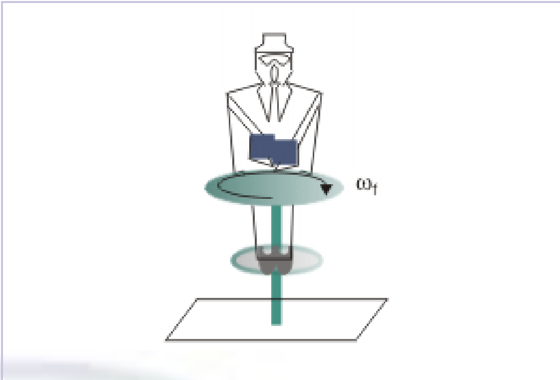

Fig. (3.2(b))

Value Addition: The animation given below demonstrates the phenomenon depicted in the above figure:

http://www.animations.physics.unsw.edu.au/jw/rotation.htm#rolling

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Multimedia Design by George Hatsidimitris

Laboratories in Waves and Sound by John Smith

3.4.1 Value Addition: **A Puzzling Question**

In the example , discussed above, we stated that since there was no torque about the vertical axis, angular momentum is conserved, i.e., $I_1\omega_1 = I_2\omega_2$. With our arms pulled in, since the moment of inertia gets reduced, angular velocity has increased. But what about the energy? With our arms pulled in, we turn faster. As a result, our energy has increased from the previous position, although angular momentum remains conserved. If we compare the kinetic energy before and after, the kinetic energy before is

$$\frac{1}{2}I_1\omega_1^2 = \frac{1}{2}J\omega_1, \qquad where \quad L = I_1\omega_1 = I_2\omega_2$$ is the angular momentum.

Afterward, we have kinetic energy , KE=$\frac{1}{2}L\omega_2$. Since $\omega_2 > \omega_1$, clearly the kinetic energy of rotation has increased. So, **what about the conservation of energy**? Have we done any work? When we lift a body, we do work against gravity. But here we move a weight horizontally, we do not do any work. If we hold a weight and pull in, we do not do any work. However, that is true only when we are not rotating. **When we are rotating, there is a centrifugal force on the weights, which are trying to move out. So while we are rotating, we have to pull the weights in against the centrifugal force.** Thus the work we do against the centrifugal force must account for the difference in kinetic energy.

Example

Consider a circular platform of mass M=100kg and radius R=2.0 m rotating in a horizontal plane about a frictionless vertical axle (Fig.3.3). This is called Merry-Go-Round. Let the angular speed of the system be2.0rad./s. Suppose there is an object lying on this rotating platform and in order to get that you walk slowly from the edge towards the centre to get the object, find the angular speed when you reach a point 0.5 m from the centre. [Take your mass m=60.0 kg and neglect the mass of the object.]

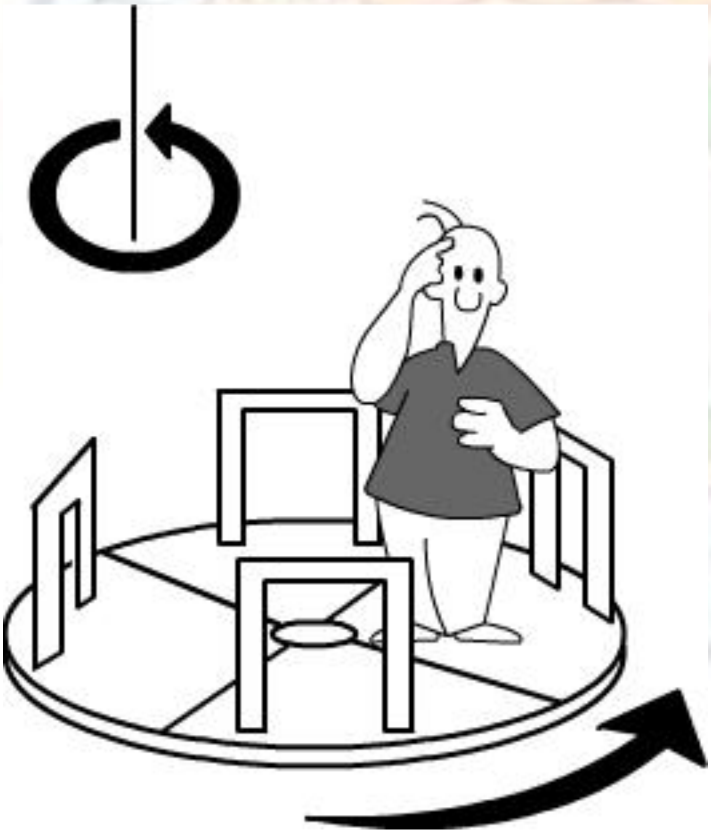


Figure.3.3

Reasoning Use the principle of conservation of angular momentum . The initial angular momentum of the system is the sum of the angular momentum of the platform plus your angular momentum when you are at the edge of the merry-go-round. The final angular momentum is the sum of the angular momentum of the platform plus your angular momentum when you are 0.50 m from the centre.

Solution

Moment of Inertia of the platform is $I_P == \frac{1}{2}MR^2 = \frac{1}{2}(100kg)(2.0m)^2 = 200kg.m^2$.

Assuming you are point particle, your initial moment of inertia is, $I_M = mR^2 = (60kg)(2.0)^2 = 240kg.m^2$.

The total initial angular momentum is $L_i = (I_P + I_M)\omega_i = (440kg.m^2)(2.0rad/s) = 880kg.m^2/s$.

After you have walked to the position 0.50 m from the centre, your moment of inertia is

$$I_M^f = mr_f^2 = (60kg)(0.5m)^2 = 15kg.m^2.$$

Note that since there is no external torque acting on the system about the axis of rotation, there is no change in the moment of inertia of the platform.

Using law of conservation of angular momentum, i.e., $L_i = L_f$

$$880kg.m^2/s = 200\omega_f + 15\omega_f$$

which gives $\omega_f = 4.09rad/s$

Thus by reaching the point 0.5 m your angular speed has nearly doubled.

3.4.2 Value Addition

Do you know how a gyroscope works?

During your childhood, you must have played with a spinning top or must have seen your friend playing with it. Do you remember having noticed the precession of the top? Have you ever thought; what is the reason of its precession? A rapidly spinning top experiences a force F due to gravity acting vertically downwards on its centre of mass. This furnishes a torque T about the point of contact with the floor as is shown in the figure (Fig.3.4 (A)). This torque is in the horizontal direction and causes the top to precess with its axis moving in a circular cone about the vertical.

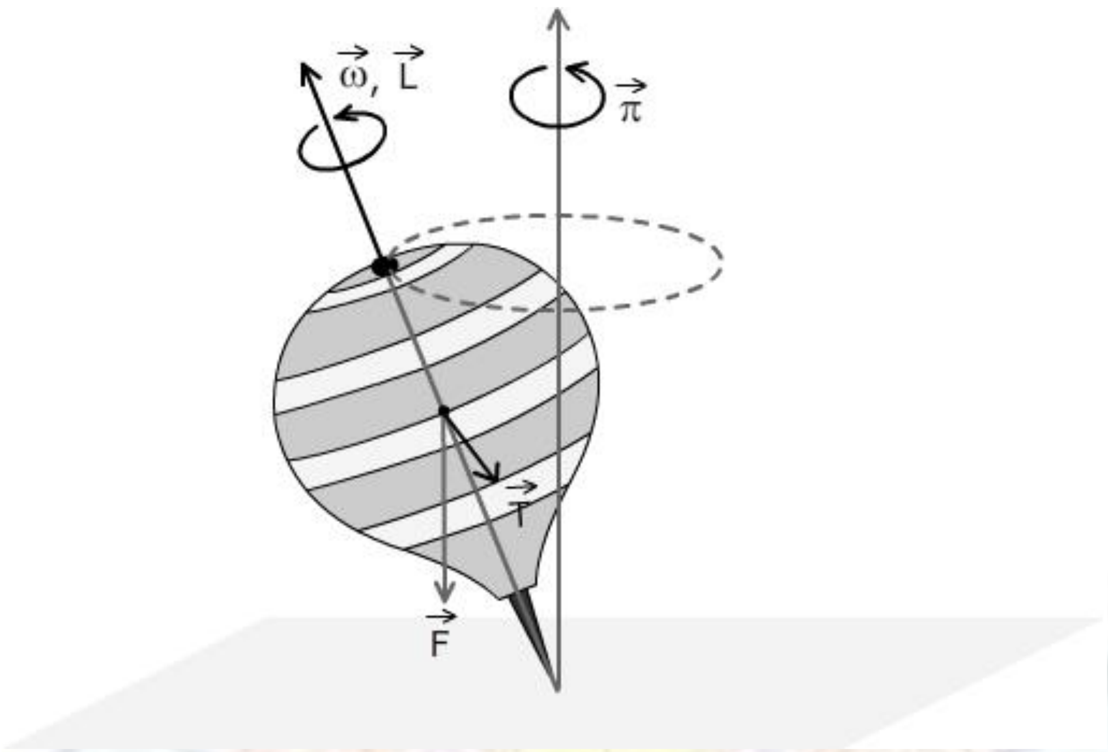


Fig. 3.4(A)

In

Precession

Let's apply **τ** = d**L**/dt to the motion of a rapidly rotating object, such as the wheel in the movie at right. At the moment shown by the upper still picture below the movie frame, the wheel is spinning clockwise when viewed from the left, so its angular momentum **L** is to the right, as the arrow shows. If we consider torques about the centre of the wheel, the weight exerts no torque about this point, but the string exerts an upwards force **F** displaced by **r** from that point, so the torque **τ** = **r** X **F** due to the string is in the direction shown. Now Δ**L**, the change in angular momentum, must be parallel to **τ**, so the **L**, which lies along the axle as shown, must move outwards towards the viewer. Further, the torque is always (approximately) perpendicular to **L**, so the motion is circular – it is called **precession**.

How fast does it precess? If we make the approximation that the shaft is horizontal, then the angle dφ through which it precesses in time dt is just

$d\varphi = dL/L \quad \text{so} \quad d\varphi/dt = (dL/dt)/L = \tau/L$

The precession rate is proportional to the torque, so increased weight makes it precess faster. But either increased mass or increased spin would increase L and thus make it precess more slowly.

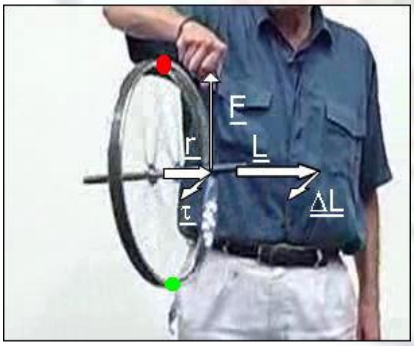
A warning: torque and angular momentum behave differently from some other vectors with regard to symmetry. For example, imagine a mirror placed to the right of this photo, and with its normal pointing to the left. The mirror image of the wheel would have an angular momentum pointing to the right. For this reason, torque and angular momentum are sometimes called pseudo-vectors.

Gyroscopes

A gyroscope consists of an object with substantial angular momentum – which usually means at has a reasonably large moment of inertia and that it is spinning with a large angular velocity. It often has a gimbal mounting, as is the case here: its axle is mounted, with low frictional torque, in a frame with an axle at right angles to it, and that mounting is mounted in another frame whose axle is again at right angles, again with low frictional torque. This allows the fast frame to be rotated in any direction with respect to that of the axis of the gyroscope without exerting much torque on the gyroscope.

Consequently, the angular momentum of the gyroscope is (approximately) conserved in an inertial frame. So, for example, an ideal gyroscope whose axis of rotation points at a distant star would continue pointing towards that star, even if the vehicle/aircraft etc. in which it was mounted turned, pitched or yawed many times.

the case of a gyroscope, watch the motion of the gyroscope in the animation shown in the website on the figure.



http://www.animations.physics.unsw.edu.au/zipped/rotation_gyro.zip

http://www.animations.physics.unsw.edu.au/zipped/rotation_wheel.zip

Credits: Authored and Presented by Joe Wolfe

Multimedia Design by George Hatsidimitris

Laboratories in Waves and Sound by John Smith

For further details, visit the following websites:

http://physics-animations.com/Physics/English/mech.htm

http://www.youtube.com/watch?v=TUgwaKebHTs

http://commons.wikimedia.org/wiki/Category:Gyroscope_animations

3.4.3 Value Addition: Do you know?

Conservation of Angular Momentum in Astrophysics

An interesting example of conservation of angular momentum can be found in astro-physics. When a massive star, at the end of its lifetime, collapses (having used up all its fuel) under the influence of gravitational forces, it causes a huge outburst of energy called a supernova explosion. The best known example of a remnant of supernova

Supernova explosion-bing videos

explosion is the Crab Nebula (See the videos on the website given here) in the form of chaotic, expanding mass of gas.

A part of star's mass in a supernova is released into space where it gets condensed into new stars and planets. Most of what is left behind collapses into what is called a neutron star. A neutron star is an extremely dense matter in a spherical shape with a diameter of about 10 km. Imagine this great reduction from the 10^6 km diameter of the original star! And yet it contains a large fraction of the star's original mass. As the moment of the system decreases during the collapse, the star's rotational speed increases. Indeed more than 700 rapidly rotating neutron stars have so far been identified. Their periods of rotation vary from millisecond to several seconds.

Summary

In this lesson you have studied

- to derive the relation of angular momentum of a rigid body rotating about a given axis in terms of its angular velocity and the moment of inertia
- define the radius of gyration of a rigid body
- state the condition under which angular momentum of a body is conserved
- describe examples from everyday life illustrating the conservation of angular momentum

Exercises:

1. Why does the moment of inertia of a given body depend on the axis of rotation about which it rotates? Explain.

Answer: Because the distribution of mass of the body varies from one axis of rotation to another. As a result, the effective distance from the axis of rotation to the point at which the whole mass of the body is assumed to be concentrated varies from one axis to another. As for example, the moment of inertia of a linear (thin, one-dimensional) uniform rod of mass M and length L about an axis passing through its centre of mass and perpendicular to its length is found to be $ML^2/12$, whereas its moment of inertia about the axis passing through one end of the rod and perpendicular to the length is given by $ML^2/3$.

2. From the statements given below, mark the ones which are true/ false. Also, justify your answer.

(a) The angular momentum vector of a body is, in general, parallel to its angular velocity vector.

(b) For symmetrical objects, rotating about the axis of symmetry, the angular momentum vector would be parallel to the angular velocity vector.

(c) If a flywheel is tilted from its axis of rotation and made to rotate, there would be a torque acting on it.

Answer: The statement (a) is false. From the expression, Eq.(3.3), obtained above in the text, it is clear that the angular momentum, $\vec{L}$, of a body is , in general, not parallel to the angular velocity, $\vec{\omega}$. Statement (b) is true, because , in the case of symmetrical object which is allowed to rotate about the axis of symmetry, the perpendicular component of one particle in the body will be cancelled by an equal amount of the angular momentum of another particle on its opposite side. In such cases, the net result is that the total angular momentum of the body will be along the axis of rotation. Statement (c) is also true. This is because the moments of inertia of the wheel about the axes, parallel and perpendicular to the symmetry axis are different, the corresponding angular momenta about these axes would also be different. And the torque is nothing but the rate of change of angular momentum. Thus when you turn the wheel, you have to turn the angular momentum vector in space, thereby exerting torque on the shaft.

3. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect, which is at rest at a point near the rim of the disc, starts moving along a diameter of the disc to reach the other end. During the journey of the insect, the angular speed of the disc:

(a) continuously increases

(b) first increases and then decreases

(c) remains unchanged

(d) continuously decreases

Answer: Since there is no external torque, angular momentum of the system is conserved, i.e., $I_1 \omega_1 = I_2 \omega_2$. As the insect approaches the centre, its radius decreases, therefore the angular velocity of the disc would increase. And as the insect moves away from the centre to reach the opposite end, the angular velocity of the disc would decrease. Correct choice is (b).

4. (a) A boy stands at the centre of a turntable with his two arms stretched out. The turntable is set rotating with angular speed of 60 rev,/min. How much is the angular speed of the child if he holds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

(b) Show that boy's new kinetic energy of rotation is more than the initial kinetic energy of rotation.

(c) How do you explain this increase in kinetic energy?

Solution

According to conservation of angular momentum,

$$L_i = I_i \omega_i = L_f = I_f \omega_f$$

Since $I_f = (2/5) I_i$,

Therefore, (a)
$$\omega_f = (5/2)\omega_i = (5/2)\ 60\ rev/\min$$
$$= 150\ rev./\min$$

(b) Kinetic energy of rotation= $\frac{1}{2} I \omega^2$

Since $I_f = (2/5) I_i$ and $\omega_f = (5/2)\omega_i$, therefore kinetic energy would increase 5/2 times.

(c) The boy uses his internal energy to increase his rotational kinetic energy.

5. In which of the following is the angular momentum conserved?

(a) A planet which moves in an elliptical orbit around the sun with the sun as one of the foci of the ellipse

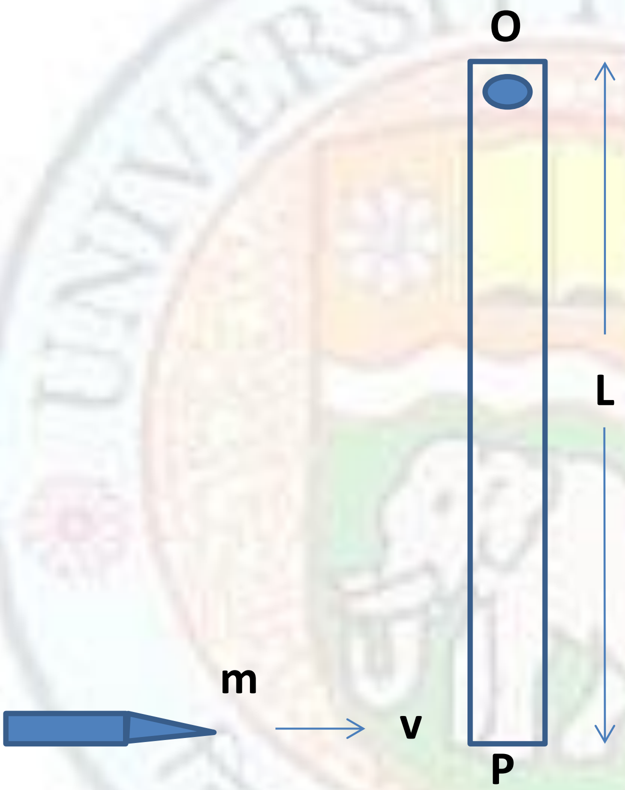
(b) An electron describing an elliptical orbit around the nucleus

(c) A boy whirls a stone tied to a string in a horizontal circle

(d) An α - particle approaching a nucleus gets scattered by the force of electrostatic repulsion between the two.

Answer: The angular momentum is conserved in all the four cases. Since the object in each case is moving under the action of central force, the torque is zero and so the angular momentum is conserved.

6. A rod of mass M and length L is suspended from O as shown in the figure. A bullet of mass m moving with velocity v in the horizontal direction strikes the end P of the rod and gets embedded in it. If I is the moment of inertia of the system, the angular velocity ω after the collision is given by



(a) ω =v L (b) ω = M v L/ I (c) ω= m v L/I (d) ω=I v L / m

Answer : From conservation of angular momentum

m v L = I ω, which gives ω= m v L/I . Thus (c) is the correct choice.

7. Why does a symmetric top while spinning about its axis start precessing after a short duration? Explain.

Answer: A rapidly spinning top experiences a force F due to gravity acting vertically downwards on its centre of mass. This furnishes a torque T about the point of contact with the floor . This torque is in the horizontal direction and causes the top to precess with its axis moving in a circular cone about the vertical.

8. A record player of mass M and radius R is rotating at angular speed ω. A coin of mass m is gently placed on the record at a distance r=R/2 from its centre. What would be its new angular speed?

Answer: The initial angular momentum of the record player is L=I ω, where I $= M R^2/2$. After the coin is placed, let its angular speed is ω'. Then the new angular momentum would be $L' = (I + mr^2)\,\omega'$. Since no external torque is acting on the system, angular momentum is conserved. Therefore

$$I\,\omega = (I + mr^2)\,\omega', \;\; which \quad gives \quad \omega' = \frac{I\,\omega}{I + mr^2}$$

Substituting for I and r=R/2, we get $\omega' = \dfrac{2\omega M}{2M + m}$.

9. A top, shown in the figure., having moment of inertia of $4.0 \times 10^{-4}\, kg.m^2$ is free to rotate about the axis A B. A string, wrapped around a peg along the axis of the top is pulled in a manner to maintain a constant tension of 5.0 N in the string. If a 90 cm of the string is pulled of the peg, (a) calculate the angular speed of the top, assuming that the string does not slip while it is being wound around the peg. (b) Given that the force of gravity acting on the centre of mass of the top furnishes a torque of 30.0×10^{-2} N-m about its point of contact with the floor, estimate the angular speed of precession of the axis.

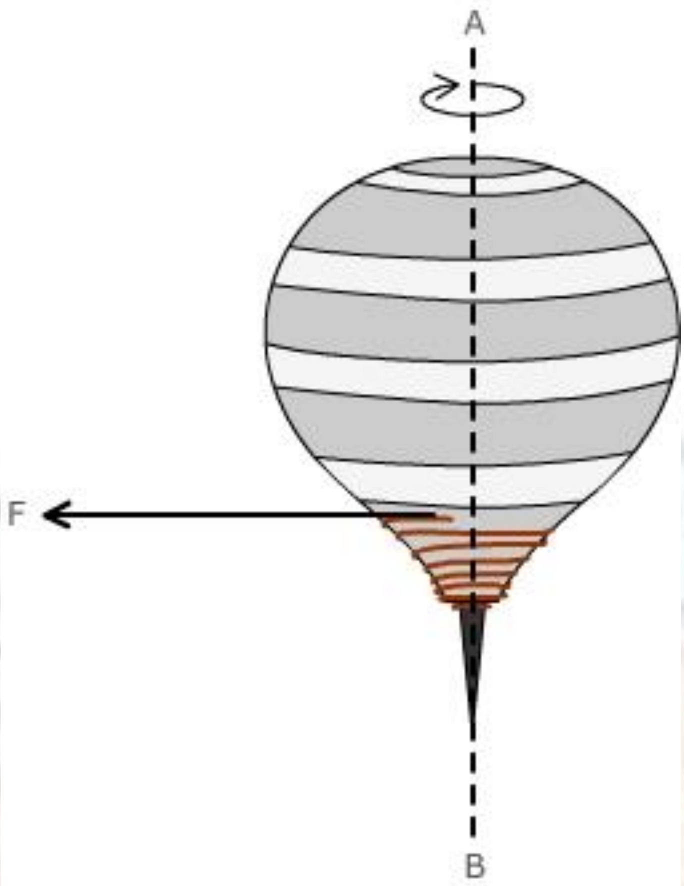


Fig. (5E)

Solution

The amount of work done on the top by the string while it was being unwound

$$=5.0\times 90\times 10^{-2}\,N-m = 4.5\,N-m$$

This work is used to impart the kinetic energy to the top due to which it rotates with angular speed, say, ω. Therefore

$$\text{K E}=\frac{1}{2}I\omega^2 = 4.5\,N-m$$

The moment of inertia I is given $4\times 10^{-4}\,kg.m^2$.

Therefore

$$\frac{1}{2}\times 4\times 10^{-4}\times\omega^2 = 4.5$$, which gives the value of

ω= 150 rad/s.

The angular momentum initially associated with the top about its axis is

$$L_0 = \omega\times I = 150\times 4\times 10^{-4} = 6\times 10^{-2}\,kg.m^2/s.$$

If ω_p is the (vertical) angular velocity of precession, then the torque

$$\vec{T} = \vec{\omega}_p\times\vec{L}_0 \quad ,$$

which is the torque provided by the force of gravity about the point of contact with the floor and causes the top to precess with its axis moving in a circular cone about the vertical. Thus

$$\omega_p L_0 = 6\times10^{-2}\omega_p = 30\times10^{-2} \quad ,$$

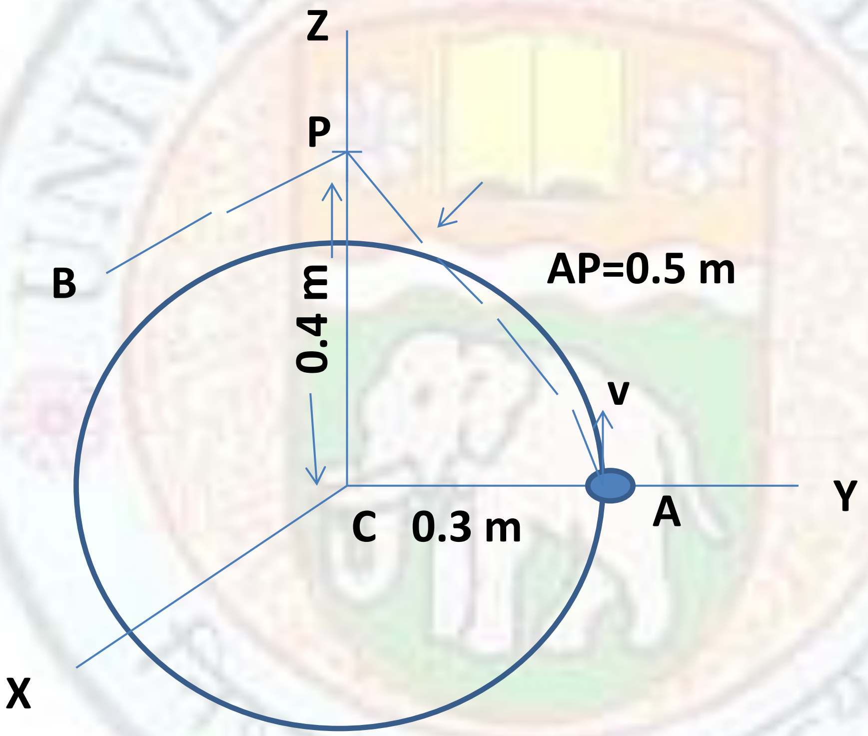
which gives ω_p =5 rad/s.

10. A body of mass 0.5 kg is moving in a circle of radius 0.3 m with constant speed of 0.2 m/s. Find out its angular momentum about (i) the centre of the circle, (ii) a point on the axis of the circle and at a distance of 0.4 m from its centre. Also determine the directions of the angular momentum in each case.

Answer : (i) The angular momentum of the body about the centre of the circle

L= m v r=0.5×0.2×0.3= 3 J-s.

Its direction is perpendicular to the plane of the circle.



(ii) L= m v r =0.5×0.2×0.5=5 J-s.

Its direction is perpendicular to the plane of the position vector (A P) and instantaneous velocity of the body, i.e., P B which is changing with time.