Discipline Course-I
Semester -I
Paper: Mechanics IB
Lesson: Work and energy (II)

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CH. 6 WORK AND ENERGY (II)

Objectives

After studying this chapter you will be able to understand:

- The elastic potential energy of an elastic medium
- The relation between work and the potential energy
- The work done by a conservative force
- The work done by a non- conservative force
- The law of conservation of energy for conservative forces
- The law of conservation of energy for non- conservative forces

1. Elastic potential energy

Consider a block of mass m attached with a spring which is fastened with a wall. The spring has energy stored in it due to the elastic properties of materials, as we slowly compress or extend the spring from its resting position. It is seen that if the surface on which the block is moving is frictionless ,the block comes back to its position of rest with the same velocity or kinetic energy with which it started compressing or extending the spring, so we can do work without changing the kinetic energy. This work gets 'stored' in the spring and we can get it back by extension or compression of the spring. This stored energy is called the *ELASTIC POTENTIAL ENERGY*.

According to Hooke's law, the linear restoring force exerted by a spring is directly proportional to the displacement measured from some fixed point and this force acts in the direction opposite to the direction of motion, so if Δx is the displacement then restoring force is given by

$$F_{restoring} = -\, k\Delta x \;,$$

where Δx is the displacement from its equilibrium (rest) position, and k is called the spring constant for that particular spring. Since we are not accelerating anything, we have to apply a force F,

$$F = -F_{restoring}$$

So the elastic potential energy stored in the spring, $U_{ELASTIC}$, is given by

$$U_{ELASTIC} = \int dU_{restoring} = \int dW = \int F\, dx$$

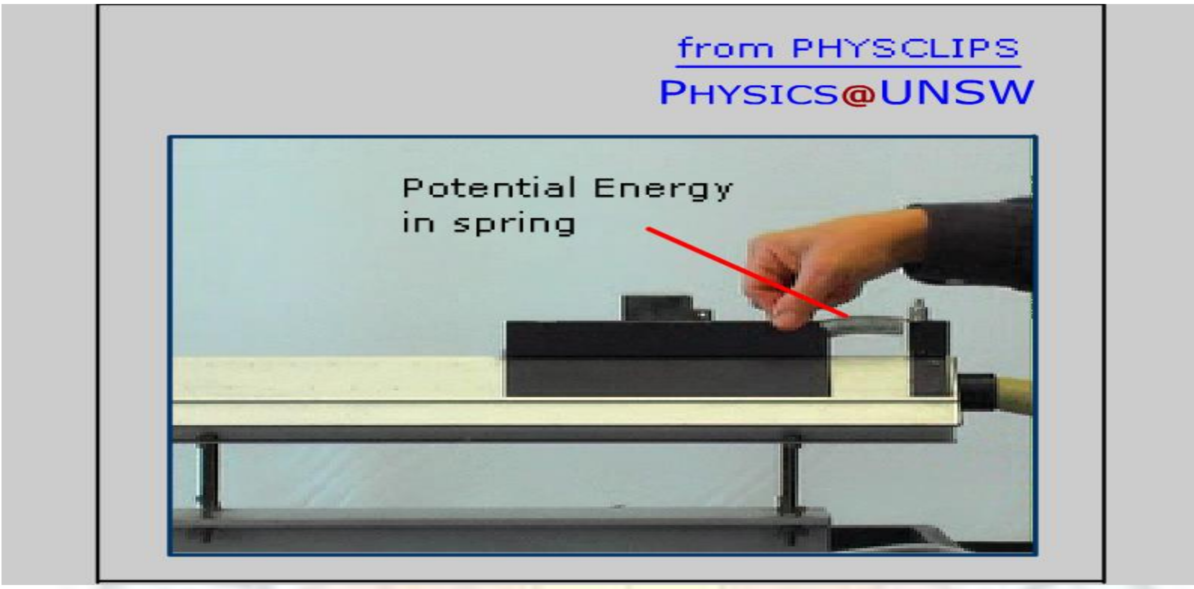
$$= -\int F_{restoring}\, dx$$

$$= \int k\Delta x\, dx,$$ here assuming rest position at the origin, so $\Delta x = x-0=x$, and we set U=0 at origin , so

$$U_{ELASTIC} = (½)kx^2$$

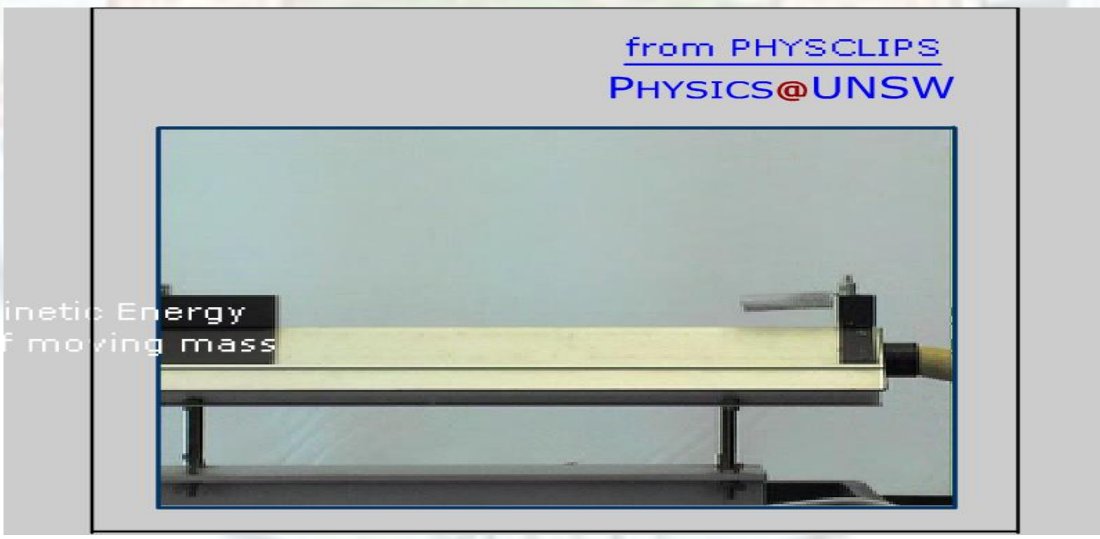
We see that with this reference value at the origin, $U_{ELASTIC}$ is always positive:
With respect to the unstressed state, both stretching ($x > 0$) and compressing ($x < 0$) require work, so the potential energy is positive in each case.

Let us study this elastic energy with the help of following multimedia:



To play the movie click Mechanics with animations and film clips: Physclips.

In the film clip, the work done is get stored as the potential energy in the spring, the spring then does work on the mass, giving it kinetic energy. Biochemical energy in our arm get converted into potential energy in the spring and then to kinetic energy.



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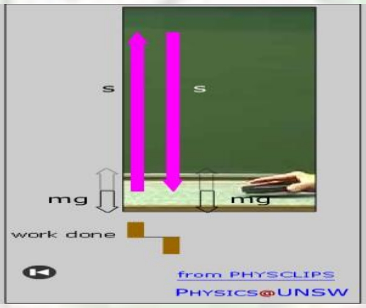
2. Work and potential energy

As we have earlier defined the potential energy is the energy stored at a position in a vector force field **F=-∇∅,** where ∅ is some scalar function, called the potential. We now, for simplicity and familiarity, take our vector force field as the gravitational force field ($F=GMm/R^2$) and the corresponding scalar potential ∅ = -GM/R, where G is Gravitational constant, M is the mass of the Earth, m is the mass of the particle, R is the radius of the earth. We now elaborate the concept by the following simple examples with multimedia.

Suppose a man slowly lift a mass m in a gravitational force field. As shown in the following multimedia, the man lifts a mass up to a height h vertically in the gravitational field of earth. The weight mg of the mass here is due to the gravitational force of earth, hence **F=W**=m**g**, so the work done here is simply given by

Work W =**F.S** = FScos0 = FS = mgh

Now this work gets stored as the potential energy of the mass at the position at height h from the ground vertically, and as the mass is lowered back to the ground this stored potential energy can do work on the floor, as the stored potential energy get converted into the kinetic energy of the moving mass.



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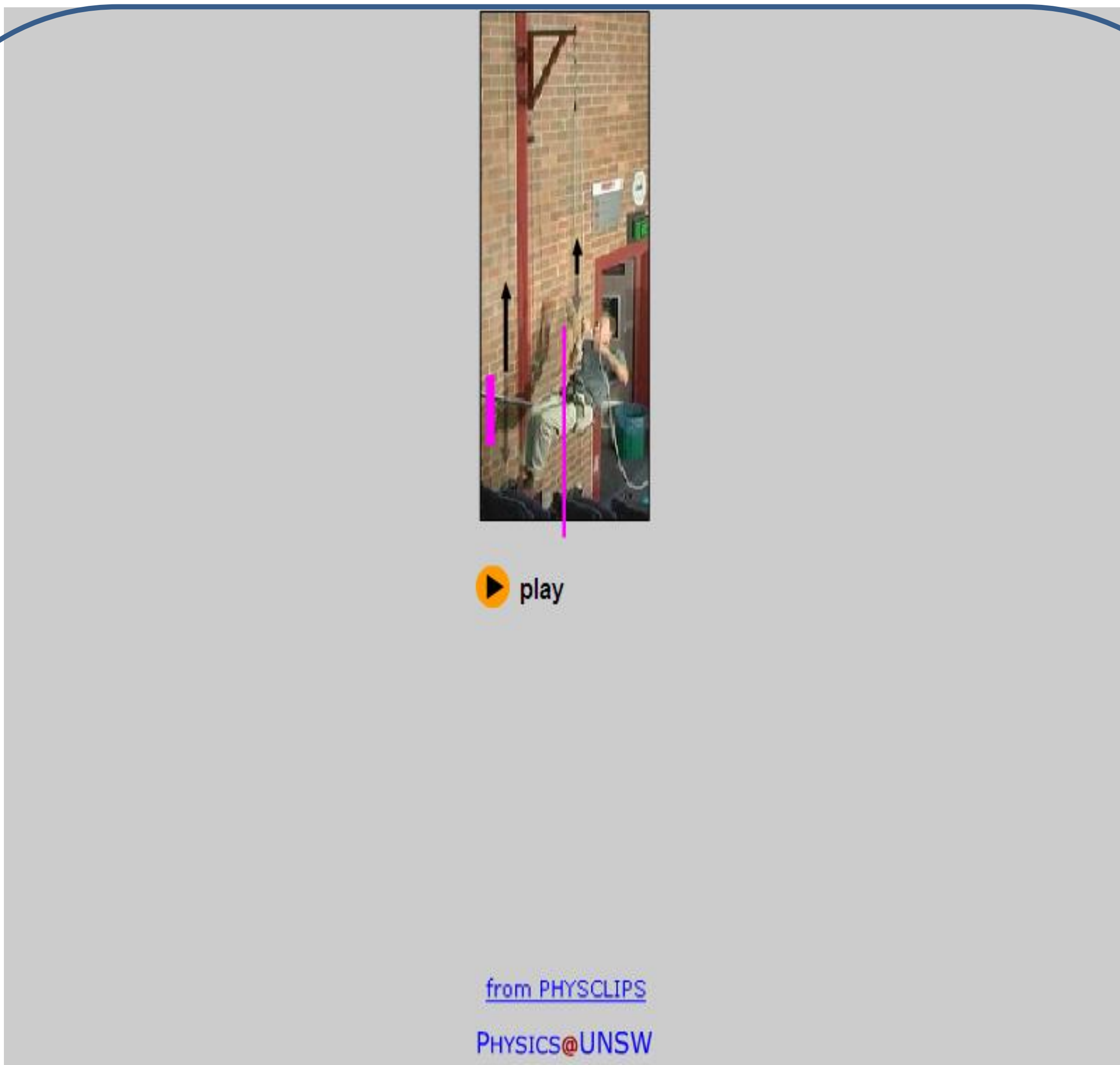

In this example a single rope goes from the support, down to the man's harness, round the pulley, back to the support, round another pulley and back to his hands. The pulleys turn easily, so the tension T in each section of the rope is the same.

Levers, blocks and pulleys don't save the work, but they can reduce (or increase) the force, which can make a task more convenient and comfortable.

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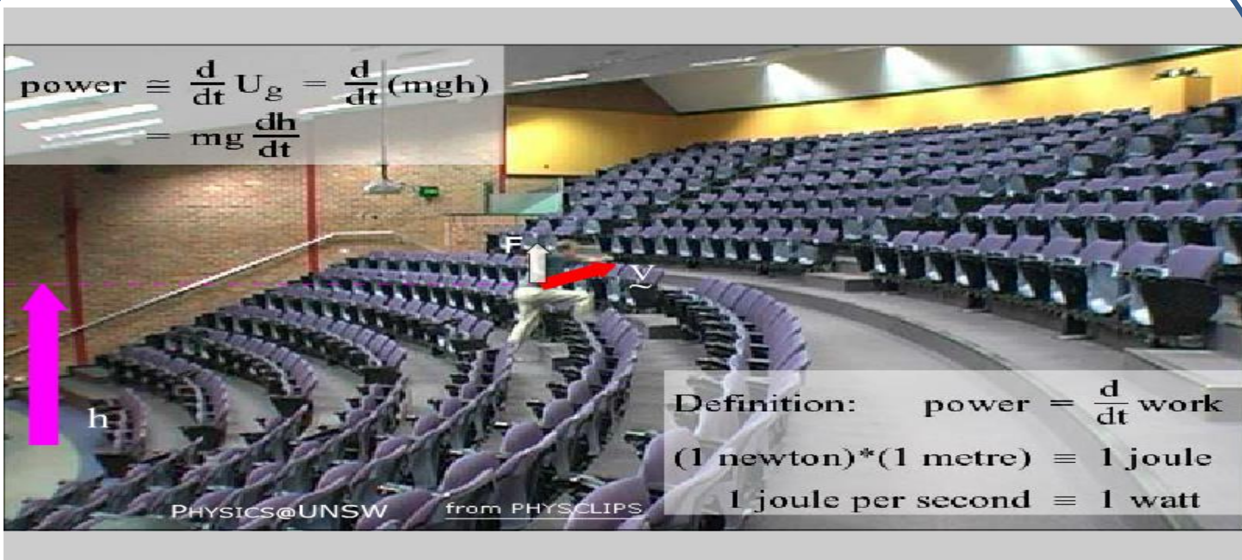

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This is the example where a man moves on a stair. For most modest displacements, g is assumed to be uniform. The potential energy U is defined by an integral, and integrals require a constant of integration. For potential energy, this constant is the reference for the zero of potential energy. If we define U_{grav} to be zero at h = 0, then we can write

$$U = \int dU_{grav} = \int dW$$

$$= \int mg\, dy = mg\Delta y = mg\Delta h$$

So $U_{grav} = mgh.$

where h is the vertical displacement.

As we know, not all forces but only conservative forces allow us to define a potential.

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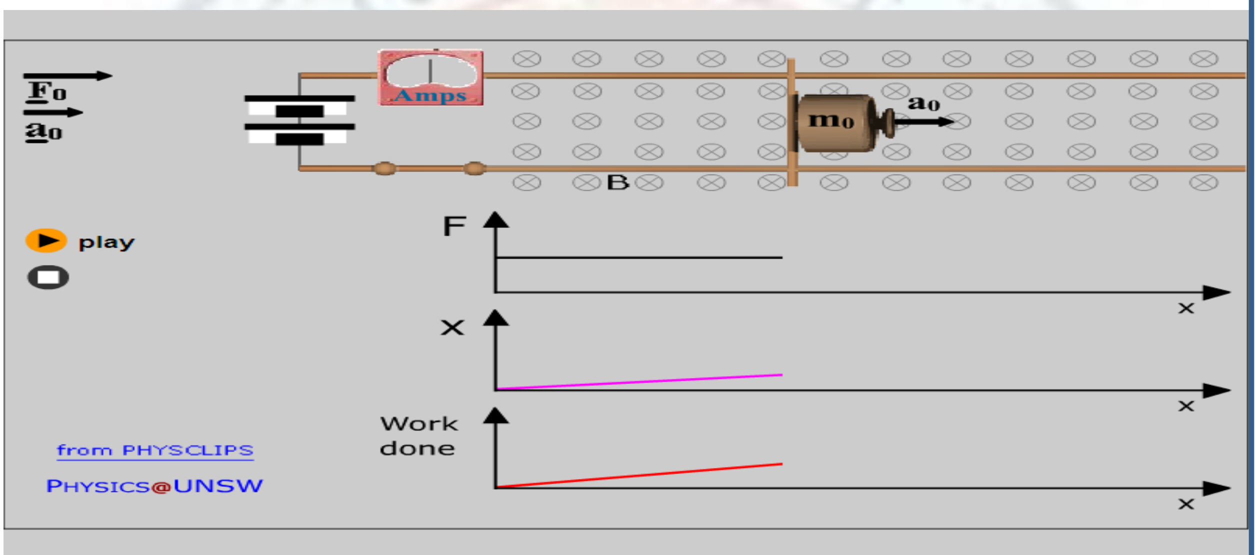
The following multimedia shows the example of a magnetic work.



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3. Work done by non-conservative forces

We know that for conservative forces the work done W_{con} is given by

$W_{con} = \Delta K = -\Delta U$.

Now we study a system where the acting forces are conservative and non-conservative force as well (for example, motion on a surface have non-ignorable frictional force). Then the work done by the resultant force is the sum of the work done by conservative forces, W_{con}, and the work done by non-conservative forces, W_{noncon}, so we have

$W_{total} = W_{con} + W_{noncon}$

Now from work-energy theorem

$W_{total} = \Delta K$

So

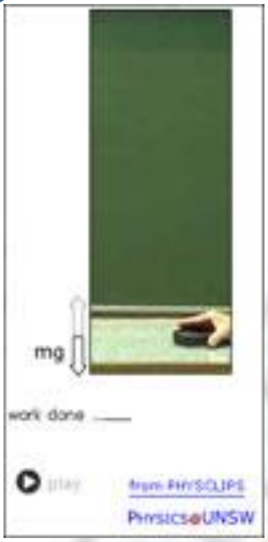
$W_{total} = W_{con} + W_{noncon} = \Delta K$

Or

$W_{noncon} = \Delta K + \Delta U = \Delta E$,

So the total mechanical energy (E=K+U) of the system is not constant but changes by the amount of work done on the system by the non-conservative forces. So here we see that when there is no non-conservative forces or there is no work done by the non-conservative forces, then only is the change in mechanical energy (ΔE) is zero, that is, the total mechanical energy is constant.
When we consider non-conservative force such as the frictional forces, we have $W_{frictional} = \Delta E$, and since friction is a dissipative force, it decreases the total mechanical energy, so there is a loss of energy. Where does this energy go?
We know that this loss of energy appears in the form of heat. So we can say that frictional work is equivalent of heat generation. Hence $W_{frictional} = -H = \Delta E$
Or $\Delta E + H = 0$
Hence we can say that the sum of total energy remains constant.

Let us study the conservative and non-conservative work with the help of a multimedia example:



Let us first consider the work done by a conservative force. Let's consider at the work done in moving a mass m in Earth's gravitational field. We assume that here mass is moved with negligible acceleration, so we assume a=0, so the force exerted by the hand and the weight of the mass add to zero, so

$$\mathbf{F}_{\mathbf{hand}} + m\mathbf{g} = \mathbf{0}$$

or

$$\mathbf{F}_{\mathbf{hand}} + \mathbf{F}_{\mathbf{grav}} = \mathbf{0}$$

Now the work done against gravity is

$$W = \int \mathbf{F}_{\mathbf{hand}}.\mathbf{dr}.$$

As we lift up the mass, $\mathbf{F}_{\mathbf{hand}}$ is upwards (positive) and **r** is also positive, so the work done by us is positive:

$$W = \int \mathbf{F}_{\mathbf{hand}}.\mathbf{dr} > 0.$$

As we lower the mass, $\mathbf{F}_{\mathbf{hand}}$ is still upwards (positive) but now **r** is negative, so the work done by us is negative:

$$W = \int \mathbf{F}_{\mathbf{hand}}.\mathbf{dr} < 0.$$

Consequently, round a complete cycle that returns the mass to its starting point,

$$W = \int \mathbf{F}_{\mathbf{hand}}.\mathbf{dr} = 0.$$

Similarly, the work done by gravity around the cycle is zero, (because $\mathbf{F}_{\mathbf{grav}} = -\mathbf{F}_{\mathbf{hand}}$).

So the gravitational force is a conservative force.
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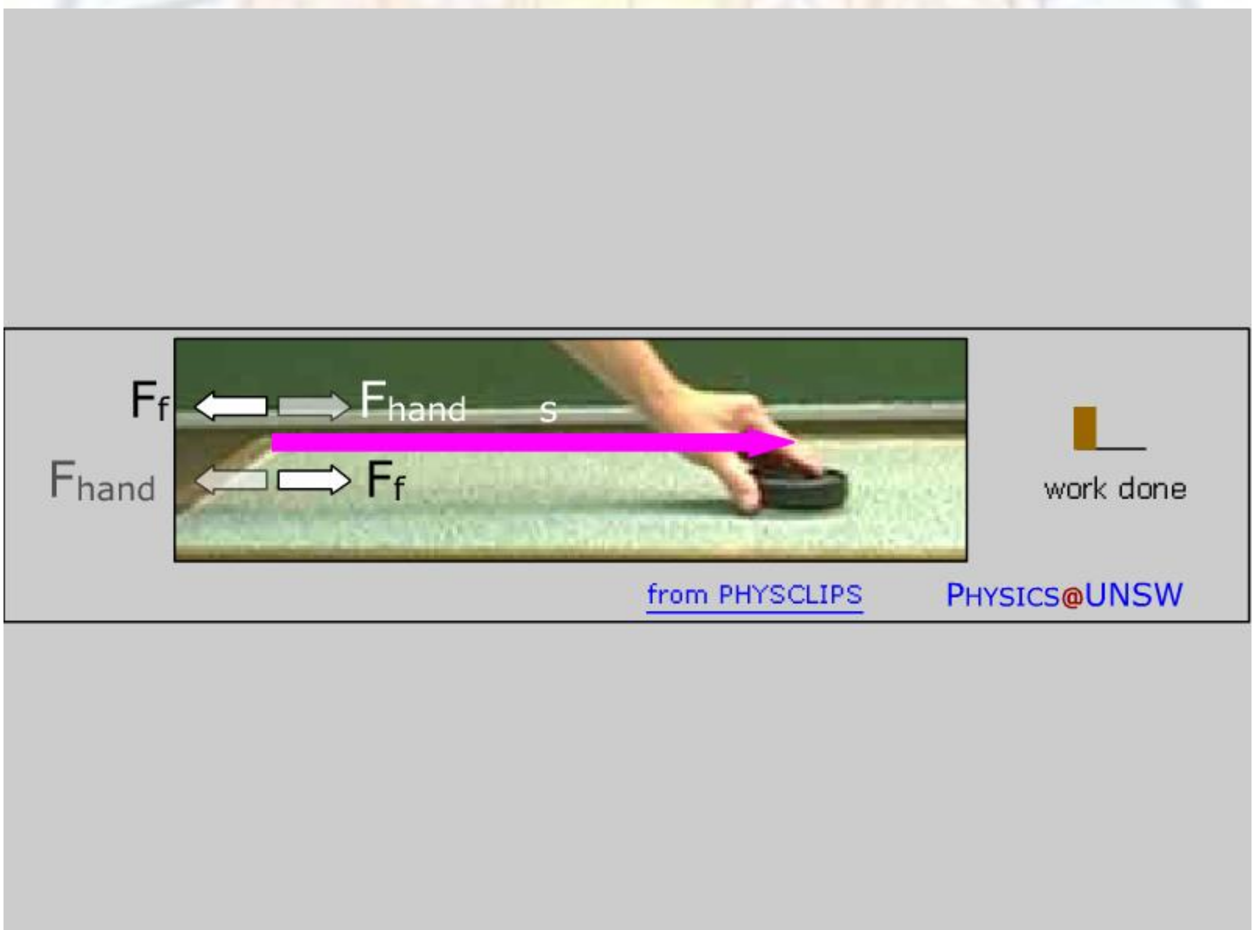
If the work done around a closed loop is not zero, then the forces are non-conservative forces. Now we study the non-conservative forces. One example of them is frictional force. We do our earlier experiment of moving a mass on a frictional surface. Let's assume again that we do this so slowly that the mass is in mechanical equilibrium, so we have

$$\mathbf{F_{hand}} + \mathbf{F_{friction}} = 0$$

Moving to the right, we apply a force to the right and the object moves to the right:
$\mathbf{F_{hand}}$ and **dr** are both positive: we do positive work and friction does negative work.
Moving to the left, we apply a force to the left and the object moves to the left:
$\mathbf{F_{hand}}$ and **dr** are both positive: we do positive work and friction does negative work.

So, around a closed loop, the work done against friction is greater than zero, so friction is a non-conservative force.



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4. Law of conservation of energy

The work energy theorem state that: "The change in the kinetic energy of a particle form initial position to finial position is equal to the work done by the force in displacing the particle from initial position to the finial position."

So, we can write

$$W_{ab} = K_f - K_i = \frac{1}{2}(mV_f^2 - mV_i^2) = \Delta K = K_b - K_a,$$

where the particle is displaced from position 'a' to the position 'b'.

Now according to the definition of the potential energy, it is given by:

$$U = W_{ab} = -\int \mathbf{F}.\mathbf{dr} = U_a - U_b$$

So we have from the above two relations

$W_{ab} = K_b - K_a = U_a - U_b$

Or $U_a + K_a = U_b + K_b = E =$ constant (total mechanical energy)

So the above relation can be read as:

"The sum of kinetic and potential energy (called the total mechanical energy) of a particle, under a conservative force field, remains constant."

This statement is known as **THE LAW OF CONSERVATION OF ENERGY**.

Now we study this law further with the help of a multimedia example.

We now study the cases where all of the forces that do the work ΔW are conservative forces:

So, the work done by those forces is minus one times the work done against them, in other words it is $-\Delta U$.
So, if the only forces that act are conservative forces, then $\Delta U + \Delta K = 0$.
Let us define the mechanical energy E by $E \equiv U + K$.
So, if the only forces that act are conservative forces, mechanical energy is conserved.

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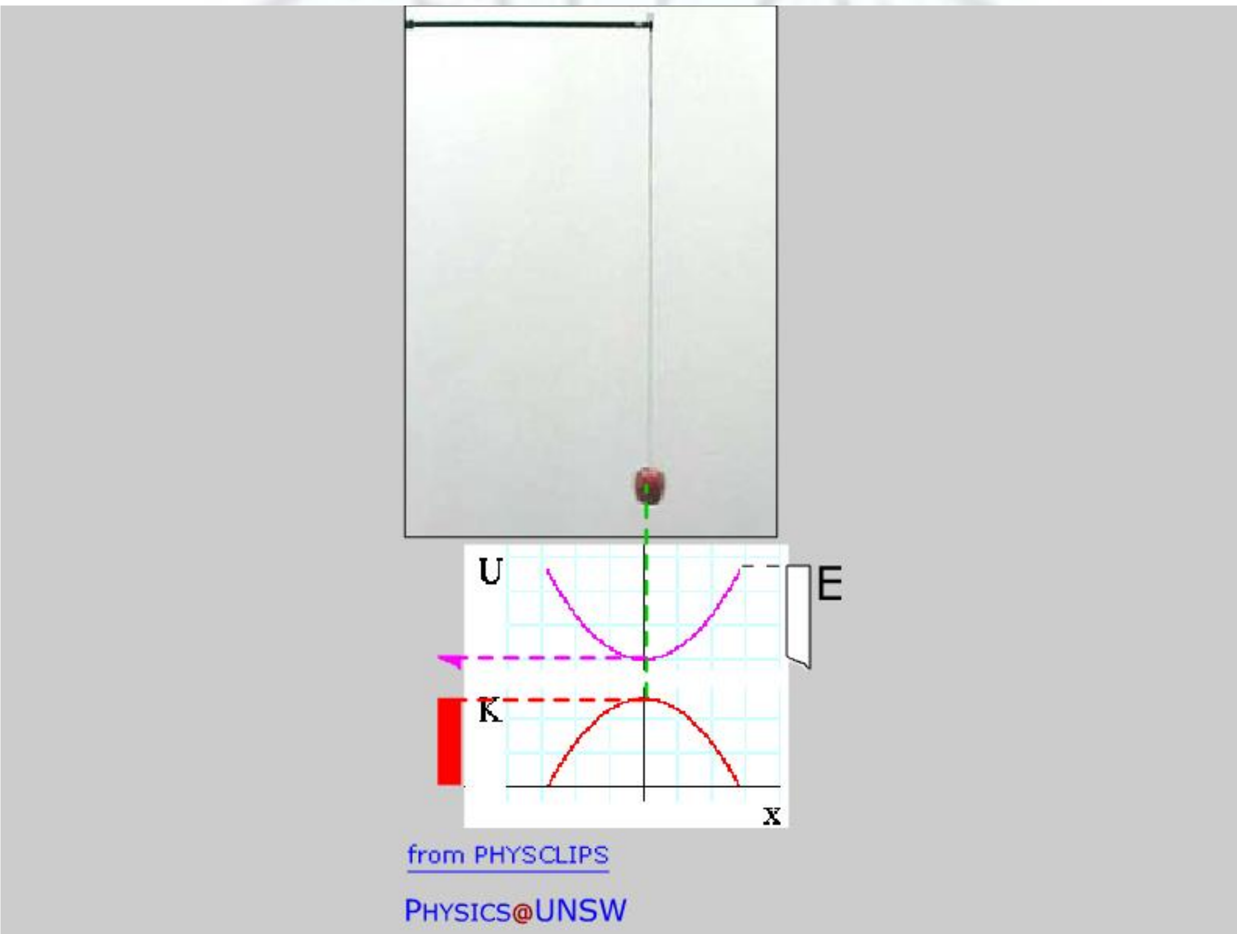
Kinetic and potential energy in the pendulum:



To play the movie click Mechanics with animations and film clips: Physclips.

This video clip shows an example of the exchange of kinetic and potential energy in a Pendulum. The kinetic energy K is shown in red: as a function of x on the graph, and as a Histogram that varies with time. Note that the K goes to zero at the extremes of the motion. The potential energy U is shown in purple. It has maxima at the extremes of the motion, when the mass is highest. Because the zero of potential energy is arbitrary, so is the zero of the total mechanical energy E = U + K. Here, E (shown in white) is constant.

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Conservation of mechanical energy:

We have seen that, if the only forces present are conservative, then mechanical energy is conserved. However, we can go further. Provided that non-conservative forces do no work, then the increase ΔK in the kinetic energy of a body is still the work done by the conservative forces, which is −ΔU.
So we can conclude that if non-conservative forces do no work then mechanical energy (E ≡ U + K) is conserved.
This statement can be written in several ways, of which here are two:

If non-conservative forces do no work, $\Delta U + \Delta K = 0$ or

$U_i + K_i = U_f + K_f$,

where i and f mean initial and final.

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Non-conservative forces:

If the forces acting are non-conservative and they also do work then the above law of conservation of mechanical energy does not hold good as we shall see in the following examples, but we can surely say that the law of conservation of the total energy (comprising of the mechanical energy, heat, electrical energy etc.) always remains conserved whether the forces are conservative or non-conservative, the law of conservation of the total energy is one of the valid laws known till date.

Let us now see whether the energy is conserved even if a non-conservative force like friction is acting on a particle. We know that frictional force is path dependent, the longer the path traversed between two given points, the greater the work done by the frictional force. So unlike the conservative force, work done in moving a particle from initial position to finial position is not equal to the reverse path and hence total work done along the forward and reverse path is non-zero. In fact there is a loss of kinetic energy in either way.
Thus, if we have both conservative and non-conservative forces acting on a particle and if work done by the two forces be $W_{conservative}$ and $W_{non\text{-}conservative}$ respectively, and ΔK be the loss in kinetic energy, we have from the work-energy theorem

$$W_{conservative} + W_{non\text{-}conservative} = \Delta K$$

Now for conservative force, we have

$$W_{conservative} = -\Delta U$$

So we have

$$W_{non\text{-}conservative} = \Delta K + \Delta U = \Delta E$$

Thus the change in total energy of the particle is no longer zero, as the case for conservative force, but equal to the work done by the non-conservative force.
If the no-conservative force is frictional force, the work done by it appears in the form of heat, H, so we have

$$W_{non\text{-}conservative} = -H$$

Therefore

$H = -\Delta E$ where E is the total mechanical energy of the particle

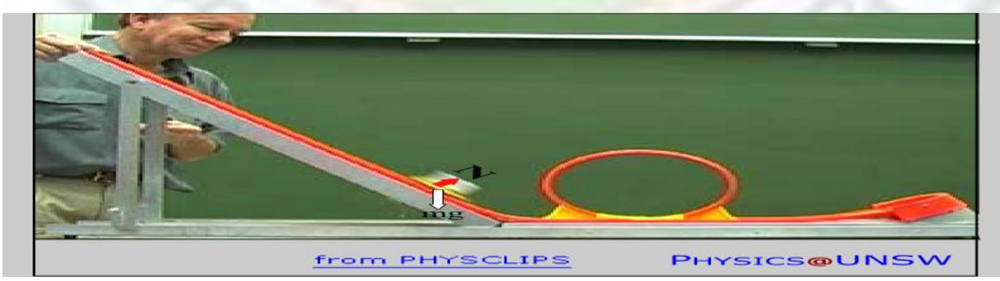
Or

$$H + \Delta E = 0$$

So the change in the total energy of the particle is zero or its total energy remains conserved.
So we can say that without exception the total energy is conserved.

Hence the general law of conservation of energy holds good in the case of conservative and non-conservative forces.

Now we study a multimedia example, called the loop the loop problem, where we can apply the law of conservation of the energy.



Loop the loop.

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This is a classic problem. A small toy car runs on wheels that turn and are assumed to turn freely and whose mass is negligible, so we can treat it as a particle. From how high must we release it so that it will loop the loop, remaining in contact with the track all the way around?
If the car retains contact with the track then, at the top of the loop, which is circular, the centripetal acceleration will be downwards and its magnitude will be v^2/r.
The forces providing this acceleration are its weight mg (acting down) and the normal force N from the track, also acting down at this point.
So, if $N > 0$, we require $v^2/r > g$, or, for the critical condition at which it just loses contact, we require

$$v_{crit}{}^2/r = g$$

or

$$v_{crit}{}^2 = rg$$

We can do this problem using the conservation of mechanical energy.

$U_{initial} + K_{initial} = U_{final} + K_{final}$

Choosing the bottom of the track as the zero for U, we could write,

$mgh_{initial} + 0 = mg.(2r) + ½mv_{final}{}^2$

and, if $v_{final} = v_{crit} = \sqrt{(rg)}$

so $\quad mgh_{initial} = 2mgr + ½mgr$

So the critical height $h_{critical}$ is 5r/2 above the bottom of the track

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6. Summary

- Elastic potential energy of a spring is given by $U_{ELASTIC} = ½kx^2$.
- Gravitational potential energy is U_{grav} = mgh.
- Friction and viscous forces are examples of non- conservative forces.
- For conservative forces, the total mechanical energy is conserved, i.e. E= K +U.
- Change in total energy for a conservative force is zero, ΔE=0

  Hence $W_{conservative} = -\Delta U$.
- Change in total energy for a non-conservative force is nonzero, but $\Delta E = W_{nonconservative}$.
- General Law of Conservation of energy says that total energy of the universe remain conserved.

8. Exercise

Q1 A car is lifted a certain distance in a service station and therefore has potential energy relative to the ground. If it were lifted twice as high, how much potential energy would it have?

Q2. Two cars are lifted to the same elevation in a service station. If one car is thrice as heavy as the other car, how do their potential energies related?

Q3. How many joules of potential energy does a 2-kg box gain when it is raised to 14 m? What is the potential energy when it is elevated to 18 m?

Q4. How many joules of kinetic energy does a 2-kg block have when it is thrown across a room at a speed of 12 m/s?

Q5. A moving car has some kinetic energy. If it speed is increased until it is moving four times as fast as earlier, how much kinetic energy does it have changed?

Q6. Compared to some original speed, how much work must the brakes of a car supply to stop a three times- as-fast car? How will the stopping distance compare?

Q7. (a) How much work do you do when you push a box horizontally with 10 N across a 20-m on a floor?

(b) If the force of friction between the box and the floor is a steady 7 N, how much KE is gained by the box after sliding 10 m?

(c) How much of the work you do converts to heat?

Q8. How does speed affect the friction between a road and a skidding tire of a car ?

Q9. What will be the kinetic energy of a sliding ball on a curved plank when it undergoes a 10-kJ decrease in potential energy?

Q10. A mango hanging from a tree has potential energy because of its height. If it falls down where do this energy goes just before it hits the ground?

Fill in the blanks:

Q11. The energy stored in an elastic spring is called ______________________.

Q12. The gravitational potential energy is given by ____________________.

Q13. The restoring force in an elastic spring is a _______________ force.

Q14. The viscous force in a fluid is an example of ________________ force.

Q15. The total mechanical energy is sum of the kinetic energy and the __________.

State whether the following statements are true or false:

Q16. All Central forces are conservative forces.

Q17. The total mechanical energy remains conserved only for the conservative forces.

Q18. We can always associate a scalar function with any force field.

Q19. Work done is always path dependent for the conservative forces.

Q20. The general law of conservation of energy holds good in the case of conservative and non-conservative forces.

Choose the most appropriate option for the following:

Q21. The elastic potential energy of a spring is given by

(A) U=mgh
(B) U= ½ mV^2
(C) U= ½ Kx^2.

Q22. The work done by a charge particle in magnetic field is given by

(A) W= mgh
(B) W= qvBx
(C) W=0.
(D)

Q23. The condition for a conservative force **F** is

(A) $\nabla . \mathbf{F} = 0$
(B) $\nabla \times \mathbf{F} = 0$
(C) $\nabla . (\nabla \times \mathbf{F}) = 0$

Q24. The law of conservation of total energy holds

(A) Only for conservative forces
(B) Only for non-conservative forces
(C) For both conservative and non-conservative forces.

Q25. For a simple pendulum

(A) The kinetic energy is maximum at the extreme positions and the potential energy is maximum at the mean position.
(B) The potential energy is maximum at the extreme positions and the kinetic energy is maximum at the mean position.
(C) The total energy is not constant.